

# Mark Scheme (Results)

January 2018

Pearson Edexcel International Advanced Subsidiary Level In Core Mathematics C12 (WMA01) Paper 01



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- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively.
   Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

#### PEARSON EDEXCEL IAL MATHEMATICS

# General Instructions for Marking

- 1. The total number of marks for the paper is 125
- 2. The Edexcel Mathematics mark schemes use the following types of marks:
- M marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- A marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- B marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

#### 3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod benefit of doubt
- ft follow through
- the symbol  $\sqrt{}$  will be used for correct ft
- cao correct answer only
- cso correct solution only. There must be no errors in this part of the question to obtain this mark
- isw ignore subsequent working
- awrt answers which round to
- SC: special case
- oe or equivalent (and appropriate)
- d... or dep dependent
- indep independent
- dp decimal places
- sf significant figures
- \* The answer is printed on the paper or ag- answer given
- or d... The second mark is dependent on gaining the first mark
- 4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

- 5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.
- 6. If a candidate makes more than one attempt at any question:
  - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
  - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.
- 7. Ignore wrong working or incorrect statements following a correct answer.

#### General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

## Method mark for solving 3 term quadratic:

#### 1. Factorisation

$$(x^2 + bx + c) = (x + p)(x + q)$$
, where  $|pq| = |c|$ , leading to  $x = \dots$   
 $(ax^2 + bx + c) = (mx + p)(nx + q)$ , where  $|pq| = |c|$  and  $|mn| = |a|$ , leading to  $x = \dots$ 

# 2. Formula

Attempt to use the <u>correct</u> formula (with values for a, b and c).

#### 3. Completing the square

Solving 
$$x^2 + bx + c = 0$$
:  $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$ ,  $q \neq 0$ , leading to  $x = \dots$ 

# Method marks for differentiation and integration:

#### 1. Differentiation

Power of at least one term decreased by 1.  $(x^n \rightarrow x^{n-1})$ 

## 2. Integration

Power of at least one term increased by 1.  $(x^n \rightarrow x^{n+1})$ 

#### Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

<u>Method mark</u> for quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values.

Where the formula is <u>not</u> quoted, the method mark can be gained by implication from <u>correct</u> working with values, but may be lost if there is any mistake in the working.

#### **Exact answers**

Examiners' reports have emphasised that where, for example, an <u>exact</u> answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

#### **Answers without working**

The rubric says that these <u>may</u> not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

| Question<br>Number | Scheme  | Notes   | Marks   |
|--------------------|---|---|---------|
| 1                  | $y = \frac{2x^{\frac{2}{3}} + 3}{6}$  |   |         |
| (a)                |   | For reducing the power of $x^{\frac{2}{3}}$ by 1 which may be   |         |
|                    | $x^{\frac{2}{3}} \rightarrow x^{-\frac{1}{3}}$  | implied by e.g. $x^{\frac{2}{3}} \to x^{\frac{2}{3}-1}$ and no other powers of $x$  | M1      |
|                    | Note that some candidates think   | $\frac{2x^{\frac{2}{3}} + 3}{6} = 2x^{\frac{2}{3}} + 3 + 6 \text{ but the M mark can still}$  |         |
|                    | sc  | core for $x^{\frac{2}{3}} \rightarrow x^{-\frac{1}{3}}$   |         |
|                    |   | Correct expression. Allow equivalent exact,   |         |
|                    | $\left(\frac{\mathrm{d}y}{\mathrm{d}x} = \right) \frac{2}{9} x^{-\frac{1}{3}}$            | simplified forms e.g. $\frac{2x^{-\frac{1}{3}}}{9}, \frac{2}{9x^{\frac{1}{3}}}, \frac{2}{9\sqrt[3]{x}}$ . Allow                         | A1      |
|                    |   | 0.222 or 0.2 with a dot over the 2 for $\frac{2}{9}$ .  |         |
|                    |   | te differentiation and ignore subsequent working ng a fully correct answer.   |         |
|                    |   |   | (2)     |
| <b>(b)</b>         | Must be integrating the given function in (b), not their answer to part (a)               |   |         |
|                    |   | Increases the power by 1 for one term from $x^{\frac{2}{3}} \rightarrow x^{\frac{5}{3}}$ or $k \rightarrow kx$ . May be implied by e.g. |         |
|                    | $x^{\frac{2}{3}} \to x^{\frac{5}{3}} \text{ or } k \to kx$                                | $x^{\frac{2}{3}} \rightarrow x^{\frac{2}{3}+1}$ . This must come from correct work, so integrating numerator and denominator e.g.       | M1      |
|                    |   | $\frac{2x^{\frac{2}{3}} + 3}{6} \to \frac{x^{\frac{5}{3}} +x}{6x} \text{ is M0}$  |         |
|                    | Note that some candidates think   | $\frac{2x^{\frac{2}{3}} + 3}{6} = 2x^{\frac{2}{3}} + 3 + 6 \text{ but the M mark can still}$  |         |
|                    | score fe  | or $x^{\frac{2}{3}} \rightarrow x^{\frac{5}{3}}$ or $k \rightarrow kx$  |         |
|                    |   | One correct term which may be un-simplified,  |         |
|                    | $\frac{3}{5} \times \frac{2}{6} x^{\frac{5}{3}}$ or $\frac{3}{6} x$                       | including the power. So, $\frac{2}{6} \times \frac{x^{1+\frac{2}{3}}}{1+\frac{2}{3}}$ would be  | A1      |
|                    |   | acceptable for this mark.   |         |
|                    |   | All correct and simplified including $+ c$ all appearing on one line. ( $c/6$ is acceptable for $c$ )                                   |         |
|                    | $\frac{1}{5}x^{\frac{5}{3}} + \frac{1}{2}x + c$   | Allow $\sqrt[3]{x^5}$ for $x^{\frac{5}{3}}$ but not $x^1$ for $x$ .   | A1      |
|                    | 3 2   | Allow 0.2 for $\frac{1}{5}$ and 0.5 for $\frac{1}{2}$   |         |
|                    | Ignore any spurious integral signs and/or $dx$ 's and ignore subsequent working following |   |         |
|                    | a1  | fully correct answer.   | (3)     |
|                    |   |   | Total 5 |
|                    | l   | I   | 1       |

| Question<br>Number | Scheme   | Notes  | Marks   |
|--------------------|--|--|---------|
| 2                  | Mark (a) and (b  | o) together  |         |
| (a)                | $u_2 = -1, u_3 = 5$  | As (a) and (b) are marked together, these can score as part of their calculation in (b) if – 1 and 5 are clearly the second and third terms. | B1, B1  |
|                    |  |  | (2)     |
| (b)                | $u_4 = 2 - 3 \times "5" (= -13)$   | Correct attempt at the 4 <sup>th</sup> term (can score anywhere) and may be implied by their calculation below)                              | M1      |
|                    | $\sum_{r=1}^{4} (r - u_r) = \pm \{ (1 - 1) + (2 - "-1") + (3 - "5") + (4 - "-13") \}$        |  |         |
|                    | or   |  | dM1     |
|                    | $\sum_{r=1}^{4} (r - u_r) = \sum_{r=1}^{4} r - \sum_{r=1}^{4} u_r = \pm \{(1 + 2 + 2 + 2)\}$ | 3+4)-(1+"-1"+"5"+"-13")}   |         |
|                    | A correct method for the sum or (– sum). Allow minor slips or mis-reads of their             |  |         |
|                    | values but the intention must be clear. Dependent on the first method mark.                  |  |         |
|                    | =18  | cso  | A1      |
|                    |  |  | (3)     |
|                    |  |  | Total 5 |

| Question<br>Number | Scheme  | Notes  | Marks   |
|--------------------|---|--|---------|
| 3(a)               | $\left(3x^{\frac{1}{2}}\right)^4 = 81x^2$                 | B1: Obtains $ax^n$ , $(a, n \neq 0)$ where $a = 81$ or $n = 2$<br>B1: $81x^2$  | B1B1    |
|                    | Do not isw so for example $\left(3x^{\frac{1}{2}}\right)$ | $\int_{0}^{1} e^{x} = 81x^{2} = 9x \text{ scores B0B0}$  |         |
|                    |   |  | (2)     |
| (b)                | $\frac{2y^7 \times (4y)^{-2}}{3y} = \frac{y^4}{24}$       | B1: Obtains $ay^{n}$ , $(a, n \neq 0)$ where $a = \frac{1}{24}$ or $n = 4$ (Allow 0.41666 or 0.416 with a dot over the 6 for $\frac{1}{24}$ )  B1: $\frac{y^{4}}{24}$ (Allow $\frac{1y^{4}}{24}$ ) | B1B1    |
|                    | Do not isw – mark t                                       | their final answer   |         |
|                    |   |  | (2)     |
|                    |   |  | Total 4 |

| Question<br>Number | Scheme  | Notes  | Marks   |
|--------------------|---|--|---------|
| 4(a)               | $b^2 - 4ac = 8^2 - 4(p-2)(p+4)$   | Attempts to use $b^2 - 4ac$ with at least two of $a$ , $b$ or $c$ correct. This could be as part of the quadratic formula or as $b^2 < 4ac$ or as $b^2 > 4ac$ or as $b^2 = 4ac$ or as $\sqrt{b^2 - 4ac}$ etc. If it is part of the quadratic formula only look for use of $b^2 - 4ac$ . There must be no $x$ 's. | M1      |
|                    | $8^2 - 4(p-2)(p+4) < 0$   | For a correct un-simplified inequality in any form that is not the final printed answer or a positive constant multiple of the final printed answer with no incorrect previous statements.   | A1      |
|                    | $64 < 4p^2 + 8p - 32$   |  |         |
|                    | $p^2 + 2p - 24 > 0*$  | Correct solution with intermediate working and no errors with the inequality sign appearing correctly before the final printed answer.   | A1*     |
|                    |   |  | (3)     |
| (b)                | $p^{2} + 2p - 24 = 0 \Rightarrow p = \dots$ $(p+1)^{2} - 1 - 24 = 0 \Rightarrow p = \dots$ $(p = )\frac{-2 \pm \sqrt{2^{2} - 4 \times 1 \times (-24)}}{2 \times 1}$ | For an attempt to solve $p^2 + 2p - 24 = 0$ (not their quadratic) leading to two critical values. See general guidance for solving a 3TQ when awarding this method mark. May be implied by their critical values.  | M1      |
|                    | <i>p</i> = 4, –6  | Correct critical values  | A1      |
|                    | p < "-6", p > "4"   | Chooses the outside region for their two critical values. Look for $p <$ their $-6$ , $p >$ their 4. This could be scored from $4  or -6 > p > 4. Evidence is to be taken from their answers not from a diagram. Allow e.g. p \le "-6", p \ge "4"$   | M1      |
|                    | p < -6 or $p > 4p < -6$ $p > 4p < -6$ , $p > 4p < -6$ ; $p > 4p < -6 \cup p > 4(-\infty, -6), (4, \infty)]-\infty, -6[, ]4, \infty[$                                | Correct inequalities e.g. answers as shown. Note that $p < -6$ and $p > 4$ would score M1A0 as would $4  or -6 > p > 4 or p < -6 \cap p > 4. Apply isw where possible.$  | A1      |
|                    |   | is of p only.  |         |
|                    | Correct answer only   | y scores full marks in (b)   | (4)     |
|                    |   |  | Total 7 |

| Question<br>Number | Scheme   | Notes  | Marks |
|--------------------|--|--|-------|
| 5(i)               | 7  | M1: Reaches $\tan = k$ where $k \neq 0$  |       |
|                    | $5\sin 3\theta - 7\cos 3\theta = 0 \Rightarrow \tan 3\theta = \frac{7}{5}$   | A1: $\tan = \frac{7}{5}$   | M1A1  |
|                    | $3\theta = 0.950$  | -  | dM1   |
|                    | $3\theta = \tan^{-1}\left(\operatorname{their}\frac{7}{5}\right)$ leading to a value of  | $3\theta$ . Must be $3\theta$ here but this may be   |       |
|                    | implied if they divide their values by 3 (you first method   | may need to check). Dependent on the   |       |
|                    | $\theta = 0.317$ or $\theta = 1.36$  | Awrt 0.317 (Allow awrt 0.101π) <b>or</b><br>Awrt 1.36 (Allow awrt 0.434π)  | A1    |
|                    | $\theta = 0.317$ and $\theta = 1.36$ only  | Awrt 0.317 (Allow awrt 0.101π) <b>or</b><br>Awrt 1.36 (Allow awrt 0.434π)  | A1    |
|                    | Alternative 1  | l for (i):   |       |
|                    | $5\sin 3\theta - 7\cos 3\theta = \sqrt{74}\sin(3\theta - 0.9505)$  | M1: Correct method using addition formula A1: $\sqrt{74} \sin(3\theta - 0.9505)$                                     | M1A1  |
|                    | $3\theta - 0.9505 = 0, \ \pi$  | $3\theta$ – their $\alpha = \sin^{-1}(0)$ . Dependent on   | dM1   |
|                    |  | the first method mark.   |       |
|                    | $\theta = 0.317$ or $\theta = 1.36$  | Awrt 0.317 (Allow awrt $0.101\pi$ ) or<br>Awrt 1.36 (Allow awrt $0.434\pi$ )   | A1    |
|                    | $\theta = 0.317$ and $\theta = 1.36$ only  | Awrt 0.317 (Allow awrt $0.434\pi$ )  Awrt 1.36 (Allow awrt $0.434\pi$ )  Awrt 1.36 (Allow awrt $0.434\pi$ )          | A1    |
|                    | Special case: If <b>both</b> answers are given in degrees allow A1A0 but needs to be awrt 18.2 <b>and</b> awrt 78.2)   |  |       |
|                    | Alternative 2 for (i):   |  |       |
|                    | $5\sin 3\theta = 7\cos 3\theta \Rightarrow 25\sin^2 \dots = 49\cos^2 \dots$  |  |       |
|                    | or   |  |       |
|                    | $5\sin 3\theta - 7\cos 3\theta = 0 \Rightarrow 25\sin^2 49\cos^2 = 0$<br>M1: Obtains $p\sin^2 = q\cos^2$ or $p\sin^2 q\cos^2 = 0$ $p,q > 0$  |  | M1    |
|                    | $\sin \dots = (\pm) \frac{7}{\sqrt{74}}  \text{or}  \cos \dots = (\pm) \frac{5}{\sqrt{74}}$ $\pm (awrt  0.8) \qquad \qquad \pm (awrt  0.6)$  | Correct value for sinor cos  | A1    |
|                    | $3\theta = 0.95054$  |  | dM1   |
|                    | $3\theta = \sin^{-1}\left(\text{their }\frac{7}{\sqrt{74}}\right) \text{ or } 3\theta = \cos^{-1}\left(\text{their }\frac{5}{\sqrt{74}}\right) \text{ leading to a value of } 3\theta.$                    |  |       |
|                    | Dependent on the first M.  |  |       |
|                    | $\theta = 0.317$ or $\theta = 1.36$  | Awrt 0.317 (Allow awrt 0.101π) <b>or</b><br>Awrt 1.36 (Allow awrt 0.434π)  | A1    |
|                    | $\theta = 0.317$ and $\theta = 1.36$ only  | Awrt 0.317 (Allow awrt 0.101π) <b>or</b><br>Awrt 1.36 (Allow awrt 0.434π)  | A1    |
|                    | Special case: If <b>both</b> answers are given in de 18.2 and awrt 78.2). If they give answers in catake precedence. For an otherwise fully cowithheld for extra answers in range. Igno Answers only score | degrees and radians, the radians answers orrect solution, the final mark can be ore extra answers outside the range. |       |
|                    |  |  | (5)   |

| 5(ii) | $9\cos^2 x + 5\cos$  | $x = 3\sin^2 x$   |          |
|-------|--|---|----------|
|       | $9\cos^2 x + 5\cos x = 3(1-\cos^2 x)$  | Uses $\sin^2 x = \pm 1 \pm \cos^2 x$  | M1       |
|       | $12\cos^2 x + 5\cos x - 3 = 0$   | Correct 3 term quadratic equation.<br>Allow equivalent equations with terms collected e.g. $12\cos^2 x + 5\cos x = 3$   | A1       |
|       | $(3\cos x - 1)(4\cos x + 3) = 0$ $\Rightarrow (\cos x) = \dots$                        | Solves their 3TQ in cos x to obtain at least one value. See general guidance for solving a 3TQ when awarding this method mark. <b>Dependent on the first method mark.</b> | dM1      |
|       | $\cos x = \frac{1}{3}, -\frac{3}{4}$   | Correct values for cos x  | A1       |
|       | x = 70.5, 289.5, 138.6, 221.4  | A1: Any 2 correct solutions (awrt) A1: All 4 answers (awrt)   | A1A1     |
|       | Special case: If all answers are given in radians allow A1A0 but needs to be awrt 1.2, |   |          |
|       | 5.1, 2.4, 3.9  |   |          |
|       | For an otherwise fully correct solution, the final mark can be withheld for extra      |   |          |
|       | answers in range. Ignore extra answers outside the range.                              |   |          |
|       | Answers only scores no marks.  |   | (6       |
|       |  |   | Total 11 |

| Question<br>Number | Scheme   | Notes  | Marks   |
|--------------------|--|--|---------|
| 6(a)               | $f(\pm 1) =$ or $f(\pm 2) =$   | Attempts $f(\pm 1)$ or $f(\pm 2)$  | M1      |
|                    | $a(-1)^3 - 8(-1)^2 + b(-1) + 6 = 0$  | Allow un-simplified but do not condone missing brackets unless later work implies a correct expression.  | A1      |
|                    | $a(2)^3 - 8(2)^2 + b(2) + 6 = -12$   | Allow un-simplified  | A1      |
|                    | $a+b=-2, 4a+b=7$ $\Rightarrow a=3, b=-5$   | M1: Solves two linear equations in <i>a</i> and <i>b</i> simultaneously to obtain values for <i>a</i> and <i>b</i> .  A1: Correct values                               | M1A1    |
|                    | Alternative by l   | ong division:  |         |
| -                  | $(ax^3 - 8x^2 + bx + 6) \div (x + 6)$  |  |         |
|                    | or   | -)   | M1      |
|                    | $\left(ax^3-8x^2+bx+6\right)\div\left(x-\frac{1}{2}\right)$  | $2) \rightarrow \text{remainder } g(a,b)$  | 1411    |
|                    | Attempts long division by either expression  | to obtain a remainder in terms of a and b  |         |
|                    | -a-b-2=0   | Allow un-simplified but do not condone missing brackets unless later work implies a correct expression.  | A1      |
|                    | 8a + 2b - 26 = -12   | Allow un-simplified  | A1      |
|                    | a+b=-2, 4a+b=7   | M1: Solves simultaneously  | M1A1    |
|                    | $\Rightarrow a=3, b=-5$  | A1: Correct values   | 1411711 |
| <i>a</i> >         |  |  | (5)     |
| (b)                | $(x+1)(ax^2+kx+)$  | Uses $(x + 1)$ as a factor and obtains at least the first 2 terms of a quadratic with an $ax^2$ term and an $x$ term. This might be by inspection or by long division. | M1      |
|                    | $(x+1)(3x^2-11x+6)$  | Correct quadratic factor   | A1      |
|                    | $3x^2 - 11x + 6 = (3x - 2)(x - 3)$   | Attempt to factorise their 3 term quadratic according to the general guidance, even if there was a remainder and $(x + 1)$ must have been used as a factor.            | M1      |
|                    | Note that $3x^2 - 11x + 6 = (x - 1)x + 6 = (x - 1)$ | $\left(-\frac{2}{3}\right)(x-3)$ scores M0 here  |         |
|                    | but $3x^2 - 11x + 6 = 3(x - 6)$  | $\left(\frac{2}{3}\right)(x-3)$ is fine for M1   |         |
|                    | $(f(x) =)(x+1)(3x-2)(x-3)$ or $(f(x) =)3(x+1)(x-\frac{2}{3})(x-3)$   | Fully correct factorisation. The factors need to appear together all on one line and no commas in between.   | A1      |
|                    | Answers with no  | working in (b):  |         |
|                    | $f(x) = 3x^3 - 8x^2 - 5x + 6 = (x+1)($   | (3x-2)(x-3) scores full marks  |         |
|                    | $f(x) = 3x^3 - 8x^2 - 5x + 6 = (x+1)(x-\frac{2}{3})(x+1)(x+1)(x+1) = (x+1)(x+1)(x+1) = (x+1)(x+1)(x+1)(x+1) = (x+1)(x+1)(x+1)(x+1)(x+1) = (x+1)(x+1)(x+1)(x+1) = (x+1)(x+1)(x+1)(x+1) = (x+1)(x+1)(x+1)(x+1) = (x+1)(x+1)(x+1)(x+1) = (x+1)(x+1)(x+1)(x+1)(x+1) = (x+1)(x+1)(x+1)(x+1)(x+1)(x+1)(x+1)(x+1)$  | (x-3) scores a special case M1A1M0A0   |         |
|                    | Just writing down roots of the cubic scores no marks.  |  |         |
|                    | Ignore any "= 0" and also ignore any subse<br>factorised for   |  |         |
|                    | ractorised for   | 111 15 50011.  | (4)     |
|                    |  |  | Total 9 |

| Correct method for the volume. It must be a correct statement for the volume. It must be a correct statement for the volume.   M1      (V) = x(375 - 80x + 4x^2) = 4x^3 - 80x^2 + 375x**     Allow the terms of $4x^3 - 80x^2 + 375x$ to be in any order.     Completes correctly to printed answer with no errors including bracketing errors   | Question<br>Number | Scheme   | Notes   | Marks        |
|--|--------------------|--|---|--------------|
| Allow the terms of $4x^3-80x^2+375x$ to be in any order.  Completes correctly to printed answer with no errors including bracketing errors  E.g. $V = 25x - 2x^2 (15 - 2x) = 4x^3 - 80x^2 + 375x$ scores M1A0  " $V = x(25 - 2x)(15 - 2x) = 4x^3 - 80x^2 + 375x$ scores M1A0 (lack of working) $V = x(25 - 2x)(15 - 2x) = (25x - 2x^2)(15 - 2x) = 4x^3 - 80x^2 + 375x$ scores M1A1 (2)  Mark (b), (c) and (d) together so that continued work with $x = 3.03$ . in (c) and (d) can be taken as evidence that the candidate has chosen this value in (b).  Allow e.g. $\frac{dy}{dx}$ for $\frac{dV}{dx}$ and/or $\frac{d^2y}{dx^2}$ for $\frac{d^2V}{dx^2}$ (b) $\frac{dV}{dx} = 0 = 2x^2 - 160x + 375$ $\frac{dV}{dx} = 0 \Rightarrow x = \frac{160 \pm \sqrt{7600}}{24}$ $\frac{dV}{dx} = 0 \Rightarrow x = \frac{160 \pm \sqrt{7600}}{24}$ $\frac{dV}{dx} = 0 \Rightarrow x = \frac{160 \pm \sqrt{7600}}{24}$ Identifies awrt 3.03 only as the required value.  (c) $\frac{d^2V}{dx^2} = 24x - 160 = 24(3.03) - 160$ $\frac{d^3V}{dx^2} = 24(3.03) - 160 \Rightarrow \frac{d^2V}{dx^2} < 0$ maximum  Fully correct proof for the maximum using a correct second derivative and using of the second derivative. A value for the second derivative is not needed and if the evaluation is incorrect, provided all the other conditions are met, this mark can be awarded. Accept statements such as "negative so x is the maximum"  Allow alternatives e.g. considers values of $V$ at, and either side of "3.03" or values of $V$ at either side of "3.03"  Allow alternatives e.g. considers values of $V$ at, and either side of "3.03" or values of $V$ at either side of "3.03"  (d) $V = 4(3.03)^3 - 80(3.03)^2 + 375(3.03)$ Anvt513 Allow that $V = 3000$ souls souls souls soul to the given $V$ or a "version" of $V$ .  Allow that $V = 3000$ souls souls soul to the given $V$ or a "version" of $V$ .  Allow that $V = 3000$ souls souls soul to the given $V$ or a "version" of $V$ .  Allow that $V = 3000$ souls souls soul to the given $V$ or a "version" of $V$ .   |                    | (V =) x(25-2x)(15-2x)  |   | M1           |
| Completes correctly to printed answer with no errors including bracketing errors  E.g. $V = 25x - 2x^2 (15 - 2x) = 4x^3 - 80x^2 + 375x scores MIAO$ " $V = w$ or e.g. "Volume = " must appear at some point. $V = x(25 - 2x)(15 - 2x) = 4x^2 - 80x^2 + 375x scores MIAO$ (lack of working) $V = x(25 - 2x)(15 - 2x) = (25x - 2x^2)(15 - 2x) = 4x^3 - 80x^2 + 375x scores MIAO$ Mark (b), (c) and (d) together so that continued work with $x = 3.03$ in (c) and (d) can be taken as evidence that the candidate has chosen this value in (b).  Allow e.g. $\frac{dy}{dx}$ for $\frac{dV}{dx}$ and/or $\frac{d^2y}{dx^2}$ for $\frac{d^2V}{dx^2}$ for $\frac{d^2V}{dx^2}$ for $\frac{dV}{dx}$ for $\frac{dV}{dx}$ and $\frac{dV}{dx} = 0 \Rightarrow x = \frac{160 \pm \sqrt{7600}}{24}$ M1: $x^3 - x^{-3}$ scen at least once  A1: Correct derivative  M1A1  (c)  (d)  (e)  (e)  (f)  (e)  (f)  (g)  (g)  A1: $\frac{d^2V}{dx^2} = 24(3.03) - 160$ (h)  A1: $\frac{d^2V}{dx^2} = 24(3.03) - 160$ A1:  |                    | · ·  | $(x^2) = 4x^3 - 80x^2 + 375x^*$                       | A1*          |
| (e)  |                    |  |   |              |
| $V = x(25-2x)(15-2x) = 4x^3 - 80x^2 + 375x \text{ scores } \text{M1A0 (lack of working)}$ $V = x(25-2x)(15-2x) = (25x-2x^2)(15-2x) = 4x^3 - 80x^2 + 375x \text{ scores } \text{M1A1}$ $(2)$ $\frac{\text{Mark (b), (c) and (d) together}}{\text{(d) can be taken as evidence that the candidate has chosen this value in (b).}}{\text{Allow e.g. }} \frac{dy}{dx} \text{ for } \frac{dV}{dx} \text{ and/or } \frac{d^2y}{dx^2} \text{ for } \frac{d^3V}{dx^2}$ $(b)$ $\frac{dV}{dx} = 0 \Rightarrow x = \frac{160 \pm \sqrt{7600}}{24}$ $\frac{dV}{dx} = 0 \Rightarrow x = \frac{160 \pm \sqrt{7600}}{24}$ $\frac{dV}{dx} = 0 \Rightarrow x = \frac{160 \pm \sqrt{7600}}{24}$ $\frac{dV}{dx} = 0 \Rightarrow x = \frac{160 \pm \sqrt{7600}}{24}$ $\frac{dV}{dx^2} = 24x - 160 = 24(3.03) - 160$ $\frac{d^2V}{dx^2} = 24x - 160 = 24(3.03) - 160$ $\frac{d^2V}{dx^2} = 24(3.03) - 160$ $d^2$   |                    | ` '  |   |              |
| $V = x(25-2x)(15-2x) = (25x-2x^2)(15-2x) = 4x^3-80x^2+375x \text{ scores M1A1}$ $\frac{\text{Mark (b), (c) and (d) together so that continued work with } x = 3.03. \text{ in (c) and (d) can be taken as evidence that the candidate has chosen this value in (b).}$ $Allow \text{ c.g. } \frac{dy}{dx} \text{ for } \frac{dV}{dx} \text{ and/or } \frac{d^2y}{dx^2} \text{ for } \frac{d^2V}{dx^2}$ $\frac{dV}{dx} = 0 \Rightarrow x = \frac{160 \pm \sqrt{7600}}{24} \text{ M1A1}$ $\frac{dV}{dx} = 0 \Rightarrow x = \frac{160 \pm \sqrt{7600}}{24} \text{ M1A1}$ $\frac{x = 3.03, 10.3}{24} \text{ but } 0 < x < 7.5 \text{ so } x = 3.03$ $\frac{d^2V}{dx^2} = 24x - 160 = 24(3.03) - 160$ $\frac{d^2V}{dx^2} = 24(3.03) - 160 \Rightarrow \frac{d^2V}{dx^2} < 0 \text{ maximum}$ $x = awr 1 3 only. There must be a substitution and there must be a reference to the sign of the second derivative evaluation is incorrect, provided all the other conditions are met, this mark can be awarded. Accept statements such as "negative so x is the maximum".  Allow alternatives e.g. considers values of V at, and either side of "3.03" or values of dV/dx either side of "3.03" or dV/dx = dV/$   |                    |  |   |              |
| $\frac{\text{Mark (b), (c) and (d) together so that continued work with } x = 3.03 \text{ in (c) and (d) can be taken as evidence that the candidate has chosen this value in (b).}$ $Allow \text{ c.g. } \frac{dy}{dx} \text{ for } \frac{dV}{dx} \text{ and/or } \frac{d^2y}{dx^2} \text{ for } \frac{d^2V}{dx^2}$ $\frac{dV}{dx} = 0 \Rightarrow x = \frac{160 \pm \sqrt{7600}}{24} \qquad \frac{dV}{dx} = 0 \text{ (may be implied) and attempts to solve a 3 term quadratic to find } x. \text{ May be implied by correct values.}}$ $x = 3.03, 10.3 \qquad \text{Identifies awrt 3.03 only as the required value.}$ $\frac{d^2V}{dx^2} = 24x - 160 = 24(3.03) - 160 \qquad \frac{d^2V}{dx^2} < 0 \text{ maximum}}{d^2V} = 0.$ $\frac{d^2V}{dx^2} = 24(3.03) - 160 \Rightarrow \frac{d^2V}{dx^2} < 0 \text{ maximum}}{d^2V} = 0.$ $\frac{d^2V}{dx^2} = 24(3.03) - 160 \Rightarrow \frac{d^2V}{dx^2} < 0 \text{ maximum}}{d^2V} = 0.$ $\frac{d^2V}{dx^2} = 24(3.03) - 160 \Rightarrow \frac{d^2V}{dx^2} < 0 \text{ maximum}}{d^2V} = 0.$ $\frac{d^2V}{dx^2} = 24(3.03) - 160 \Rightarrow \frac{d^2V}{dx^2} < 0 \text{ maximum}}{d^2V} = 0.$ $\frac{d^2V}{dx^2} = 24(3.03) - 160 \Rightarrow \frac{d^2V}{dx^2} < 0 \text{ maximum}}{d^2V} = 0.$ $\frac{d^2V}{dx^2} = 24(3.03) - 160 \Rightarrow \frac{d^2V}{dx^2} < 0 \text{ maximum}}{d^2V} = 0.$ $\frac{d^2V}{dx^2} = 24(3.03) - 160 \Rightarrow \frac{d^2V}{dx^2} < 0 \text{ maximum}}{d^2V} = 0.$ $\frac{d^2V}{dx^2} = 24(3.03) - 160 \Rightarrow \frac{d^2V}{dx^2} < 0 \text{ maximum}}{d^2V} = 0.$ $\frac{d^2V}{dx^2} = 24(3.03) - 160 \Rightarrow \frac{d^2V}{dx^2} < 0 \text{ maximum}}{d^2V} = 0.$ $\frac{d^2V}{dx} = 0.$ $\frac{d^2V}{dx$  |                    |  |   |              |
| $\frac{\text{Mark (b), (c) and (d) together}}{\text{(d) can be taken as evidence that the candidate has chosen this value in (b).}}{\text{Allow e.g. }} \frac{dy}{dx} \text{ for } \frac{dV}{dY} \text{ and/or } \frac{d^2y}{dx^2} \text{ for } \frac{dV}{dY}$ $\frac{dV}{dx} = 0 \Rightarrow x = \frac{160 \pm \sqrt{7600}}{24} \qquad \frac{\text{M1: } x^n \to x^{n-1} \text{ seen at least once}}{\text{A1: Correct derivative}} \qquad \text{M1A1}$ $\frac{dV}{dx} = 0 \Rightarrow x = \frac{160 \pm \sqrt{7600}}{24} \qquad \frac{\text{Puts } \frac{dV}{dx} = 0 \text{ (may be implied) and attempts to solve a 3 term quadratic to find } x. \text{ May be implied by correct values.}} \qquad \text{M1}$ $\frac{d^2V}{dx^2} = 24x - 160 = 24(3.03) - 160 \qquad \frac{\text{Attempts the second derivative}}{(x^n \to x^{n-1}) \text{ and substitutes at least one}} \qquad \text{M1}$ $\frac{d^2V}{dx^2} = 24(3.03) - 160 \Rightarrow \frac{d^2V}{dx^2} < 0 \text{ maximum}} \qquad \text{M1}$ $\frac{d^2V}{dx} = 24(3.03) - 160 \Rightarrow \frac{d^2V}{dx^2} < 0 \text{ maximum}} \qquad \text{M1}$ $x = avri 3 only. There must be a substitution and there must be a reference to the sign of the second derivative A value for the second derivative and using a correct second derivative and using of the second derivative and the sign of the second derivative and th$   |                    | $V = x(25-2x)(15-2x) = (25x-2x^2)(15-2x)$  | $(5-2x) = 4x^3 - 80x^2 + 375x$ scores M1A1            |              |
| (d) can be taken as evidence that the candidate has chosen this value in (b).  Allow e.g. $\frac{dy}{dx}$ for $\frac{dV}{dx}$ and/or $\frac{d^2y}{dx^2}$ for $\frac{d^2V}{dx^2}$ (b) $\frac{dV}{dx} = 12x^2 - 160x + 375$ $\frac{dV}{dx} = 0 \Rightarrow x = \frac{160 \pm \sqrt{7600}}{24}$ $\frac{dV}{dx} = 0 \Rightarrow x = \frac{160 \pm \sqrt{7600}}{24}$ $\frac{x = 3.03, 10.3}{\text{but } 0 < x < 7.5 \text{ so } x = 3.03}$ $\frac{dV}{dx^2} = 24x - 160 = 24(3.03) - 160$ $\frac{d^2V}{dx^2} = 24(3.03) - 160$ $\frac{d^2V}{dx^2} = 24(3.03) - 160 \Rightarrow \frac{d^2V}{dx^2} < 0 \therefore \text{maximum}$ Fully correct proof for the maximum using a correct second derivative and using $x = \text{awrt } 3 \text{ only}$ . There must be a substitution and there must be a reference to the sign of the second derivative. A value for the second derivative is not needed and if the evaluation is incorrect, provided all the other conditions are met, this mark can be awarded. Accept statements such as "negative so x is the maximum."  Allow alternatives e.g. considers values of V at, and either side of "3.03" or values of $dV/dx$ either side of "3.04" and either side of "3.05" or values of $dV/dx$ either side of "3.05" or values of $dV/dx$ either side of "3.05" or values  |                    | Moult (b) (a) and (d) together so that a   | $\begin{array}{c ccccccccccccccccccccccccccccccccccc$ | (2)          |
| (b) $\frac{dV}{dx} = 12x^2 - 160x + 375$ $\frac{dV}{dx} = 0 \Rightarrow x = \frac{160 \pm \sqrt{7600}}{24}$ $\frac{dV}{dx} = 0 \Rightarrow x = \frac{160 \pm \sqrt{7600}}{24}$ $\frac{dV}{dx} = 0 \Rightarrow x = \frac{160 \pm \sqrt{7600}}{24}$ $\frac{dV}{dx} = 0 \Rightarrow x = \frac{160 \pm \sqrt{7600}}{24}$ $\frac{dV}{dx} = 0 \Rightarrow x = \frac{160 \pm \sqrt{7600}}{24}$ $\frac{dV}{dx} = 0 \Rightarrow x = \frac{160 \pm \sqrt{7600}}{24}$ $\frac{dV}{dx} = 0 \Rightarrow x = \frac{160 \pm \sqrt{7600}}{24}$ $\frac{dV}{dx} = 0 \Rightarrow x = \frac{160 \pm \sqrt{7600}}{24}$ $\frac{dV}{dx} = 0 \Rightarrow x = \frac{160 \pm \sqrt{7600}}{24}$ $\frac{dV}{dx} = 0 \Rightarrow x = \frac{160 \pm \sqrt{7600}}{24}$ $\frac{dV}{dx} = 0 \Rightarrow x = \frac{160 \pm \sqrt{7600}}{24}$ $\frac{dV}{dx} = 0 \Rightarrow x = \frac{160 \pm \sqrt{7600}}{24}$ $\frac{dV}{dx} = 0 \Rightarrow x = \frac{160 \pm \sqrt{7600}}{24}$ $\frac{dV}{dx} = 0 \Rightarrow x = \frac{160 \pm \sqrt{7600}}{24}$ $\frac{dV}{dx} = 0 \Rightarrow x = \frac{160 \pm \sqrt{7600}}{24}$ $\frac{dV}{dx} = 0 \Rightarrow x = \frac{160 \pm \sqrt{7600}}{24}$ $\frac{dV}{dx} = 0 \Rightarrow x = \frac{160 \pm \sqrt{7600}}{24}$ $\frac{dV}{dx} = 0 \Rightarrow x = \frac{160 \pm \sqrt{7600}}{24}$ $\frac{dV}{dx} = 0 \Rightarrow x = \frac{160 \pm \sqrt{7600}}{24}$ $\frac{dV}{dx} = 0 \Rightarrow x = \frac{160 \pm \sqrt{7600}}{24}$ $\frac{dV}{dx} = 0 \Rightarrow x = \frac{160 \pm \sqrt{7600}}{24}$ $\frac{dV}{dx} = 0 \Rightarrow x = \frac{160 \pm \sqrt{7600}}{24}$ $\frac{dV}{dx} = 0 \Rightarrow x = \frac{160 \pm \sqrt{7600}}{24}$ $\frac{dV}{dx} = 0 \Rightarrow x = \frac{160 \pm \sqrt{7600}}{24}$ $\frac{dV}{dx} = 0 \Rightarrow x = \frac{160 \pm \sqrt{7600}}{24}$ $\frac{dV}{dx} = 0 \Rightarrow x = \frac{160 \pm \sqrt{7600}}{24}$ $\frac{dV}{dx} = 0 \Rightarrow x = \frac{160 \pm \sqrt{7600}}{24}$ $\frac{dV}{dx} = 0 \Rightarrow x = \frac{160 \pm \sqrt{7600}}{24}$ $\frac{dV}{dx} = 0 \Rightarrow x = \frac{160 \pm \sqrt{7600}}{24}$ $\frac{dV}{dx} = 0 \Rightarrow x = \frac{160 \pm \sqrt{7600}}{24}$ $\frac{dV}{dx} = 0 \Rightarrow x = \frac{160 \pm \sqrt{7600}}{24}$ $\frac{dV}{dx} = 0 \Rightarrow x = \frac{160 \pm \sqrt{7600}}{24}$ $\frac{dV}{dx} = 0 \Rightarrow x = \frac{160 \pm \sqrt{7600}}{24}$ $\frac{dV}{dx} = 0 \Rightarrow x = \frac{160 \pm \sqrt{7600}}{24}$ $\frac{dV}{dx} = 0 \Rightarrow x = \frac{160 \pm \sqrt{7600}}{24}$ $\frac{dV}{dx} = 0 \Rightarrow x = \frac{160 \pm \sqrt{7600}}{24}$ $\frac{dV}{dx} = 0 \Rightarrow x = \frac{160 \pm \sqrt{7600}}{24}$ $\frac{dV}{dx} = 0 \Rightarrow x = \frac{160 \pm \sqrt{7600}}{24}$ $\frac{dV}{dx} = 0 \Rightarrow x = \frac{160 \pm \sqrt{7600}}{24}$ $\frac{dV}{dx} = 0 \Rightarrow x = \frac{160 \pm \sqrt{7600}}{24}$ $\frac{dV}{dx} = 0 \Rightarrow x = \frac{160 \pm \sqrt{7600}}{24}$ $\frac{dV}{dx} = 0 \Rightarrow x = \frac{160 \pm \sqrt{7600}}{24}$ $\frac{dV}{dx} = 0 \Rightarrow x = \frac{160 \pm \sqrt{7600}}{24}$ $\frac{dV}{dx} = 0 \Rightarrow x = \frac{160 \pm \sqrt{7600}}{24}$ $\frac{dV}{dx} = 0 \Rightarrow x = \frac{160 \pm \sqrt{7600}}{24}$ $\frac{dV}{dx} = 0 \Rightarrow x = \frac{160 \pm \sqrt{7600}}{24}$ $\frac{dV}{dx} = 0 \Rightarrow x = 160 \pm \sqrt$   |                    |  |   |              |
| (b) $ \frac{dV}{dx} = )12x^2 - 160x + 375 $ $ \frac{M1: x^0 \rightarrow x^{n-1} \text{ seen at least once}}{A1: \text{ Correct derivative}} $ $ \frac{dV}{dx} = 0 \Rightarrow x = \frac{160 \pm \sqrt{7600}}{24} $ $ \frac{dV}{dx} = 0 \text{ (may be implied) and attempts to solve a 3 term quadratic to find x. May be implied by correct values.} $ $ \frac{x = 3.03, 10.3}{\text{but } 0 < x < 7.5 \text{ so } x = 3.03} $ $ \frac{d^2V}{dx^2} = 24x - 160 = 24(3.03) - 160 $ $ \frac{d^2V}{dx^2} = 24(3.03) - 160 \Rightarrow \frac{d^2V}{dx^2} < 0 \therefore \text{ maximum} $ $ \frac{d^2V}{dx^2} = 24(3.03) - 160 \Rightarrow \frac{d^2V}{dx^2} < 0 \therefore \text{ maximum} $ Fully correct proof for the maximum using a correct second derivative and using x = awrt 3 only. There must be a substitution and there must be a reference to the sign of the second derivative. A value for the second derivative is not needed and if the evaluation is incorrect, provided all the other conditions are met, this mark can be awarded. Accept statements such as "negative so x is the maximum"  Allow alternatives e.g. considers values of V at, and either side of "3.03" or values of dV/dx either side of "3.03"  (d) $V = 4(3.03)^3 - 80(3.03)^2 + 375(3.03)$ Substitutes a (positive) x from their $\frac{dV}{dx} = 0$ into the given V or a "version" of V.  Note that $V = \text{awrt } 513$ only scores M1A1  (2)   |                    |  |   |              |
| Al: Correct derivative   Al: Correct derivative   Al: Correct derivative   |                    | Allow e.g. $\frac{1}{dx}$ for $\frac{1}{dx}$   | and/or $\frac{1}{dx^2}$ for $\frac{1}{dx^2}$          |              |
| $\frac{dV}{dx} = 0 \Rightarrow x = \frac{160 \pm \sqrt{7600}}{24}$ Puts $\frac{dV}{dx} = 0$ (may be implied) and attempts to solve a 3 term quadratic to find x. May be implied by correct values. $x = 3.03, 10.3$ but $0 < x < 7.5$ so $x = 3.03$ Identifies awrt 3.03 only as the required value. $\left(\frac{d^2V}{dx^2}\right) = 24x - 160 = 24(3.03) - 160$ Attempts the second derivative $\left(x^n \to x^{n-1}\right)$ and substitutes at least one positive value of x from their $\frac{dV}{dx} = 0$ Fully correct proof for the maximum using a correct second derivative and using x = awrt 3 only. There must be a substitution and there must be a reference to the sign of the second derivative. A value for the second derivative is not needed and if the evaluation is incorrect, provided all the other conditions are met, this mark can be awarded. Accept statements such as "negative so x is the maximum"  Allow alternatives e.g. considers values of V at, and either side of "3.03" or values of $dV/dx$ either side of "3.03" or values of $dV/dx$ either side of "3.03"  (2)  (d)  V = $4(3.03)^3 - 80(3.03)^2 + 375(3.03)$ Substitutes a (positive) x from their $\frac{dV}{dx} = 0$ into the given V or a "version" of V.  V = 513  Note that $V = \text{awrt 513}$ only scores M1A1   | (b)                | $\left(\frac{dV}{dx}\right) = 12x^2 - 160x + 375$                                      |   | M1A1         |
| $\frac{d^{V}}{dx} = 0 \Rightarrow x = \frac{160 \pm \sqrt{7600}}{24}$ attempts to solve a 3 term quadratic to find x. May be implied by correct values. $x = 3.03, 10.3$ $\text{but } 0 < x < 7.5 \text{ so } x = 3.03$ Identifies awrt 3.03 only as the required value. $(x^{0}) \Rightarrow x = \frac{d^{2}V}{dx^{2}} = 24x - 160 = 24(3.03) - 160$ Attempts the second derivative $(x^{0}) \Rightarrow x^{0} \Rightarrow x$ | -                  | $\left( dx \right)$  |   |              |
| $\frac{d^{V}}{dx} = 0 \Rightarrow x = \frac{160 \pm \sqrt{7600}}{24}$ attempts to solve a 3 term quadratic to find x. May be implied by correct values. $x = 3.03, 10.3$ $\text{but } 0 < x < 7.5 \text{ so } x = 3.03$ Identifies awrt 3.03 only as the required value. $(x^{0}) \Rightarrow x = \frac{d^{2}V}{dx^{2}} = 24x - 160 = 24(3.03) - 160$ Attempts the second derivative $(x^{0}) \Rightarrow x^{0} \Rightarrow x$ |                    |  | Puts $\frac{dV}{dx} = 0$ (may be implied) and         |              |
| to find x. May be implied by correct values. $x = 3.03, 10.3$ Identifies awrt 3.03 only as the required value.  (c) Attempts the second derivative $\left(x^n \to x^{n-1}\right)$ and substitutes at least one positive value of x from their $\frac{dV}{dx} = 0$ $\frac{d^2V}{dx^2} = 24(3.03) - 160 \Rightarrow \frac{d^2V}{dx^2} < 0 : \text{maximum}$ Fully correct proof for the maximum using a correct second derivative and using $x = \text{awrt } 3 \text{ only}$ . There must be a substitution and there must be a reference to the sign of the second derivative. A value for the second derivative is not needed and if the evaluation is incorrect, provided all the other conditions are met, this mark can be awarded. Accept statements such as "negative so x is the maximum"  Allow alternatives e.g. considers values of $V$ at, and either side of "3.03" or values of $V$ at, and either side of "3.03" or values of $V$ at, and either side of "3.03" or values of $V$ at, and either side of "3.03" or values of $V$ at, and either side of "3.03" or values of $V$ at, and either side of "3.03" or values of $V$ at, and either side of "3.03" or values of $V$ at, and either side of "3.03" or values of $V$ at, and either side of "3.03" or values of $V$ at, and either side of "3.03" or values of $V$ at, and either side of "3.03" or values of $V$ at an $V$ at $V$ and $V$ are side of "3.03" or values of $V$ at $V$ and $V$ are side of "3.03" or values of $V$ at $V$ and $V$ are side of "3.03" or values of $V$ at $V$ and $V$ are side of "3.03" or values of $V$ at $V$ and $V$ are side of "3.03" or values of $V$ at $V$ and $V$ are side of "3.03" or values of $V$ at $V$ and $V$ are side of "3.03" or values of $V$ at $V$ and $V$ are side of "3.03" or values of $V$ at $V$ and $V$ are side of "3.03" or values of $V$ at $V$ and $V$ are side of "3.03" or values of $V$ and $V$ are side of "3.03" or values of $V$ and $V$ are side of "3.03" or values of $V$ and $V$ are side of "3.03" or values of $V$ and $V$ are side of "3.03" or values of $V$ and $V$ are side of "3.03" or values of $V$ and $V$ are si  |                    | $\frac{\mathrm{d}V}{\mathrm{d}t} = 0 \Rightarrow x = \frac{160 \pm \sqrt{7600}}{1000}$ | GOV .   | M1           |
| Cc   Cc   Cc   Cc   Cc   Cc   Cc   Cc  |                    | dx 24  | to find x. May be implied by correct                  |              |
| tension but $0 < x < 7.5$ so $x = 3.03$ required value.  At tempts the second derivative $ \left(\frac{d^2V}{dx^2}\right) = 24x - 160 = 24(3.03) - 160 $ At tempts the second derivative $ \left(x^n \to x^{n-1}\right) \text{ and substitutes at least one} \\ \text{positive value of } x \text{ from their } \frac{dV}{dx} = 0 $ At the provided and if the evaluation is incorrect, provided all the other conditions are met, this mark can be awarded. Accept statements such as "negative so $x$ is the maximum"  Allow alternatives e.g. considers values of $V$ at, and either side of "3.03" or values of $dV/dx$ either side of "3.03"  (d) $V = 4(3.03)^3 - 80(3.03)^2 + 375(3.03)$ $V = 513$ Substitutes a (positive) $x$ from their were side of "3.03"  Allow that $V = \text{awrt } 513$ only scores M1A1  Note that $V = \text{awrt } 513$ only scores M1A1   |                    |  |   |              |
| (c)  |                    | •  | •   | A1           |
|  |                    |  |   | (4)          |
| positive value of $x$ from their $\frac{dV}{dx} = 0$ $\frac{d^2V}{dx^2} = 24(3.03) - 160 \Rightarrow \frac{d^2V}{dx^2} < 0 \therefore \text{ maximum}$ Fully correct proof for the maximum using a correct second derivative and using $x = \text{awrt } 3$ only. There must be a substitution and there must be a reference to the sign of the second derivative. A value for the second derivative is not needed and if the evaluation is incorrect, provided all the other conditions are met, this mark can be awarded. Accept statements such as "negative so $x$ is the maximum"  Allow alternatives e.g. considers values of $V$ at, and either side of "3.03" or values of $V$ at an either side of "3.03" or values of $V$ at a considering a considering value of $V$ at a considering value of $V$ at $V$ and $V$ at $V$ and $V$ at $V$ and $V$ at $V$ at $V$ and $V$ are the following values of $V$ at $V$ and $V$ are the following values of $V$ at $V$ and $V$ are the following values of $V$ at $V$ and $V$ are the following values of $V$ at $V$ and $V$ are the following values of $V$ at $V$ and $V$ are the following values of $V$ at $V$ and $V$ are the following values of $V$   | (c)                |  | =   |              |
| Positive value of x from their $\frac{d^2V}{dx} = 0$   $\frac{d^2V}{dx^2} = 24(3.03) - 160 \Rightarrow \frac{d^2V}{dx^2} < 0$ :. maximum   Fully correct proof for the maximum using a correct second derivative and using $x = \text{awrt } 3 \text{ only}$ . There must be a substitution and there must be a reference to the sign of the second derivative. A value for the second derivative is not needed and if the evaluation is incorrect, provided all the other conditions are met, this mark can be awarded. Accept statements such as "negative so x is the maximum"   Allow alternatives e.g. considers values of V at, and either side of "3.03" or values of $dV/dx$ either side of "3.03"   (2)   Substitutes a (positive) x from their   $dV = 4(3.03)^3 - 80(3.03)^2 + 375(3.03)$   $dV = 0$   into the given V or a   M1   "version" of V.   $dV = 513$   Awrt 513   Awrt 513   All   Note that $V = \text{awrt } 513 \text{ only scores } M1A1$   (2)   |                    | $\left(\frac{d^2V}{dx^2}\right) = 24x - 160 = 24(3.03) - 160$                          | $(x^n \to x^{n-1})$ and substitutes at least one      | M1           |
| Fully correct proof for the maximum using a correct second derivative and using $x = \text{awrt } 3$ only. There must be a substitution and there must be a reference to the sign of the second derivative. A value for the second derivative is not needed and if the evaluation is incorrect, provided all the other conditions are met, this mark can be awarded. Accept statements such as "negative so $x$ is the maximum"  Allow alternatives e.g. considers values of $V$ at, and either side of "3.03" or values of $dV/dx$ either side of "3.03"  (2)  (d) $V = 4(3.03)^3 - 80(3.03)^2 + 375(3.03)$ Substitutes a (positive) $x$ from their $\frac{dV}{dx} = 0$ into the given $V$ or a "version" of $V$ . $V = 513$ Awrt $513$ All  Note that $V = \text{awrt } 513$ only scores M1A1  | _                  |  | $\mathbf{d}x$   |              |
| Fully correct proof for the maximum using a correct second derivative and using $x = \text{awrt } 3$ only. There must be a substitution and there must be a reference to the sign of the second derivative. A value for the second derivative is not needed and if the evaluation is incorrect, provided all the other conditions are met, this mark can be awarded. Accept statements such as "negative so $x$ is the maximum"  Allow alternatives e.g. considers values of $V$ at, and either side of "3.03" or values of $dV/dx$ either side of "3.03"  (2)  (d) $V = 4(3.03)^3 - 80(3.03)^2 + 375(3.03)$ Substitutes a (positive) $x$ from their $\frac{dV}{dx} = 0$ into the given $V$ or a "version" of $V$ . $V = 513$ Awrt $513$ All  Note that $V = \text{awrt } 513$ only scores M1A1  |                    | $\frac{d^2V}{dt^2} = 24(3.03) - 160 \implies \frac{d^2V}{dt^2} < 0$ : maximum          |   |              |
| $x = \text{awrt 3 only.}$ There must be a substitution and there must be a reference to the sign of the second derivative. A value for the second derivative is not needed and if the evaluation is incorrect, provided all the other conditions are met, this mark can be awarded. Accept statements such as "negative so $x$ is the maximum"  Allow alternatives e.g. considers values of $V$ at, and either side of "3.03" or values of $dV/dx$ either side of "3.03"  Substitutes a (positive) $x$ from their $\frac{dV}{dx} = 0$ into the given $V$ or a "version" of $V$ . $V = 513$ Awrt $513$ All  Note that $V = \text{awrt 513 only scores M1A1}$  |                    | Cr. Cr.  |   |              |
| evaluation is incorrect, provided all the other conditions are met, this mark can be awarded. Accept statements such as "negative so $x$ is the maximum"  Allow alternatives e.g. considers <b>values</b> of $V$ at, and either side of "3.03" or <b>values</b> of $dV/dx$ either side of "3.03"  (d) $V = 4(3.03)^3 - 80(3.03)^2 + 375(3.03)$ Substitutes a (positive) $x$ from their $\frac{dV}{dx} = 0$ into the given $V$ or a "version" of $V$ . $V = 513$ Awrt 513 All  Note that $V = \text{awrt 513 only scores M1A1}$   |                    |  |   |              |
| awarded. Accept statements such as "negative so x is the maximum"  Allow alternatives e.g. considers values of V at, and either side of "3.03" or values of $dV/dx$ either side of "3.03"  (2)  (d) $V = 4(3.03)^3 - 80(3.03)^2 + 375(3.03)$ Substitutes a (positive) x from their $\frac{dV}{dx} = 0$ into the given V or a "version" of V. $V = 513$ Awrt $513$ All  Note that $V = \text{awrt } 513 \text{ only scores } M1A1$  |                    |  |   |              |
| Allow alternatives e.g. considers <b>values</b> of $V$ at, and either side of "3.03" or <b>values</b> of $dV/dx$ either side of "3.03"  (d) $V = 4(3.03)^3 - 80(3.03)^2 + 375(3.03)$ Substitutes a (positive) $x$ from their $\frac{dV}{dx} = 0$ into the given $V$ or a "version" of $V$ . $V = 513$ Awrt 513 All Note that $V = \text{awrt 513 only scores M1A1}$  |                    | · •  |   |              |
| (d) $V = 4(3.03)^{3} - 80(3.03)^{2} + 375(3.03)$ Substitutes a (positive) x from their $\frac{dV}{dx} = 0  \text{into the given } V \text{ or a}$ $\text{"version" of } V.$ $V = 513  \text{Awrt } 513  \text{Al}$ Note that $V = \text{awrt } 513 \text{ only scores } M1A1$ (2)  |                    |  |   |              |
| (d) $V = 4(3.03)^{3} - 80(3.03)^{2} + 375(3.03)$ Substitutes a (positive) x from their $\frac{dV}{dx} = 0  \text{into the given } V \text{ or a}$ $\text{"version" of } V.$ $V = 513  \text{Awrt } 513  \text{Al}$ Note that $V = \text{awrt } 513 \text{ only scores } M1A1$ (2)  | -                  |  |   | (2)          |
| $V = 4(3.03)^{3} - 80(3.03)^{2} + 375(3.03)$ $\frac{dV}{dx} = 0 \text{ into the given } V \text{ or a}$ $\text{"version" of } V.$ $V = 513$ $\text{Awrt } 513$ $\text{Note that } V = \text{awrt } 513 \text{ only scores } M1A1$ $(2)$  | (4)                |  | Substitutes a (nositive) v from their                 | (2)          |
| "version" of $V$ . $V = 513$ Awrt $513$ Al  Note that $V =$ awrt $513$ only scores $M1A1$ (2)  | (u)                | W 4(2.02) <sup>3</sup> 00(2.02) <sup>2</sup> 277(2.02)                                 | 4   | 2.61         |
| $V = 513 \qquad \text{Awrt } 513 \qquad \text{A1}$ Note that $V = \text{awrt } 513 \text{ only scores M1A1}$ $(2)$   |                    | V = 4(3.03) - 80(3.03) + 375(3.03)   | $\frac{-}{\mathrm{d}x} = 0$ into the given $V$ or a   | MI           |
| Note that $V = \text{awrt } 513 \text{ only scores M1A1}$ (2)  |                    |  |   |              |
| (2)  |                    |  |   | A1           |
|  |                    | Note that $V = \text{awrt 513 only scores M1A1}$                                       |   | (4)          |
|  |                    |  |   | (2) Total 10 |

| Question<br>Number | Scheme  | Notes                              | Marks   |
|--------------------|---|------------------------------------|---------|
| 8(a)               | (-4, 7)   | 5                                  |         |
|                    | <del>-6</del> (-  | 1, 3)                              |         |
|                    | Reflection in the <i>y</i> -axis. Needs to be a positive minimum in the second quadrant. The curlet should be a curve and not a   | rve must at least reach both axes. | B1      |
|                    | Passes through (-6, 0) and (0, 5). Allow -6 and 5 to be marked in the correct places and allow (0, -6) and (5, 0) as long as they are in the correct places. There must be a sketch but this mark can be awarded if the correct coordinates are given in the body of the script provided they correspond with the sketch. Ignore any other intercepts.  If there is any ambiguity, the sketch takes precedence but if the correct coordinates are seen in the script, allow sign errors when transferring them to the sketch.  Maximum at (-4, 7) and minimum at (-1, 3) in the second quadrant. Must be seen as correct coordinate pairs or as numbers marked on the axes that clearly indicate the position of the maximum or minimum. There must be a sketch but this mark can be awarded if the correct coordinates are given in the body of the script provided they correspond with the sketch. Ignore any other turning points.  If there is any ambiguity, the sketch takes precedence but if the correct coordinates are seen in the script, allow sign errors when transferring them to the sketch. |                                    | B1      |
|                    |   |                                    | B1      |
|                    | the sheet   | ••                                 | (3)     |
| (b)                | 5 (0.5,   | 3)                                 |         |
|                    | A stretch in the x direction. Need to see $(x, y) \rightarrow (kx, y)$ where $k \ne 1$ for all points seen. There must be no evidence of a change in ant y coordinates.  The curve must at least reach both axes. It should be a curve and not a set of straight lines.   |                                    | B1      |
|                    | Passes through (3, 0) and (0, 5). Allow 3 and 5 to be marked in the correct places and allow (0, 3) and (5, 0) as long as they are in the correct places. There must be a sketch but this mark can be awarded if the correct coordinates are given in the body of the script provided they correspond with the sketch. Ignore any other intercepts.  If there is any ambiguity, the sketch takes precedence.  |                                    | B1      |
|                    | Minimum at $\left(\frac{1}{2}, 3\right)$ and maximum at $(2, 7)$ . in the first quadrant. Must be seen as correct coordinate pairs or as numbers marked on the axes that clearly indicate the position of the maximum or minimum. There must be a sketch but this mark can be awarded if the correct coordinates are given in the body of the script provided they correspond with the sketch.  Ignore any other turning points.  |                                    | B1      |
|                    | If there is any ambiguity, the sk   | Cicii tanes pi eccuence.           | (3)     |
|                    |   |                                    | Total 6 |

| Question<br>Number | Scheme  | Notes   | Marks       |
|--------------------|---|---|-------------|
| 9(a)               | $t_5 = ar^{n-1} = 20 \times 0.9^{5-1} = 13.122$   | M1: Use of a correct formula with $a = 20$ , $r = 0.9$ and $n = 5$ . Can be implied by a correct answer.  A1: 13.122 or $\frac{6561}{500}$ . Apply isw but just 13.1 is A0. | M1A1        |
|                    | MR: Some are misreading fifth as fifteenth or fiftieth and find $t_{15} = ar^{n-1} = 20 \times 0.9^{15-1} = 4.57$ or $t_{15} = ar^{n-1} = 20 \times 0.9^{50-1} = 0.114$ Allow M1A0 in these cases.  Listing: Need to see a fully correct attempt to find the fifth term e.g. 20, 18, 16.2, 14.58, 13.122 Must reach awrt 13 and intermediate decimals may |   |             |
|                    | not bee se  | ,   |             |
| -                  | Just 13.122 with no working   | ng scores both marks  | (2)         |
| (b)                | $S_8 = \frac{a(1-r^n)}{1-r} = \frac{20(1-0.9^8)}{1-0.9} = 113.9$  | M1: Use of a correct formula with $a = 20$ , $r = 0.9$ and $n = 8$ A1: 113.9 <b>only</b>  | (2)<br>M1A1 |
| -                  | Listing: Need to see a fu<br>e.g. 20 + 18 + 16.2 + 14.58 ++ 9.565938  | lly correct method  |             |
| -                  |   |   | (2)         |
| (c)                | $S_{\infty} = \frac{20}{1 - 0.9} (= 200)$   | Correct $S_{\infty}$ which can be simplified or un-simplified.  | B1          |
|                    | $200 - \frac{20(1 - 0.9^N)}{1 - 0.9} < 0.04$  | M1: Attempts $S_{\infty} - S_N < 0.04$ (allow $n$ for $N$ ) using $a = 20$ and $r = 0.9$<br>A1: Correct inequality in any form in terms of $N$ or $n$ only.                 | M1A1        |
|                    | Note that $\frac{20}{1-0.9} - \frac{20(1-0.9^{N})}{1-0.9} < 0.04 \text{ scores B1M1A1}$   |   |             |
|                    | $0.9^{N} < 0.0002*$   | Reaches the printed answer with intermediate working and with no errors or incorrect statements   | A1*         |
| (d)                | $(N >) \frac{\log 0.0002}{\log 0.9} \Rightarrow N = 81$   | M1: Correct attempt to find $N$ ignoring what they use for ">" i.e. they could be using < or =. Look for $(N =) \frac{\log 0.0002}{\log 0.9}$ or $(N =) \log_{0.9} 0.0002$  | (4)<br>M1A1 |
|                    | Q1 only with no working   | May be implied by awrt 81 A1: 81 only. Accept 81 only or $N/n = 81$ but not $N/n > 81$ .  |             |
|                    | 81 <u>only</u> with no working  | Scores Doth marks   | (2)         |
|                    |   |   | Total 10    |

| Question<br>Number | Scheme   | Notes  | Marks    |
|--------------------|--|--|----------|
| 10(i)              | Examples:  |  |          |
| 10(1)              | $3\log_8 2 = \log_8 2^3$ , $3\log_8 2 = \log_8 8$                        | Demonstrates a law or property of logs on  | B1       |
|                    | $3\log_8 2 = 1$ , $\log_8 2 = \frac{1}{3}$ , $2 = \log_8 64$             | either of the constant terms.  |          |
|                    | Examples: $(7-x)$  |  |          |
|                    | $\log_8(7-x) - \log_8 x = \log_8 \frac{(7-x)}{x}$                        | Demonstrates the addition or subtraction law of logs on two terms, at least one of   | B1       |
|                    | $\log_8 64 + \log_8 x = \log_8 64x$                                      | which is in terms of $x$ .   |          |
|                    | $\log_8 8 + \log_8 (7 - x) = \log_8 8(7 - x)$                            |  |          |
|                    |  | as described and award the marks where nd some incorrect work, do not look to  |          |
|                    | -  | ncorrect statements.   |          |
|                    | •  | $\frac{1}{x} = 1$ , $\log_8 \frac{(7-x)}{8x} = 0$ , $\log_8 \frac{8(7-x)}{x} = 2$  | M1       |
|                    |  | e of these equations or the equivalent.  a correct equation.   | 1V11     |
|                    | $\boldsymbol{x}$   | $=8, \frac{7-x}{8x}=1, \frac{8(7-x)}{x}=64$  | A1       |
|                    | Correct equation   | n with logs removed  |          |
|                    | $x = \frac{7}{9}$  | Accept equivalents but must be exact e.g. 56   | A1       |
|                    | 9  | $\frac{56}{72}$ or 0.777 or 0.7 with a dot over the 7  |          |
| (ii)               | 22v  | $3^{y+1} = 10$   | (5)      |
| (11)               |  | <u> </u>   |          |
|                    | , ,  | $(3^y)^2 + 3 \times 3^y = 10 \text{ or } x = 3^y \implies x^2 + 3x = 10$<br>adratic in x (or 3y)   | B1       |
|                    | 1  |  |          |
|                    | $x^2 + 3x - 10 = 0 \Longrightarrow x = \dots$                            | Correct attempt to solve a quadratic equation of the form $ax^2 + bx \pm 10 = 0$ (may be a letter other than x or may be $3^y$ etc.)                                 | M1       |
|                    | x=2  or  x=2  and  -5  | Correct values.  | A1       |
|                    | $3^{y} = 2 \Rightarrow y = \log_{3} 2 \text{ or } \frac{\log 2}{\log 3}$ | Correct use of logs. Need to see $3^y = k \Rightarrow y = \log_3 k$ or $\frac{\log k}{\log 3}$ , $k > 0$ which may be implied by awrt 0.63. Allow lg and ln for log. | dM1      |
|                    | $y = \log_3 2 \text{ or } y = \frac{\log 2}{\log 3}$                     | Cao (And no incorrect work using "-5"). Give BOD but penalise very sloppy notation e.g. log3(2) for log <sub>3</sub> 2 if necessary.                                 | A1       |
|                    |  |  | (5)      |
|                    |  |  | Total 10 |

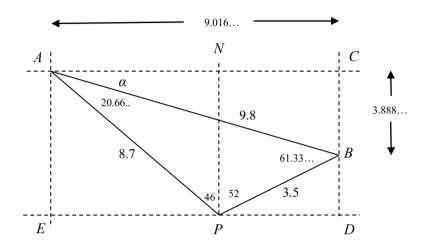
| (ii)  | $3^{2y} + 3^{y+1} = 10$   |  |      |  |
|-------|---|--|------|--|
| Way 2 | $3^{2y} + 3^{y+1} = (3^2)^y + 3(9)^{0.5y}$ $\Rightarrow 9^y + 3(9)^{0.5y} = 10$ | Correct quadratic in 9 <sup>0.5y</sup>   | B1   |  |
|       | $x^2 + 3x - 10 = 0 \Rightarrow x = 2(\text{or } -5)$                            | M1: Correct attempt to solve a quadratic equation of the form $ax^2 + bx - 10 = 0$ (may be a letter other than x or may be $9^{0.5y}$ etc.)  A1: Correct solution(s) | M1A1 |  |
|       | $9^{0.5y} = 2 \Rightarrow 0.5y = \log_9 2 \text{ or } \frac{\log 2}{\log 9}$    | Correct use of logs. Need to see $9^{0.5y} = k \Rightarrow 0.5y = \log_9 k$ or $\frac{\log k}{\log 9}, k > 0$  | dM1  |  |
|       | $y = 2\log_9 2 \text{ or } y = \frac{2\log 2}{\log 9}$                          | Cao (And no incorrect work using "-5")   | A1   |  |
|       |   |  | (5)  |  |

| Question<br>Number | Scheme  | Notes   | Marks |  |
|--------------------|---|---|-------|--|
|                    | Mark (a)(i) and (ii) together   |   |       |  |
| 11(a)(i)           | $(x\pm 4)^2$ and  | M1  |       |  |
|                    | Attempts to complete the square on $x$ and $y$ o  | r sight of $(x\pm 4)^2$ and $(y\pm 5)^2$ . May be   |       |  |
|                    | implied by a centre of $(\pm 4, \pm 5)$ . Or if considering $x^2 + y^2 + 2gx + 2fy + c = 0$ , centre is |   |       |  |
|                    | $(\pm g, \pm f)$ .  |   |       |  |
|                    | Centre is (4, 5)  | Correct centre  | A1    |  |
|                    | Correct answer scor   | es both marks   |       |  |
| (ii)               | $r^2 = (\pm "4")^2 + (\pm "5")^2 - 16$ (Must be -16)  |   |       |  |
|                    | Must rea  |   |       |  |
|                    | $r^2 = \text{their} (\pm 4)^2 + \text{their} (\pm 5)^2 - 16 \text{ or } r$                              | $=\sqrt{\text{their}(\pm 4)^2 + \text{their}(\pm 5)^2 - 16}$  |       |  |
|                    | or if using $x^2 + y^2 + 2gx + 2fx + c = 0$ , r   | <u></u> _   |       |  |
|                    | Must clearly be identifying   | •   |       |  |
|                    | May be implied by a   | correct radius.   |       |  |
|                    | r = 5   |   | A1    |  |
|                    | Correct answer scor   | es both marks   | (4)   |  |
| (b)                | $MT^2 = (20 - "4")^2 + (12 - "5")^2 (= 305)$  | Fully correct method using Pythagoras for <i>MT</i> or <i>MT</i> <sup>2</sup>   | M1    |  |
|                    | Other methods may be seen for finding $MT$ .  |   |       |  |
|                    | E.g. $\tan \theta = \frac{7}{16} \Rightarrow \theta = 23.6, MT = \frac{7}{\sin \theta} = 17.46$         |   |       |  |
|                    | Needs a fully correct   | method for MT   |       |  |
|                    | $MT = \sqrt{305}$   | Must be exact   | A1    |  |
|                    | Beware incorrect work leading to a correct answer e.g.  |   |       |  |
|                    | $MT^2 = \sqrt{(20-4)^2} + \sqrt{(12-5)^2} = \sqrt{256} + \sqrt{49} = \sqrt{305}$ scores M0              |   |       |  |
|                    |   |   | (2)   |  |
| (c)                | $\left(MP^2\right) = MT^2 - "5"^2$  | Correct method for MP or MP <sup>2</sup> where MT > "5"   | M1    |  |
|                    | Area $MTP = \frac{1}{2} \times "5" \times "\sqrt{280}"$ $5\sqrt{70}$                                    | Correct triangle area method  | M1    |  |
|                    | 5√70  | cao   | A1    |  |
|                    |   |   | (3)   |  |
|                    | Alternative for (c):  |   |       |  |
|                    | $\cos PTM = \frac{"5"}{\sqrt{"305"}} \sin PMT = \frac{"5"}{\sqrt{"305"}}$                               | Correct method for angle $PTM$ or $PMT$ (NB $PTM = 73.36, PMT = 16.63)$   | M1    |  |
|                    | Area $MTP = \frac{1}{2} \times "5" \times "\sqrt{305}" \times \sqrt{\frac{56}{61}}$                     | Correct triangle area method. May not work with exact values but needs to be a fully correct method using their values. | M1    |  |
|                    | 5√70  | Cao. Note that $5\sqrt{70} = 41.83$ which might imply a correct method.   | A1    |  |
|                    |   |   |       |  |

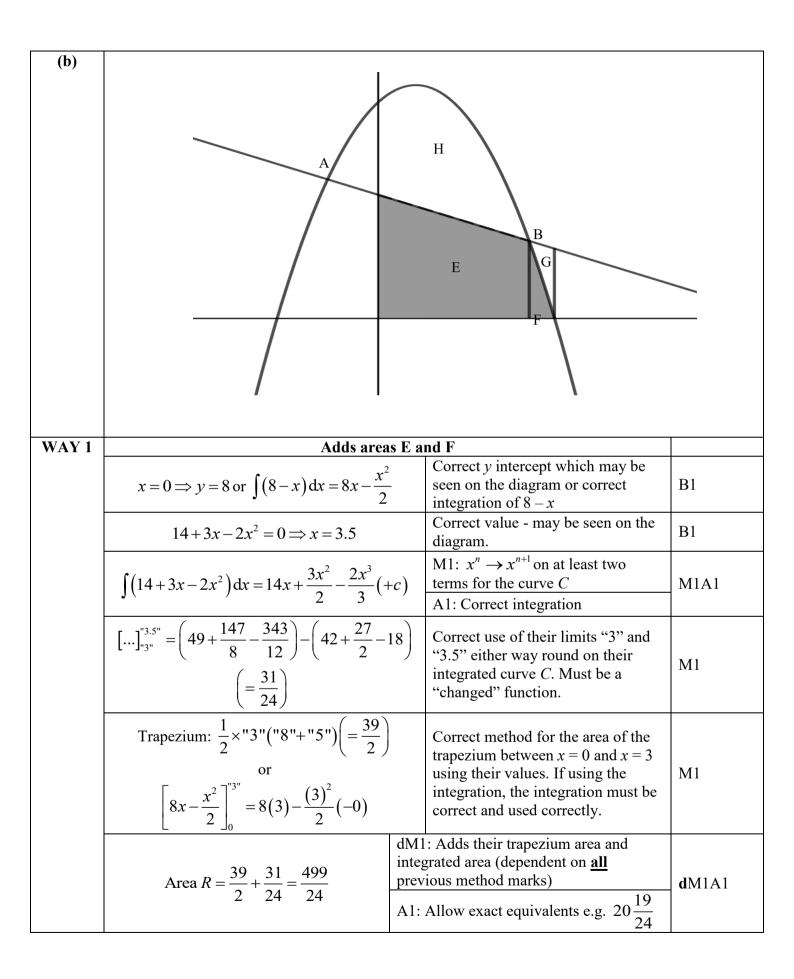
|                    |   |  | Total 9 |
|--------------------|---|--|---------|
|                    | y = 3x - 3  | values   | A1      |
|                    | $3(2)-3+c=0 \Rightarrow c=-3$   | Correct method to find c using their   | M1      |
|                    | 3x - y + c = 0  | B1: " $3x - y$ " M1: $3x - y + c = 0$  | B1M1    |
|                    | Alternative for last  | 4 marks of (c):  |         |
|                    |   |  | (5)     |
|                    | y = 3x - 3  | cao  | A1      |
|                    | $y = mx + x \Rightarrow "3" = "3" \times "2" + c \Rightarrow c = \dots$                                   | gradient. If using $y = mx + c$ , they must reach as far as a value for $c$ .  |         |
|                    | y-"3"="3"(x-"2") or   | Correct straight line method using their midpoint and a "changed"  | M1      |
|                    | Perpendicular gradient = 3  | Correct perpendicular gradient rule. This can be awarded for a correct value or a correct method e.g. $m = \frac{-1}{-\frac{1}{3}} \text{ or } \frac{-1}{3} \times m = -1 \Rightarrow m = \dots$ | M1      |
|                    | Gradient of $l_1 = -\frac{1}{3}$  | Correct gradient of $l_1$ . Allow equivalent exact expressions. May be implied by a correct perpendicular gradient.  | B1      |
| (c)                | $M = \left(\frac{-1 + 5}{2}, \frac{4}{2}\right) = (2, 3)$   | Correct midpoint method. May be implied by at least one correct coordinate if no working is shown.   | M1      |
|                    | (115) - 2 (10   | 2V10 only  |         |
| -                  | $\frac{\sqrt{(AR)} = 2\sqrt{10}}{\sqrt{10}}$  | $2\sqrt{10}$ only  | A1      |
| (b)                | $AB^{2} = ("4"-2)^{2} + (-1 - "5")^{2}$ or $AB = \sqrt{("4"-2)^{2} + (-1 - "5")^{2}}$ $(AB) = 2\sqrt{10}$ | Correct Pythagoras method using $(-1, "4")$ and $("5", 2)$ to find $AB$ or $AB^2$  | M1      |
|                    |   |  | (2)     |
|                    | p=4 and $q=5$   | Both correct values. May be implied<br>by e.g. when $x = -1$ , $y = 4$<br>and when $y = 2$ , $x = 5$   | B1      |
| 12(a)              | p = 4 or $q = 5$  | One correct value. May be implied<br>by e.g. when $x = -1$ , $y = 4$<br>or when $y = 2$ , $x = 5$  | B1      |
| Question<br>Number | Scheme  | Notes  | Marks   |

| Question<br>Number | Scheme  | Notes  | Marks   |
|--------------------|---|--|---------|
| 13(a)              | $(APN =) 360^{\circ} - 314^{\circ} = 46^{\circ}$ $(APB =) 46^{\circ} + 52^{\circ} = 98^{\circ}$ or $(Reflex APB) = 314^{\circ} - 52^{\circ} = 262^{\circ}$ $(APB =) 360^{\circ} - 262^{\circ} = 98^{\circ}$ or Shows on a sketch the 314 and 46 And states $46^{\circ} + 52^{\circ} = 98^{\circ}$ | Correct explanation that <b>explains why</b> $APN$ is $46^{\circ}$ (e.g. $360^{\circ} - 314^{\circ}$ ) and adds that to $52^{\circ}$ <b>or</b> shows/states that reflex $APB = 262^{\circ}$ and so $APB = 360^{\circ} - 262^{\circ} = 98^{\circ}$ . Do not be overly concerned how they use the letters to reference angles as long as the correct calculations are seen. <b>Do not allow the use of</b> $AB = 9.8$ <b>from (b).</b> | B1      |
|                    |   |  | (1)     |
| (b)                | $(AB^2 = )8.7^2 + 3.5^2 - 2 \times 8.7 \times 3.5 \cos 98^\circ$  | Correct use of cosine rule. You can ignore the lhs for this mark so just look for $8.7^2 + 3.5^2 - 2 \times 8.7 \times 3.5 \cos 98^\circ$  | M1      |
|                    | AB = 9.8  (km)  | Awrt 9.8 km (you can ignore their intermediate value for $AB^2$ provided awrt 9.8 is obtained for $AB$ )   | A1      |
|                    |   |  | (2)     |
| (c)<br>Way 1       | $\frac{"9.8"}{\sin 98^{\circ}} = \frac{3.5}{\sin PAB}$ or $3.5^{2} = 8.7^{2} + "9.8"^{2} - 2 \times 8.7 \times "9.8" \cos PAB$ $\Rightarrow PAB =$  | Correct sine or cosine rule method to obtain angle <i>PAB</i> . May be implied by awrt 21°   | M1      |
|                    | <i>PAB</i> = 20.66°   | Allow awrt 21°. May be implied by a correct bearing.   | A1      |
|                    | Bearing is 180° – "20.66° " – 46°   | Fully correct method   | M1      |
|                    | = 113° or 114°  | Awrt 113° or awrt 114°   | A1      |
| (c)<br>Way 2       | $\frac{"9.8"}{\sin 98^{\circ}} = \frac{8.7}{\sin PBA}$ or $8.7^{2} = 3.5^{2} + "9.8"^{2} - 2 \times 3.5 \times "9.8" \cos PBA$ $\Rightarrow PBA =$  | Correct sine or cosine rule method to obtain angle <i>PBA</i> . May be implied by awrt 61° or 62°  | M1      |
|                    | $PBA = 61.33^{\circ}$   | Allow awrt 61° or awrt 62°. May be implied by a correct bearing.   | A1      |
|                    | Bearing is 52° + "61.33°"   | Fully correct method   | M1      |
|                    | = 113° or 114°  | Awrt 113° or awrt 114°   | A1      |
|                    |   |  | (4)     |
| (c)<br>Way 3       | Let $\alpha = \text{Bearing} - 90^{\circ}$  |  |         |
| -                  | $\tan \alpha = \frac{BC}{AC} = \frac{8.7\cos 46^{\circ} - 3.5\cos 52^{\circ}}{8.7\sin 46^{\circ} + 3.5\sin 52^{\circ}}$   | Correct method for α   | M1      |
|                    | $\alpha = 23.33^{\circ}$  | Allow awrt 23°. May be implied by a correct bearing.   | A1      |
|                    | Bearing is 90° + "23.33°"   | Fully correct method   | M1      |
|                    | = 113° or 114°  | Awrt 113° or awrt 114°   | A1      |
|                    |   |  | (4)     |
|                    |   |  | Total 7 |

# Diagram for Q13



| Question<br>Number | Scheme   | Notes  | Marks  |
|--------------------|--|--|--------|
| 14                 | $y = 8 - x$ , $y = 14 + 3x - 2x^2$   |  |        |
| (a)                | $8-x = 14+3x-2x^{2}$ or $y = 14+3(8-y)-2(8-y)^{2}$                               | Uses the given line and curve to obtain an equation in one variable.   | M1     |
|                    | $2x^{2}-4x-6=0 \Rightarrow x = \dots$ or $2y^{2}-28y+90=0 \Rightarrow y = \dots$ | Solves their 3TQ as far as $x =$ or $y =$ Dependent on the first method mark.  | dM1    |
|                    | x = -1, $x = 3$ or $y = 5$ , $y = 9$   | Correct x values or correct y values   | A1     |
|                    | (-1, 9) (3, 5)   | ddM1: Solves for y or x using at least one value of x or y.  Dependent on both previous method marks.  A1: Correct coordinates which do not need to be paired so just look for correct values. | ddM1A1 |
|                    | Special case: Fully correct answers only with no working scores M0M0A0M1A1       |  |        |
|                    |  |  | (5)    |



| WAY 2 | Adds areas E, F and H and subtracts area H  |   |       |  |
|-------|---|---|-------|--|
|       | $\pm (\text{curve-line}) = \pm (14 + 3x - 2x^2 - (8 - x))$  |   | B1    |  |
|       | $14 + 3x - 2x^2 = 0 \Longrightarrow x = 3.5$  | Correct value - may be seen on the diagram.   | B1    |  |
|       | $\int (14+3x-2x^2) dx = 14x + \frac{3x^2}{2} - \frac{2x^3}{3} (+c)$ or  | M1: $x^n \to x^{n+1}$ on at least two terms for the curve $C$ or their $\pm$ (curve—line)   | MIAI  |  |
|       | $\int \pm (\text{curve-line}) dx = \pm \left( \text{"}6x + 2x^2 - \frac{2x^3}{3} \text{"} \right) (+c^3)$   | A1: Correct integration but allow correct ft integration for slips on their ±(curve-line)(ignore + c)   | M1A1  |  |
|       | $\left[\dots\right]_0^{"3.5"} = \left(49 + \frac{147}{8} - \frac{343}{12}\right) - (0)\left(=\frac{931}{24}\right)$   | Correct use of their upper limit "3.5" and 0 (which may be implied) either way round on their integrated curve <i>C</i> . Must be a "changed" function. | M1    |  |
|       | $\left[6x + 2x^2 - \frac{2x^3}{3}\right]_0^{3} = 6(3) + 2(3)^2 - \frac{2(3)^3}{3}(-0)$ Correct use of their "3" and 0 (which may be implied) either way round on their integrated $\pm$ (curve – line). Must be a "changed" function. |   | M1    |  |
|       |   |   |       |  |
|       | Area $R = \frac{931}{24} - 18 = \frac{499}{24}$   | M1: Subtracts (curve – line) area from arve area (dependent on <u>all</u> previous ethod marks)  1: Allow exact equivalents e.g. $20\frac{19}{24}$      | dM1A1 |  |

| WAY 3 | Adds areas E, F and G and subtracts area G   |   |   |          |
|-------|--|---|---|----------|
|       | $x = 0 \Rightarrow y = 8$ or $\pm (\text{line - curve}) = \pm \left(8 - x - \left(14 + 3x - x\right)\right)$ or $\cot \int (8 - x) dx = 8x - \frac{x^2}{2}$              | $(2x^2)$                                    | Correct y intercept - may be seen on the diagram. Or correct $\pm$ (curve-line) or correct integration of $8-x$                         | B1       |
|       | $14 + 3x - 2x^2 = 0 \Longrightarrow x = 3.5$   | Correct diagram                             | value - may be seen on the  | B1       |
|       | $\int \pm (\text{line} - \text{curve}) dx = \pm \left(\frac{2x^3}{3} - 6x - 2x^2\right) (+c)$  | their ±                                     | $x^{n+1}$ on at least two terms for (curve-line)<br>rect integration but allow correct ration for slips on their $x^{n+1}$ (ignore + c) | M1A1     |
|       | $\left[ "\frac{2x^3}{3} - 6x - 2x^2" \right]_{"3"}^{"3.5"} = \frac{2("3.5")^3}{3} - 6("3.5") - 2("3.5")^2 - \left(\frac{2("3")^3}{3} - 6("3") - 2("3")^2\right)$         |   |   | M1       |
|       | Correct use of their "3" and "3.5" either way round on their integrated $\pm$ (curve – line). Must be a "changed" function.  |   |   |          |
|       | Trapezium:<br>$\frac{1}{2} \times "3.5" ("8" + "4.5") \left( = \frac{175}{8} \right)$ or $\left[ 8x - \frac{x^2}{2} \right]_0^{"3.5"} = 8(3.5) - \frac{(3.5)^2}{2} (-0)$ | Correct<br>trapezit<br>using th<br>integrat | method for the area of the arm between $x = 0$ and $x = "3.5"$ neir values. If using the and used correctly.                            | M1       |
|       | Area $R = \frac{175}{8} - \frac{13}{12} = \frac{499}{24}$  | trapeziu<br>previou                         | ubtracts (line – curve) area from area (dependent on <b>all</b> as method marks)  ow exact equivalents e.g. $20\frac{19}{24}$           | dM1A1    |
|       |  |   |   | (8)      |
|       |  |   |   | Total 13 |

# Q14(b) COMBINED SCHEME

B1  $x = 0 \rightarrow y = 8$  (May be seen on the diagram)

OR: Correct integration of 8 - x, giving  $8x - \frac{x^2}{2}$ 

OR: 
$$\pm (curve - line) = \pm (14 + 3x - 2x^2 - (8 - x))$$

- B1  $14 + 3x 2x^2 = 0 \rightarrow x = 3.5$  (May be seen on the diagram).
- M1 Integration of the curve quadratic or their  $\pm (curve line)$  quadratic expression with  $x^n \rightarrow x^{n+1}$  for at least two terms.
- A1 Completely correct integration of the quadratic expression, even if mistakes have been made in 'simplifying' their quadratic expression. Ignore "+ c". (So the M1A1 is essentially given for correct integration).

N.B. "integrated curve" = "
$$\left(14x + \frac{3x^2}{2} - \frac{2x^3}{3}\right)$$
"

"integrated (curve – line)" = "
$$\left(6x + 2x^2 - \frac{2x^3}{3}\right)$$
"

Next two M marks for any one of the following three variations, with correct use of their limits on their integrated function (must be a "changed" function) or correct method for the appropriate trapezium using their values:

M1 1(i) ["integrated curve"] 
$${3.5}$$
" = ...  $\left(\frac{31}{24}\right)$ 

M1 1(ii) 
$$\left[8x - \frac{x^2}{2}\right]^{"3"}_{0} = \cdots \text{ or } \frac{1}{2} \times "3" \times ("8 + "5")$$
  $\left(\frac{39}{2}\right)$ 

M1 2(i) ["integrated curve"] 
$$\frac{3.5}{0}$$
 = ...  $\left(\frac{931}{24}\right)$ 

M1 
$$2(ii)$$
 ["integrated  $\pm (curve - line)$ "]  $\frac{3}{0}$  = ... (18)

M1 3(i) ["integrated 
$$\pm$$
 (line – curve)"]"3.5" =  $\cdots$   $\left(\frac{13}{12}\right)$ 

M1 3(ii) 
$$\left[8x - \frac{x^2}{2}\right]^{"3.5"} = \cdots$$
 or  $\frac{1}{2} \times "3.5" \times ("8 + "4.5")$   $\left(\frac{175}{8}\right)$ 

dM1 (Dependent on all previous method marks). Attempts the correct combination, which must be either 1(i) + 1(ii), or 2(i) - 2(ii), or 3(ii) - 3(i).

A1 
$$\frac{499}{24}$$
 or exact equivalent, e.g.  $20\frac{19}{24}$ 

| Question<br>Number | Scheme  | Notes   | Marks       |
|--------------------|---|---|-------------|
| 15                 | $(1+kx)^{n} = 1 + nkx + \frac{n(n-1)}{2}k^{2}x^{2}$   |   |             |
| (a)                | $\frac{n(n-1)}{2}k^2 = 126k \text{ or } \frac{n(n-1)}{2}k = 126k \text{ or } {}^nC_2k^2 = 126k \text{ or } {}^nC_2k = 126k$ Compares $x^2$ terms using one of these forms, with or without the $x^2$ .  |   | M1          |
|                    | $kn(n-1) = 252*$ Obtains the printed equation from $\frac{n(n-1)}{2}k^2 = 126k$ or $\frac{n(n-1)}{2}k^2x^2 = 126kx^2$   |   |             |
|                    | Note that these are acceptable proofs:<br>$\frac{n(n-1)}{2}k^2x^2 \text{ followed by } \frac{n(n-1)}{2}k = 126 \Rightarrow nk(n-1) = 252$ $\frac{n(n-1)}{2}k^2x^2 \text{ followed by } n(n-1)k^2 = 252k \Rightarrow nk(n-1) = 252$                                |   |             |
| (b)                | nk = 36   | Correct equation (oe). Can score anywhere.  | (2)<br>B1   |
|                    | $36(n-1) = 252$ or $36(\frac{36}{k} - 1) = 252$   | Uses a valid method with their $nk = 36$ and the given equation to obtain an equation in $n$ or $k$ only. It must be a correct algebraic method allowing for sign and/or arithmetic slips only. | M1          |
|                    | $36n - 36 = 252 \Rightarrow n = 8$ or $\frac{36}{k} - 1 = 7 \Rightarrow k = 4.5$  | dM1: Solves, using a correct method, to obtain a value for <i>n</i> or <i>k</i> A1: Correct value for <i>n</i> or <i>k</i>  | dM1A1       |
|                    | $n=8 \Rightarrow k=4.5 \text{ or } k=4.5 \Rightarrow n=8$   | Correct values for <i>n</i> and <i>k</i>  | A1          |
|                    | Special Case: Some candidates have a second term of $nx$ which gives $n = 36$ and then solve $kn(n-1) = 252$ to give $k = 0.2$ . This scores a special case of B1.  Generally, to score the method marks, candidates must be solving 2 equations in $n$ and $k$ . |   |             |
| (c)                | $\frac{n(n-1)(n-2)}{3!}k^3(x^3)$  | Correct coefficient. May be implied by $56k^3$ or "8" $C_3$ " $k$ " with or without $x^3$ . If no working is shown, you may need to check their values.   | (5)<br>B1ft |
|                    | $=\frac{8(8-1)(8-2)}{3!}4.5^{3}=\dots$  | Substitutes their values correctly including integer $n$ , $n > 3$ , to obtain a value for the coefficient of $x^3$ . Must be a correct calculation for the $x^3$ coefficient for their values. | M1          |
|                    | = 5103  | Allow 5103x <sup>3</sup>  | A1          |
|                    | Answer only of 5103 s   | scores BIMIAl   | (3)         |
|                    |   |   | Total 10    |