C12 October 2017 (MA)

Q1a) $8x + 2y - 15 = 0$

$2y = 15 - 8x$

$\div 2: \quad y = \frac{15}{2} - 4x$

so $\boxed{m = -4}$ = gradient

b) $m_{l_2} = -4$ (parallel) $\quad (-\frac{3}{4}, 16)$

$\therefore \quad y - 16 = -4(x + \frac{3}{4})$

$y = -4x - 3 + 16$

$\boxed{y = -4x + 13}$

Q2a) $f(x+2)$ indicates a translation of $+2$ in the negative x-direction

so $\boxed{(0, 3)}$

b) $-f(x) \rightarrow$ multiply all y by -1

so $\boxed{(2, -3)}$

c) $y = \frac{1}{2}f(x) \longrightarrow$ multiply all y by $\frac{1}{2}$.

so $\boxed{(2, \frac{3}{2})}$

d) $y = f(x) - 4$ → translation of $+4$ in the negative y-dir.

$\boxed{(2,-1)}$

Q3a) $\frac{x^3+4}{2x^2} = \frac{x^3}{2x^2} + \frac{4}{2x^2} = \frac{x}{2} + \frac{2}{x^2}$

$= \boxed{\frac{1}{2}x^1 + 2x^{-2}}$

b) $\int \left[\frac{x^3+4}{2x^2}\right] dx = \int \left(\frac{1}{2}x + 2x^{-2}\right) dx$

$= \frac{x^2}{4} + \frac{2x^{-1}}{-1} (+c) = \boxed{\frac{1}{4}x^2 - \frac{2}{x} + c}$

Q4a) Area $\Delta = \frac{1}{2}ab\sin C = \frac{1}{2}(3x)(x)\sin 60$

$\Rightarrow \frac{3x^2}{2}\sin 60 = 24\sqrt{3}$

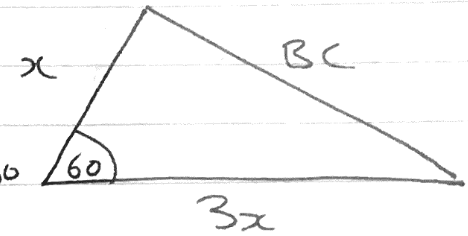
$\Rightarrow \frac{3x^2}{2} \times \frac{\sqrt{3}}{2} = 24\sqrt{3}$

$\Rightarrow x^2\left(\frac{3\sqrt{3}}{4}\right) = 24\sqrt{3}$

$\Rightarrow x^2 = \frac{24\sqrt{3}}{\frac{3\sqrt{3}}{4}} = 32$ //

so $x = \sqrt{32} = \boxed{4\sqrt{2}}$

b) <u>cosine rule</u>



$BC^2 = x^2 + (3x)^2 - 2(x)(3x)\cos 60$

$BC^2 = 10x^2 - 6x^2\left(\frac{1}{2}\right)$

$BC^2 = 7x^2 = 7(4\sqrt{2})^2 = 224$ //

so $BC = \sqrt{224} = \boxed{4\sqrt{14}}$

Q5a) $y = 27x^{\frac{1}{2}} - 2x^2$

$\frac{dy}{dx} = \boxed{\frac{27}{2}x^{-\frac{1}{2}} - 4x}$

b) $\frac{dy}{dx} = 0$ at P

$\frac{27}{2}x^{-\frac{1}{2}} - 4x = 0$

$4x = \frac{27}{2}x^{-\frac{1}{2}}$

$\therefore\ 4x^{\frac{3}{2}} = \frac{27}{2}$

so $x^{\frac{3}{2}} = \frac{27}{8}$

$x = \sqrt[3/2]{\frac{27}{8}} = \frac{9}{4}$ //

at $x = \frac{9}{4}$, $y = 27\left(\frac{9}{4}\right)^{\frac{1}{2}} - 2\left(\frac{9}{4}\right)^2 = \frac{243}{8}$ //

so $\boxed{P\left(\frac{9}{4}, \frac{243}{8}\right)}$

Q6a) year 1 year 2 year 3

600 680 760

$\begin{pmatrix} a = 600 \\ d = 80 \end{pmatrix}$

n^{th} term $= a + (n-1)d$

N^{th} term $= 600 + (N-1)\times 80 = 1000$

$80(N-1) = 400$

$N - 1 = 5$

$\therefore \boxed{N = 6}$

b) we require sum of the first 15 terms.

$S_n = \frac{n}{2}\left(2a + (n-1)d\right)$

$n = 15$: $S_{15} = \frac{15}{2}\left(2(600) + (14)(80)\right)$

$= \boxed{£17400}$

c) year 1 year 2 year 3

A 2A 3A

$\begin{pmatrix} a = A \\ d = A \end{pmatrix}$

Sum of first 15 terms is equal for Saima and Lin.

$S_{15} = \frac{15}{2}\left(2A + (15-1)A\right) = 17400$ //

So $\frac{15}{2}(2A + 14A) = 17400$

$$120A = 17400$$

$$A = \frac{17400}{120} = \boxed{145}$$

(Q7a) $g(x) = 2x^3 + ax^2 - 18x - 8$

$(x+2)$ is a factor of $g(x)$ so $g(-2) = 0$

$$g(-2) = 2(-2)^3 + a(-2)^2 - 18(-2) - 8 = 0$$

$$-16 + 4a + 36 - 8 = 0$$

$$4a = -12 \quad \therefore \boxed{a = -3}$$

b)

$$\begin{array}{r l}
 & 2x^2 - 7x - 4 \\
x+2 & \overline{)\,2x^3 - 3x^2 - 18x - 8} \\
 & 2x^3 + 4x^2 \\
 & \overline{0 \quad -7x^2 - 18x} \\
 & \quad -7x^2 - 14x \\
 & \quad \overline{0 \quad -4x - 8} \\
 & \qquad -4x - 8 \\
 & \qquad \overline{0 \quad 0}
\end{array}$$

$\therefore g(x) = (x+2)(2x^2 - 7x - 4)$

but $2x^2 - 7x - 4 = (2x+1)(x-4)$

so $g(x) = \boxed{(x+2)(2x+1)(x-4)}$

c) $2\sin^3\theta - 3\sin^2\theta - 18\sin\theta - 8 = 0$

but notice this is exactly the same as $[g(x) = 0]$ if we let $x = \sin\theta$.

so let $x = \sin\theta$:

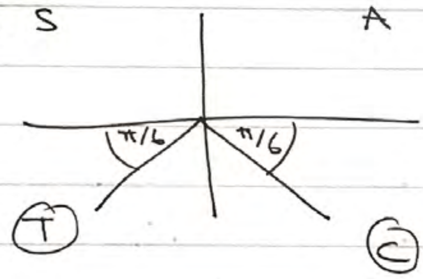
$$2x^3 - 3x^2 - 18x - 8 = 0 \quad (= g(x))$$

$$\therefore \quad (x+2)(2x+1)(x-4) = 0$$

$x + 2 = 0$
$x = -2$
$\sin\theta = -2$
NO SOLUTIONS
$-1 \le \sin\theta \le 1$

$2x + 1 = 0$
$x = -\frac{1}{2}$
$\sin\theta = -\frac{1}{2}$
$\theta = \sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$

$x - 4 = 0$
$x = 4$
$\sin\theta = 4$
NO SOLUTIONS
$-1 \le \sin\theta \le 1$



$\theta = \pi + \frac{\pi}{6}, \quad 2\pi - \frac{\pi}{6}$

$$\boxed{\theta = \frac{7\pi}{6}, \ \frac{11\pi}{6}}$$

Q8a) $L = \boxed{r\theta = 6}$

$\frac{1}{2}r^2\theta$ = Area OAB = 20

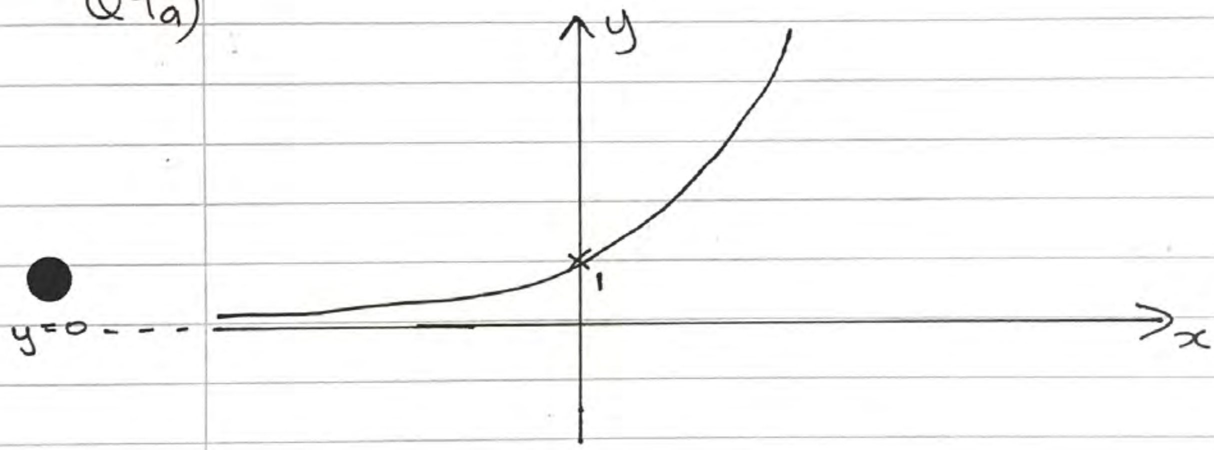
$20 = \frac{1}{2}r^2\theta \longrightarrow \boxed{40 = r^2\theta}$

b) $r\theta = 6 \longrightarrow 40 = 6 \times r$

$r = \frac{40}{6} = \boxed{\frac{20}{3}}$

$\theta = \frac{6}{r} = \frac{6}{\frac{20}{3}} = \boxed{\frac{9}{10}}$

Q9a)



b) $h = \frac{b-a}{n} = \frac{4--4}{4} = 2$

Area $\simeq \frac{1}{2} \times 2 \left[0.0625 + 16 + 2(0.25+1+4)\right]$

$\simeq \boxed{26.56}$

c i) $2^{x+2} = 2^x(2^2) = 2^2 \times 2^x$

so $\int_{-4}^{4} 2^{x+2}\,dx = 2^2 \times 26.56 \approx \boxed{106}$

or $\frac{425}{16}$ (exact)

ii) $\int_{-4}^{4} (3+2^x)\,dx = \int_{-4}^{4} (3)\,dx + \int_{-4}^{4} (2^x)\,dx$

$= \left[3x\right]_{-4}^{4} + 26.56$

$= [3\times4] + [3\times4] + 26.56 \approx \boxed{50.6}$

or $\frac{809}{16}$ (exact)

Q10a) $A(7,-3)$ and $D(10,5)$
$C(p,q)$

midpoint $= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

since D is the midpoint . . .

$\hookrightarrow \left(\frac{7+p}{2}, \frac{q-3}{2}\right) = (10, 5)$

$\frac{7+p}{2} = 10 \quad \therefore \; p = 20 - 7 = \boxed{13}$

$\frac{q-3}{2} = 5 \quad \therefore \; q = 10 + 3 = \boxed{13}$

b) $m_{AC} = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{13 - -3}{13 - 7} = \dfrac{16}{6} = \dfrac{8}{3}$

but l is perp to AC so $m_l = -\dfrac{3}{8}$ //

$\left(-\dfrac{3}{8} \times \dfrac{8}{3} = -1\right)$ //

$\Rightarrow y - 5 = -\dfrac{3}{8}(x - 10)$

$\Rightarrow y = -\dfrac{3}{8}x + \dfrac{30}{8} + 5$

$\Rightarrow y = -\dfrac{3}{8}x + \dfrac{35}{4}$

$\xrightarrow{\times 8 :}$ $8y = -3x + 70$

$\rightarrow \boxed{3x + 8y = 70}$

c) AB is simply the line $x = 7$ (for $-3 \leq y \leq 20$)

So sub $x = 7$: $3(7) + 8y = 70$

$8y = 49$

$\boxed{y = \dfrac{49}{8}}$

so $\boxed{E\left(7, \dfrac{49}{8}\right)}$ //

Q11a) $(3+ax)^5 \simeq (3)^5 + \binom{5}{1}(3)^4(ax)^1 + \binom{5}{2}(3)^3(ax)^2$

$\simeq \boxed{243 + 405ax + 270a^2x^2}$

b) $f(x) = (a-x)(243 + 405ax + 270a^2x^2)$

$= 243 + \underline{405a^2x} + 270a^3x^2 - \underline{243x} - 405ax^2 - 270a^2x^3$

x terms are $(+405a^2x)$ and $(-243x)$

coeff. of x is 0 so $405a^2 - 243 = 0$

$\Rightarrow a^2 = \frac{243}{405} = \frac{3}{5}$

$\therefore a = \boxed{\sqrt{\frac{3}{5}}}$

Q12i) $3\sin(\theta+30) = 2\cos(\theta+30)$

$\div \cos(\theta+30)$: $3\tan(\theta+30) = 2$

$\tan(\theta+30) = \frac{2}{3}$

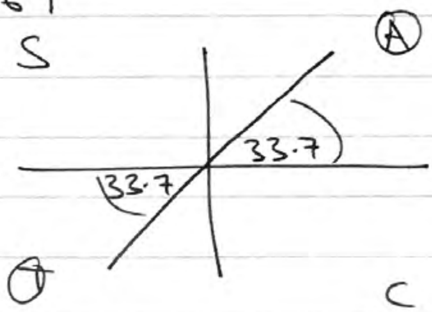
so $\theta + 30 = \tan^{-1}\left(\frac{2}{3}\right) = 33.69°..$

range ... $30° < \theta + 30 \leq 390°$

$\theta + 30° = 33.69°, 180 + 33.69°$

$\theta + 30 = 33.69°, 213.69°$

$\boxed{\theta = 3.69°, 183.69°}$



iia) $\frac{\cos^2 x + 2\sin^2 x}{1 - \sin^2 x} = 5$

$\frac{\cos^2 x + 2\sin^2 x}{\cos^2 x} = 5$

$1 + 2\tan^2 x = 5$

$2\tan^2 x = 4$

$\therefore \boxed{\tan^2 x = 2}$ $(k = 2)$

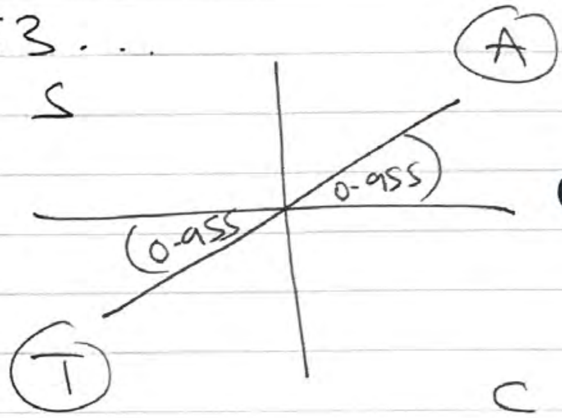
$0 < x \leq 2\pi$

b) we need to solve : $\tan^2 x = 2$

$\tan x = \pm\sqrt{2}$

$\tan x = \sqrt{2}$

$x = \tan^{-1}(\sqrt{2}) = 0.9553...$

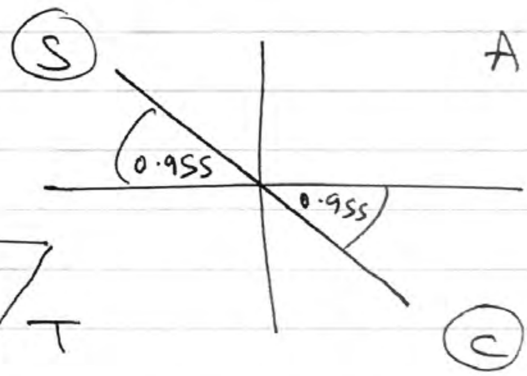


$x = 0.955^c, 4.10^c, ...$

$\boxed{x = 0.955^c, 4.097^c}$

$\tan x = -\sqrt{2}$

$x = \tan^{-1}(-\sqrt{2}) = -0.9553...$



$x = \pi - 0.9553,$
$2\pi - 0.9553$

$\boxed{x = 2.186^c, 5.328^c}$

1

Q13a i) $(x-3)^2 + (y+4)^2 = 30$

centre $\boxed{(3,-4)}$

ii) radius $\boxed{\sqrt{30}}$

b) since P lies inside the circle we know that the distance from the centre to P will be less than the radius.

so $\sqrt{(6-3)^2 + (4+k)^2} < \sqrt{30}$

square both sides ...

$$(6-3)^2 + (k+4)^2 < 30$$

$$9 + k^2 + 16 + 8k < 30$$

$$k^2 + 8k + 25 - 30 < 0$$

$$k^2 + 8k - 5 < 0$$

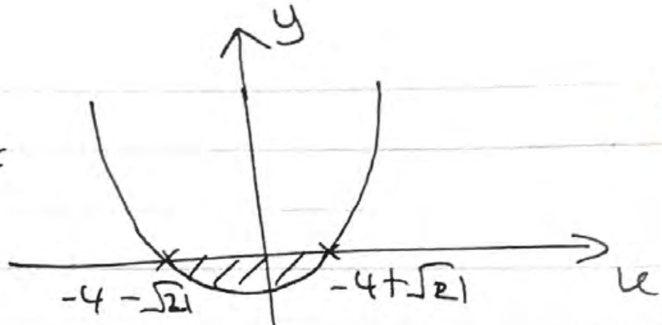
c) we want the values of k for which

$$k^2 + 8k - 5 < 0$$

critical values: $k^2 + 8k - 5 = 0$

By Quadratic formula, $k = -4 \pm \sqrt{21}$ //

we want where
y is less than 0



So $\boxed{-4-\sqrt{21} < k < -4+\sqrt{21}}$

(Q14a)

year 1	year 2	year 3
8000	$8000 \times (0.85)$	$8000 \times (0.85)^2$

$$\begin{pmatrix} a = 8000 \\ r = 0.85 \end{pmatrix}$$

So 6^{th} year $\longrightarrow$ $ar^{n-1} = 8000(0.85)^5$

$$= 3549.6$$

$$\simeq \boxed{3550}$$

b) $\boxed{r < 1}$ as $n \to \infty$, $S_n \to$ constant.
(ie S_∞)

c) $S_\infty = \dfrac{a}{1-r} = \dfrac{8000}{1-0.85} \approx \boxed{53333}$

d) $S_N = \dfrac{a(1-r^n)}{1-r} = 40000$

$$\frac{8000(1-0.85^N)}{0.15} = 40000$$

$$8000(1-0.85^N) = 6000$$

$$1-0.85^N = \frac{3}{4}$$

$$0.85^N = \frac{1}{4}$$

$$\log(0.85^N) = \log\left(\frac{1}{4}\right)$$

$$N\log 0.85 = \log\frac{1}{4}$$

$$N = \frac{\log\frac{1}{4}}{\log 0.85} = 8.53\ldots$$

so $\boxed{N=9}$ (in the 8^{th} year, total mass < 40000) so $N=9$

Q15ai) $\underline{x=0}$: $y = \frac{(-3)^2(4)}{2} = \boxed{18}$

ii) $\underline{y=0}$: $\frac{(x-3)^2(x+4)}{2} = 0$

$\boxed{x=3}$ $(x>0)$

b) $\underline{y=18}$: $18 = \frac{(x-3)^2(x+4)}{2}$

$$36 = (x-3)^2(x+4)$$

$(x^2 - 6x + 9)(x + 4) = 36$

$x^3 - 2x^2 - 15x + \cancel{36} = \cancel{36}$

$x^3 - 2x^2 - 15x = 0$

$x(x^2 - 2x - 15) = 0$

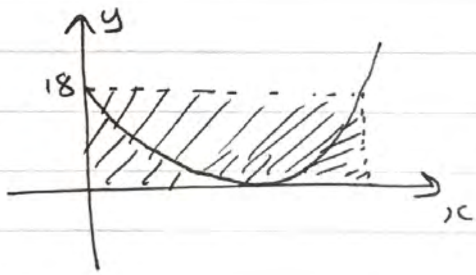
$x > 0 \quad \therefore \quad \boxed{x^2 - 2x - 15 = 0}$

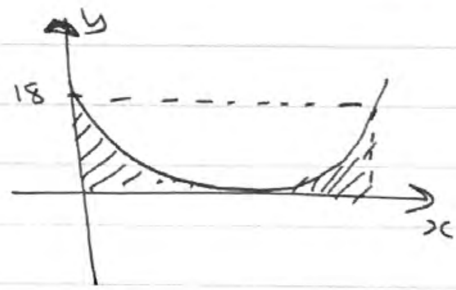
c) $(x-5)(x+3) = 0$

$x = 5$
$x = -3$

$x > 0$ so $\boxed{x = 5}$

hence $\boxed{Q(5, 18)}$

d) $S =$ 

$-$ 

$S =$ 

$- \displaystyle\int_0^5 f(x)\,dx$

$$\therefore\ S = (18 \times 5) - \frac{1}{2}\int_0^5 \left[x^3 - 2x^2 - 15x + 36\right] dx$$

$$= 90 - \frac{1}{2}\left[\frac{x^4}{4} - \frac{2x^3}{3} - \frac{15x^2}{2} + 36x\right]_0^5$$

$$= 90 - \frac{1}{2}\left[\frac{785}{12}\right] = \boxed{\frac{1375}{24}}$$