Write your name here

Surname | Other names

Pearson Edexcel
International
Advanced Level

Centre Number | Candidate Number

Core Mathematics C12

Advanced Subsidiary

Wednesday 23 May 2018 – Morning
Time: 2 hours 30 minutes

Paper Reference
WMA01/01

You must have:
Mathematical Formulae and Statistical Tables (Blue)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 125.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

Pearson

C12 June 2018.

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1. The table below shows corresponding values of x and y for $y = \frac{1}{\sqrt{(x+1)}}$, with the values for y rounded to 3 decimal places where necessary.

x	0	3	6	9	12	15
y	1	0.5	0.378	0.316	0.277	0.25.

(a) Complete the table by giving the value of y corresponding to $x = 15$

(1)

(b) Use the trapezium rule with all the values of y from the completed table to find an approximate value for

$$\int_0^{15} \frac{1}{\sqrt{(x+1)}}\,dx$$

giving your answer to 2 decimal places.

(4)

(b) $\underline{h}$

$3-0=3.$

$6-3=3.$

$\therefore \underline{h=3}$

$\frac{1}{2}\times h\times (a+b)$

$= \frac{1}{2}\times 3+(1+0.25+2(0.5+0.378$
$+0.316+0.277))$

$= \underline{\underline{6.29}}$

Leave blank

2. $$f(x) = ax^3 + 2x^2 + bx - 3$$

where a and b are constants.

When $f(x)$ is divided by $(2x - 1)$ the remainder is 1

(a) Show that

$$a + 4b = 28$$

(2)

When $f(x)$ is divided by $(x + 1)$ the remainder is -17

(b) Find the value of a and the value of b.

(4)

$f(\frac{1}{2}) = 1.$

$a(\frac{1}{2})^3 + 2(\frac{1}{2})^2 + b(\frac{1}{2}) - 3 = 1$

$\frac{1a}{8} + \frac{1}{2} + \frac{b}{2} - 3 = 1.$

$8 \times \frac{1a}{8} + \frac{1}{2}b = \frac{7}{2}. \quad \times 8$

$a + 4b = 28$ as req

(b) $f(-1) = -17.$

$-a + 2(-1)^2 + b(-1) - 3$

$-a + 2 - b - 3 = -17.$

$-a - b = -16.$

$+ \quad a + 4b = 28.$

$3b = 12$

$b = 4$

$a = 16 - b$

$a = 16 - 4 = 12$

Leave blank

3. The line l_1 passes through the points $A(-1, 4)$ and $B(5, -8)$

(a) Find the gradient of l_1

(2)

The line l_2 is perpendicular to the line l_1 and passes through the point $B(5, -8)$

(b) Find an equation for l_2 in the form $ax + by + c = 0$, where a, b and c are integers.

(4)

(a) $\frac{y_2-y_1}{x_2-x_1} = \frac{-8-4}{5--1}$

$= \frac{-12}{6} = \underline{\underline{-2}}$

(b) grad of ~~⊥~~ line

$= \frac{1}{2}$.

$y = \frac{1}{2}x + c \quad (5, -8)$

$-8 = \frac{5}{2} + c$

$c = -\frac{21}{2}$.

$y = \frac{1}{2}x - \frac{21}{2}$.

$2y = x - 21$

$2y - x + 21 = 0$.

Leave blank

4. Given that

$$y = \frac{64x^6}{25}, \quad x > 0$$

express each of the following in the form kx^n where k and n are constants.

(a) $y^{-\frac{1}{2}}$

(3)

(b) $(25y)^{\frac{2}{3}}$

(2)

$y^{-1/2}$

$= \left(\frac{64x^6}{25}\right)^{-1/2}$

$\frac{64^{-1/2} \cdot x^{-3}}{25^{-1/2}}$

$= \frac{5}{8}x^{-3}$

(b) $\frac{\cancel{25} \cdot 64x^6}{\cancel{25}}$

$(64x^6)^{2/3}$

$= 64^{2/3} \cdot x^4$

$= 16x^4$

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5. (a) Find the first 4 terms, in ascending powers of x, of the binomial expansion of

$$\left(1+\frac{x}{3}\right)^{18}$$

giving each term in its simplest form.

(4)

(b) Use the answer to part (a) to find an estimated value for $\left(\frac{31}{30}\right)^{18}$, stating the value of x that you have used and showing your working. Give your estimate to 4 decimal places.

(3)

(a) $1+\binom{18}{1}\left(\frac{x}{3}\right)+\binom{18}{2}\left(\frac{x}{3}\right)^2+\binom{18}{3}\left(\frac{x}{3}\right)^3$

$= 1+6x+17x^2+\frac{272}{9}x^3 \ldots$

(b) $1+\frac{x}{3}=\frac{31}{30}$

$x=0.1$

$1+6(0.1)+17(0.1)^2+\frac{272}{9}(0.1)^3$

$=1.8002$

Leave blank

6. Find the exact values of x for which

$$2\log_5(x+5) - \log_5(2x+2) = 2$$

Give your answers as simplified surds.

(7)

$\log_5 (x+5)^2 - \log_5 (2x+2)$

$\log_5 \left(\frac{(x+5)^2}{(2x+2)}\right) = 2.$

$25 = \frac{(x+5)^2}{2(x+1)}$

$50(x+1)\cancel{(x+5)^2} = (x+5)^2$

$50x+50 = x^2+10x+25.$

$x^2 - 40x - 25 = 0$

$\frac{40 \pm \sqrt{40^2 - 4(-25)}}{2 \times 1}$

$x = \underline{\underline{20 \pm \sqrt{17}}}$

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7. A sequence is defined by

$$u_1 = 3$$
$$u_{n+1} = u_n - 5, \qquad n \geqslant 1$$

Find the values of

(a) u_2, u_3 and u_4 **(2)**

(b) u_{100} **(3)**

(c) $\sum_{i=1}^{100} u_i$ **(3)**

(a) $u_{1+1} = u_1 - 5$

$= 3 - 5$

$u_2 = -2$

$u_3 = u_2 - 5$

$= -2 - 5$

$u_3 = -7$

$u_4 = u_3 - 5$

$= -7 - 5$

$u_4 = -12$

(b) $a + d(n-1) = 3 + (-5)(99) = -492$

(~~b~~c) $a = 3$.

$d = -5$

$\frac{1}{2}n\left[2a + d(n-1)\right]$.

$= \frac{100}{2}\left[2(3) + (-5)(99)\right]$

$= -24{,}450$

(~~c~~)= $= -24{,}450$

8. The equation $(k-4)x^2 - 4x + k - 2 = 0$, where k is a constant, has no real roots.

(a) Show that k satisfies the inequality

$$k^2 - 6k + 4 > 0$$

(3)

(b) Find the exact range of possible values for k.

(4)

(a) $b^2 - 4ac < 0.$

$(-4)^2 - 4(k-4)(k-2) < 0$

$16 + [-4k + 16(k-2)]$

$\downarrow$

$-4k^2 + 8k + 16k - 32$

$16 + [-4k^2 + 24k - 32] < 0$

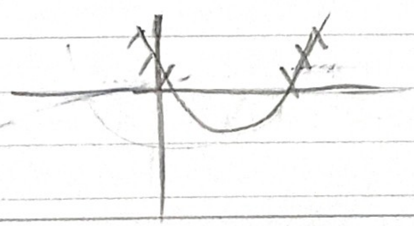
$$\frac{-4k^2 + 24k - 16 < 0}{-4} \quad \overline{-4}.$$

$k^2 - 6k + 4 > 0$ as req.

(b) solve sketch range.

$$\frac{6 \pm \sqrt{6^2 - 4(4)}}{2}$$

$k = 3 \pm \sqrt{5}$



$k < 3 - \sqrt{5}$

OR

$k > 3 + \sqrt{5}$

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9. A cyclist aims to travel a total of 1200 km over a number of days.
He cycles 12 km on day 1
He increases the distance that he cycles each day by 6% of the distance cycled on the previous day, until he reaches the total of 1200 km.

(a) Show that on day 8 he cycles approximately 18 km.
(3)

He reaches his total of 1200 km on day N, where N is a positive integer.

(b) Find the value of N.
(4)

The cyclist stops when he reaches 1200 km.

(c) Find the distance that he cycles on day N. Give your answer to the nearest km.
(2)

$a = 12.$

$ar = 12.72 \quad \therefore \; \underline{r = 1.06}$

ar^{n-1}

$= 12 \times 1.06^{7} = 18.04$

$\approx \underline{18\,km}$

(b) $\dfrac{a(1-r^n)}{1-r}$

$\dfrac{12(1-1.06^{n})}{1-1.06} = 1200.$

$1 - 1.06^{n} = -6.$

$7 = 1.06^{n}$

$n = \dfrac{\log(7)}{\log(1.06)} = 33.39$

$\therefore \; \underline{\underline{N = 34}}$

(c) $1200 - \dfrac{12(1.06^{34-1} - 1)}{1.06 - 1}$

$= \underline{\underline{32km}}$

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10.

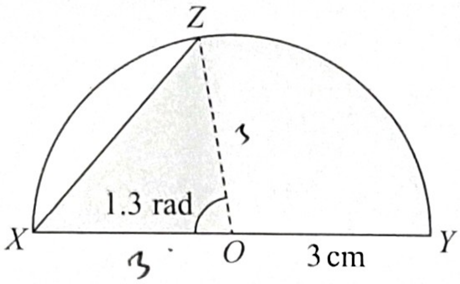

Diagram not drawn to scale

Figure 1

Figure 1 shows a semicircle with centre O and radius 3 cm. XY is the diameter of this semicircle. The point Z is on the circumference such that angle $XOZ = 1.3$ radians. The shaded region enclosed by the chord XZ, the arc ZY and the diameter XY is a template for a badge.

Find, giving each answer to 3 significant figures,

(a) the length of the chord XZ, **(2)**

(b) the perimeter of the template $XZYX$, **(4)**

(c) the area of the template. **(4)**

a) cosine rule

$a^2 = 3^2 + 3^2 - (2\times3\times3\cos(1\cdot3))$

$a = 3\cdot63$ cm

(b) Arc length ZY.

$l = r\theta$

$= 3\times(\pi - 1\cdot3)$

$= 5\cdot52$

$3\cdot63 + 5\cdot52 + 6$

$= 15\cdot2$ cm

(c) Area of triangle.

$\frac{1}{2}\times3\times3\sin(1\cdot3)$

$= \cancel{4\cdot19}\ 4\cdot34$

Area of sector

$\frac{1}{2}\times3^2\times(\pi-1\cdot3)$

$= 8\cdot29$

$= 4\cdot34 + 8\cdot29$

$= 12\cdot6$ cm^2

Leave blank

11. The curve C has equation $y = \mathrm{f}(x)$, $x > 0$, where

$$\mathrm{f}'(x) = \frac{5x^2 + 4}{2\sqrt{x}} - 5$$

It is given that the point $P(4, 14)$ lies on C.

(a) Find $\mathrm{f}(x)$, writing each term in a simplified form.

(6)

(b) Find the equation of the tangent to C at the point P, giving your answer in the form $y = mx + c$, where m and c are constants.

(4)

(a) $\frac{5x^2}{2\sqrt{x}} + \frac{4}{2\sqrt{x}} - 5$

$= \frac{5}{2}x^{3/2} + 2x^{-1/2} - 5.$

Integrate.

$\int \frac{5}{2}x^{3/2} + 2x^{-1/2} - 5 \, dx.$

$\frac{\frac{5}{2}x^{5/2}}{5/2} + \frac{2x^{1/2}}{1/2} - 5x + c.$

$f(x) = x^{5/2} + 4x^{1/2} - 5x + c$

When $x = 4$ $y = 14$.

$14 = (4)^{5/2} + 4(4)^{1/2} - 5(4) + c$

$c = -6.$

$\therefore f(x) = x^{5/2} + 4x^{1/2} - 5x - 6.$

(b) $f'(4)$

$= \frac{5(4)^2 + 4}{2\sqrt{4}} - 5$

$= \frac{84}{4} - 5 = \underline{\underline{16}}$

$y = 16x + c$ $\quad (4, 14)$

$14 = 64 + c$

$c = -50.$

$y = 16x - 50$

Leave blank

12. [*In this question solutions based entirely on graphical or numerical methods are not acceptable.*]

(i) Solve for $0 \leqslant x < 360°$,

$$5\sin(x + 65°) + 2 = 0$$

giving your answers in degrees to one decimal place.

(4)

(ii) Find, for $0 \leqslant \theta < 2\pi$, all the solutions of

$$12\sin^2\theta + \cos\theta = 6$$

giving your answers in radians to 3 significant figures.

(6)

(i) $\sin(x+65) = \frac{-2}{5}$

$x+65 = \sin^{-1}\left(-\frac{2}{5}\right)$

$x+65 = 203.6, 336.4,$ ~~23.6~~

$x = 138.6°, 271.4°$

ii) $12(1-\cos^2\theta)+\cos\theta = 6$

$12-12\cos^2\theta+\cos\theta = 6$

$12\cos^2\theta - \cos\theta - 6 = 0$

$\frac{1 \pm \sqrt{1^2 - 4(12\times -6)}}{2\times 12}$

$\cos\theta = -2/3$ or $3/4$

When $\cos\theta = -2/3$

$\theta = 2.30, 3.98$

when $\cos\theta = 3/4$

$\theta = 0.723, 5.56$

Leave blank

13. The point $A(9, -13)$ lies on a circle C with centre the origin and radius r.

(a) Find the exact value of r. **(2)**

(b) Find an equation of the circle C. **(1)**

A straight line through point A has equation $2y + 3x = k$, where k is a constant.

(c) Find the value of k. **(1)**

This straight line cuts the circle again at the point B.

(d) Find the exact coordinates of point B. **(6)**

(a) $\cancel{(x-9)^2+(y+13)^2=}$

$x^2+y^2=r^2$.

~~tot~~ length OA.

$\sqrt{(9-0)^2+(-13-0)^2}$

$=5\sqrt{10}$

$\therefore\ r=5\sqrt{10}$

(b) $x^2+y^2=250$.

(c) $2(-13)+3(9)=k$.

$k=1$

$2y=-3x+1$

$y=-\frac{3}{2}x+\frac{1}{2}$.

$x^2+\left[\frac{1}{2}(-3x+1)\right]^2=250$.

$x^2+\frac{1}{4}(-3x+1)^2=250$

$x^2+\frac{1}{4}(9x^2-6x+1)=250$.

$4x^2+9x^2-6x+1=1000$.

$13x^2-6x-999=0$.

$\frac{6\pm\sqrt{6^2-4(13\times-999)}}{2\times13}$

$x=\frac{-111}{13}$ or 9.

$x=\frac{-111}{13}$ $y=\frac{173}{13}$

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14.

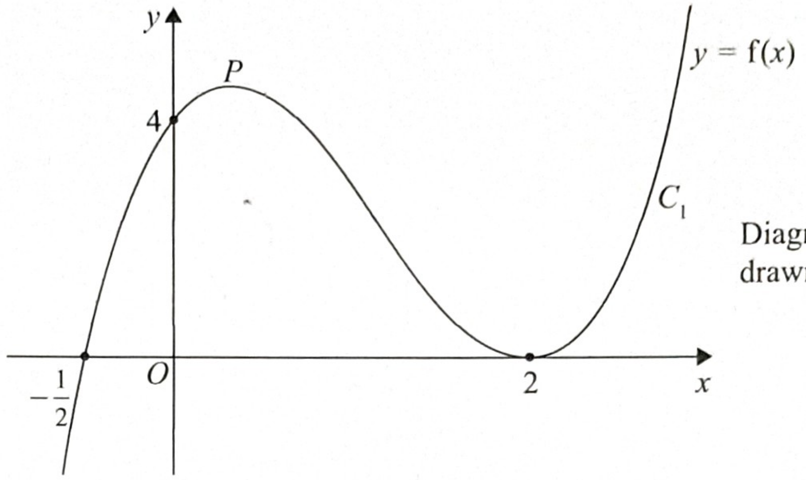


Figure 2

Figure 2 shows a sketch of the curve C_1 with equation $y = f(x)$ where

$$f(x) = (x-2)^2(2x+1), \qquad x \in \mathbb{R}$$

The curve crosses the x-axis at $\left(-\frac{1}{2}, 0\right)$, touches it at (2, 0) and crosses the y-axis at (0, 4). There is a maximum turning point at the point marked P.

(a) Use $f'(x)$ to find the exact coordinates of the turning point P.

(7)

A second curve C_2 has equation $y = f(x+1)$.

(b) Write down an equation of the curve C_2
You may leave your equation in a factorised form.

(1)

(c) Use your answer to part (b) to find the coordinates of the point where the curve C_2 meets the y-axis.

(2)

(d) Write down the coordinates of the two turning points on the curve C_2

(2)

(e) Sketch the curve C_2, with equation $y = f(x+1)$, giving the coordinates of the points where the curve crosses or touches the x-axis.

(3)

$f(x) = (x-2)^2(2x+1)$

Using chain rule.

$2(x-2)(2x+1) + (x-2)^2 \cdot 2$

$f'(x) = 0.$

$2(x-2)(2x+1) + 2(x-2)^2 = 0.$

$(x-2)\left[2(2x+1) + 2(x-2)\right] = 0$

$\underline{\underline{x = 2}}$ (not required point from diagram).

Leave blank

Question 14 continued

$4x+2+2x-4=0.$

$6x-2=0.$

$x=\frac{1}{3}.$

$f(1/3)=\left(\frac{1}{3}-2\right)^2\left(2\left(\frac{1}{3}\right)+1\right)$

$=\frac{125}{27}$

$\therefore P \Rightarrow \left(\frac{1}{3},\frac{125}{27}\right)$

(b) $(x-1)^2(2x+3)$

(c) y int $x=0.$

$(-1)^2(3)$

$=3 \quad (0,3).$

(d) $\frac{dy}{dx}.$

$2(x-1)(2x+3)+2(x-1)^2$

$=(x-1)\left[2(2x+3)+2(x-1)\right]=0$

$x=1 \quad y=0$

or

$4x+6+2x-2=0.$

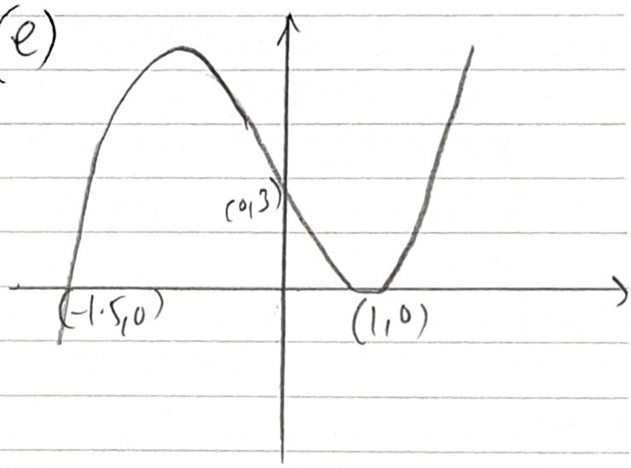
$6x+4=0$

$\cancel{x=1/2} \quad x=-2/3.$

$y=\frac{125}{27}.$

$(1,0) \quad \left(-2/3,\frac{125}{27}\right).$

(e)



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15.

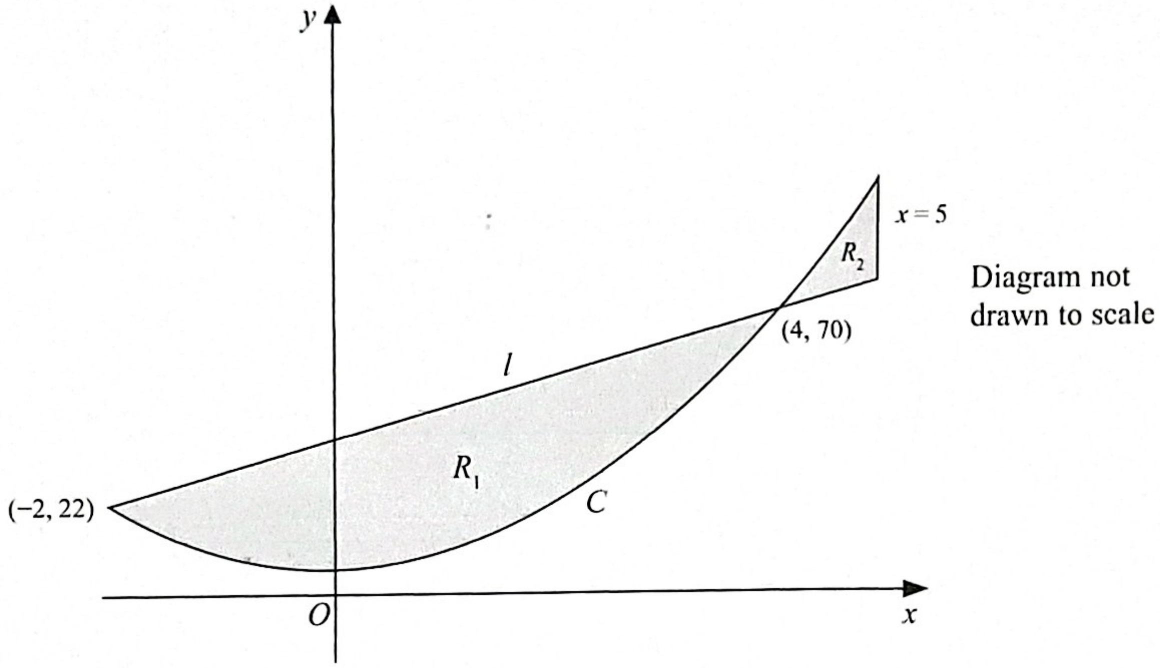

Figure 3

A design for a logo consists of two finite regions R_1 and R_2, shown shaded in Figure 3.

The region R_1 is bounded by the straight line l and the curve C.

The region R_2 is bounded by the straight line l, the curve C and the line with equation $x = 5$

The line l has equation $y = 8x + 38$

The curve C has equation $y = 4x^2 + 6$

Given that the line l meets the curve C at the points (–2, 22) and (4, 70), use integration to find

(a) the area of the larger lower region, labelled R_1 **(6)**

(b) the exact value of the total area of the two shaded regions. **(3)**

Given that

$$\frac{\text{Area of } R_1}{\text{Area of } R_2} = k$$

(c) find the value of k. **(1)**

Leave blank

Question 15 continued

(a) Area Under Curve:

$\int 4x^2+6.$

$\left[\frac{4x^3}{3}+6x\right]_{-2}^{4}$

$\left[\frac{4(3)^3}{3}+6(4)\right]-\left[\frac{4(-2)^3}{3}+6(-2)\right]$

$= \underline{132}$

Area under line.

$\frac{1}{2}\times(2 \cancel{+} 4)\times(22+70)$

$= 276.$

$276-132$

$= \underline{\underline{144}}$

(b) Area of R2

$\left[\frac{4x^3}{3}+6x\right]_{4}^{5}$

$\left[\frac{4(5)^3}{3}+6(5)\right]-\left[\frac{4(4)^3}{3}+6(4)\right]$

$\frac{262}{3}.$

Area under line.

$\frac{(5-4)(70+78)}{2}$

$= 74.$

$\frac{262}{3} - 74$

$= \underline{\underline{\frac{40}{3}}}$

$\therefore \overset{144}{\cancel{132}} + \frac{40}{3} = \cancel{\frac{436}{3}}\ \frac{472}{3}$

(c) $\underline{\underline{10.8}}$