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1. The first three terms in ascending powers of x in the binomial expansion of $(1 + px)^8$ are given by

$$1 + 12x + qx^2$$

where p and q are constants.

Find the value of p and the value of q.

$$P = 12 \Rightarrow P = \frac{3}{2}$$

2

$$28p^2 = q \Rightarrow q = 63$$

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2. Find the range of values of x for which

(a)
$$4(x-2) \le 2x+1$$

(2)

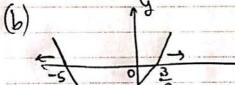
(b)
$$(2x-3)(x+5) > 0$$

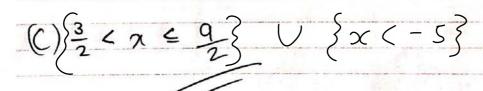
(3)

(c) both
$$4(x-2) \le 2x+1$$
 and $(2x-3)(x+5) > 0$

(1)









3. Answer this question without a calculator, showing all your working and giving your answers in their simplest form.

(i) Solve the equation

$$4^{2x+1} = 8^{4x}$$

(3)

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(ii) (a) Express

$$3\sqrt{18} - \sqrt{32}$$

in the form $k\sqrt{2}$, where k is an integer.

(2)

(b) Hence, or otherwise, solve

$$3\sqrt{18} - \sqrt{32} = \sqrt{n}$$

(2)

$$\therefore \left(2^2\right)^{2n+1} = \left(2^3\right)^{4n}$$

$$= 952 - 452$$

$$= 552 \quad K = 5$$

(c)
$$5\Gamma_2 = \Gamma_0$$

 $(5\Gamma_2)^2 = (5n)^2 = 7$ $n = 50$

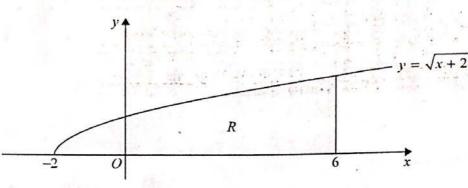


Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \sqrt{x+2}$, $x \ge -2$

The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis and the line x = 6

The table below shows corresponding values of x and y for $y = \sqrt{x+2}$

x	-2	0	2	4	6
у	0	1.4142	2	2.4495	2.8284

- (a) Complete the table above, giving the missing value of y to 4 decimal places.
- (1)
- (b) Use the trapezium rule, with all of the values of y in the completed table, to find an approximate value for the area of R, giving your answer to 3 decimal places.

(3)

Use your answer to part (b) to find approximate values of

(c) (i)
$$\int_{-2}^{6} \frac{\sqrt{x+2}}{2} \, dx$$

(ii)
$$\int_{-2}^{6} (2 + \sqrt{x+2}) dx$$

(4)

(b) Area R
$$\propto$$
 ($\frac{1}{3}$ ($\frac{1}{3}$)[0+2(1.4142+2+2.4495)]
= 14.556 (34)

4.

Question 4 continued

Question 4 continued

(C)(i)
$$\int_{-2}^{6} \frac{\sqrt{n+2}}{2} dn = \frac{1}{2} \int_{2}^{6} \sqrt{n+2} dx \approx \frac{1}{2} \times 14.556$$

$$= 7.278$$

(ii)
$$\int_{2}^{6} t \sqrt{n+2} \, dn = \int_{2}^{6} 2 \, dn + \int_{2}^{6} \sqrt{n+2} \, dn$$

$$\approx [2n]_{-2} + 14.556$$

$$= 16 + 14 - 556 = 30.556$$

$$U_{n+1} = \frac{U_n}{U_n - 3}, \quad n \geqslant 1$$

Given $U_1 = 4$, find

(a) U,

(1)

(b)
$$\sum_{n=1}^{100} U_n$$

(2)

(ii) Given

$$\sum_{r=1}^{n} (100 - 3r) < 0$$

find the least value of the positive integer n.

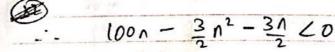
(3)

(b)
$$U_1 = U_2 = U_3 = U_4 = U_5 = ... = U_n = 4$$

$$\int_{\Lambda=1}^{100} U_{\Lambda} = \frac{100}{\Lambda=1} = \frac{400}{100}$$

$$\binom{11}{11} = \frac{1}{100-30} = \frac{1}{100} = \frac$$

$$= (000 - \frac{30}{2}(0+1))$$



$$C-V$$
 $n = \frac{197}{3} = 65-66...$

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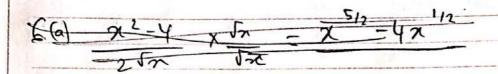
- 6. (a) Show that $\frac{x^2-4}{2\sqrt{x}}$ can be written in the form $Ax^p + Bx^q$, where A, B, p and q are constants to be determined.
- (3)

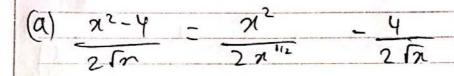
(b) Hence find

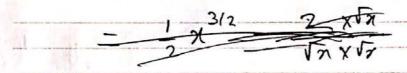
$$\int \frac{x^2 - 4}{2\sqrt{x}} dx, \quad x > 0$$

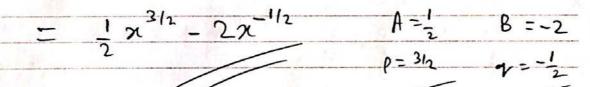
giving your answer in its simplest form.

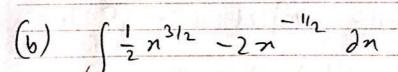
(4)











$$=\frac{1}{5}x - 4x + C$$

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7.

$$f(x) = 3x^3 + ax^2 + bx - 10$$
, where a and b are constants.

Given that (x-2) is a factor of f(x),

(a) use the factor theorem to show that
$$2a + b = -7$$

Given also that when f(x) is divided by (x + 1) the remainder is -36

f(x) can be written in the form

$$f(x) = (x - 2)Q(x)$$
, where $Q(x)$ is a quadratic function.

- (c) (i) Find Q(x).
 - (ii) Prove that the equation f(x) = 0 has only one real root.

You must justify your answer and show all your working.

(4)

(4)

(a) (n-2) as a factor implies n=2 is a root of f(n)=0

$$f(2) = 3(2)^3 + a(2)^2 + 2b - 10 = 0$$

$$f(2) = 24 + 4a + 2b - 10$$

(b)
$$f(-1) = -36$$

 $f(-1) =$
=) -3 + a -6 -10 = -36
 $a = b = -22$

$$a = -10$$

(c)(i)

$$3n^3 - 10n^2 + 13n - 10 = (n-2)[3x^2 + Kn + 5]$$

$$3n^{3}-10n^{2}+13n=3n^{3}+kn^{2}+5n-6n^{2}-2kn-10$$

$$=3n^{3}+(k-6)n^{2}+(5-2k)n-10$$

1 (a) 8

$$=) K = -10$$

$$=) K = -4'$$

$$-)$$
 $\chi^{-2} = 0$: $\chi = 2$

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8. In this question the angle θ is measured in degrees throughout.

(a) Show that the equation

$$\frac{5+\sin\theta}{3\cos\theta}=2\cos\theta,\qquad \theta\neq(2n+1)90^{\circ},\quad n\in\mathbb{Z}$$

may be rewritten as

$$6\sin^2\theta + \sin\theta - 1 = 0 \tag{3}$$

(b) Hence solve, for $-90^{\circ} < \theta < 90^{\circ}$, the equation

$$\frac{5+\sin\theta}{3\cos\theta}=2\cos\theta$$

Give your answers to one decimal place, where appropriate.

 $8(a). \quad 5+\sin\theta = 2\cos\theta$

(6) (3sino-1)(2sino+1) = 0

$$SINO = -\frac{1}{2} + \frac{S}{C} = -\frac{A}{30}$$



PMT

9. The first term of a geometric series is 6 and the common ratio is 0.92

For this series, find

- (a) (i) the 25th term, giving your answer to 2 significant figures,
 - (ii) the sum to infinity.

(4)

The sum to n terms of this series is greater than 72

(b) Calculate the smallest possible value of n.

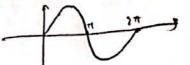
(4)

9 (a) (A)

$$u_{25} = 6 \times 0.92^2$$

(11)
$$S_{0} = \frac{6}{1-6.92}$$

- 10. The curve C has equation $y = \sin\left(x + \frac{\pi}{4}\right)$, $0 \leqslant x \leqslant 2\pi$
 - (a) On the axes below, sketch the curve C.



- (b) Write down the exact coordinates of all the points at which the curve C meets or intersects the x-axis and the y-axis.
- (3)

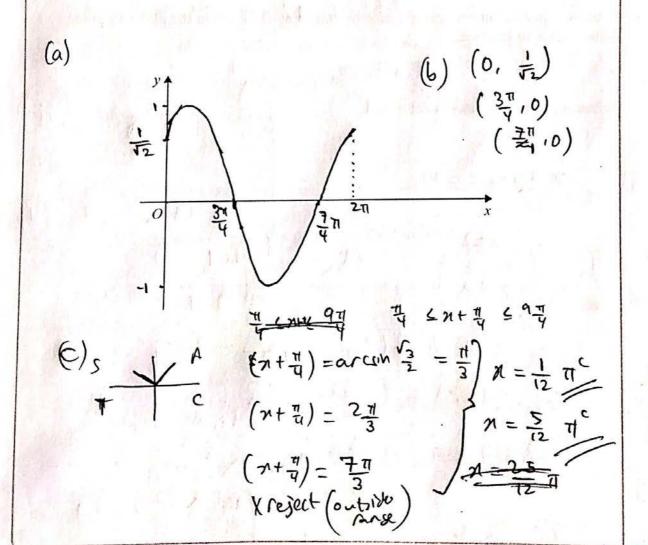
(2)

(c) Solve, for $0 \le x \le 2\pi$, the equation

$$\sin\left(x + \frac{\pi}{4}\right) = \frac{\sqrt{3}}{2}$$

giving your answers in the form $k\pi$, where k is a rational number.

(4)



11.

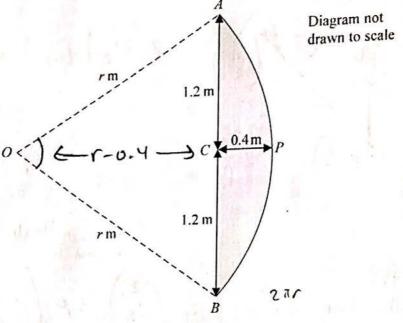


Figure 2

Figure 2 shows the design for a sail APBCA.

The curved edge APB of the sail is an arc of a circle centre O and radius r m.

The straight edge ACB is a chord of the circle.

The height AB of the sail is 2.4 m.

The maximum width CP of the sail is 0.4 m.

(a) Show that r = 2

(2)

(b) Show, to 4 decimal places, that angle AOB = 1.2870 radians.

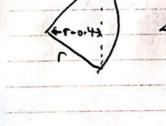
(2)

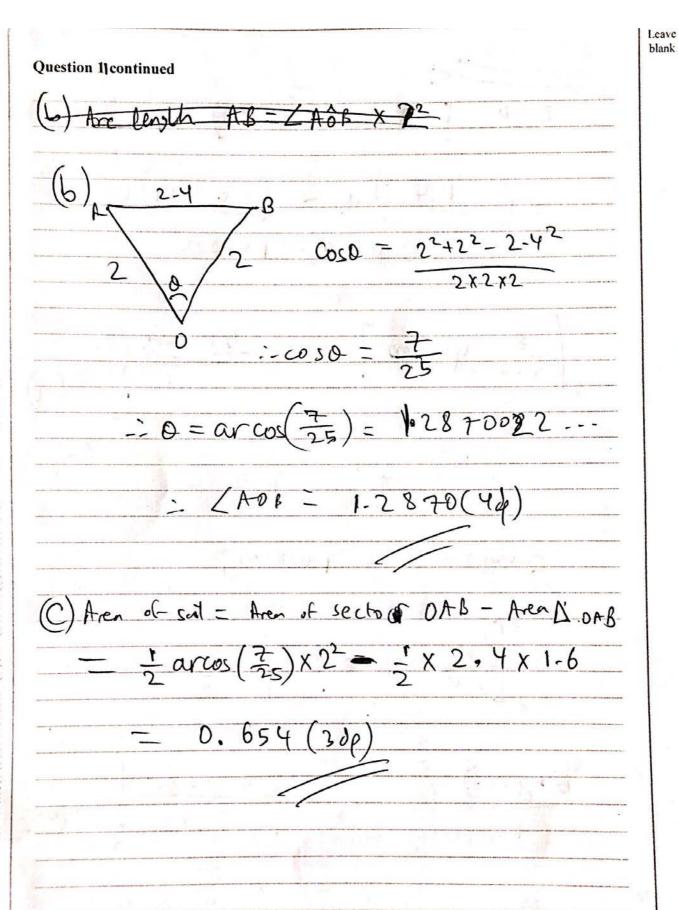
(c) Hence calculate the area of the sail, giving your answer, in m², to 3 decimal places.

(a) (1-2=) (1-0-4)2+1.22=12 (1-0-4)=1.6=12

= 0.8r=1.6

=) r= 2 as required





PMT

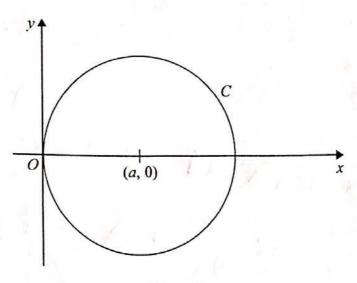


Figure 3

Figure 3 shows a circle C

C touches the y-axis and has centre at the point (a, 0) where a is a positive constant.

(a) Write down an equation for C in terms of a

(2)

Given that the point P(4, -3) lies on C,

(b) find the value of a

(3)

(b)
$$(4-a)^2 + 9 = a^2$$

13. (a) Show that the equation

$$2\log_2 y = 5 - \log_2 x$$
 $x > 0, y > 0$

may be written in the form $y^2 = \frac{k}{x}$ where k is a constant to be found.

(3)

Leave blank

(b) Hence, or otherwise, solve the simultaneous equations

$$2\log_2 y = 5 - \log_2 x$$

$$\log_x y = -3$$

for
$$x > 0, y > 0$$

(5)

$$: (\log_2(9^2\pi) = 5\log_2 2 = \log_2 2^5$$

$$=) y^2 n = 32$$

$$y^2 = \frac{32}{2}$$
 $k = 32$

7/

(L)
$$\log_n y = -3\log_n x = \log_n x^{-3} \Rightarrow y = \frac{1}{n^3}$$

 $= \frac{1}{n^3} = \frac{32}{n}$

$$y = \frac{1}{\pi^2} + 4 + y^2 = \frac{32}{\pi}$$

$$= \left(\frac{1}{n^3}\right)^2 = \frac{32}{n} = \frac{32}{n^6} = \frac{32}{n}$$

$$1 = 32 \pi^5 = \frac{1}{32}$$

$$= x = \frac{1}{2} = y = \frac{1}{(\frac{1}{2})^3}$$

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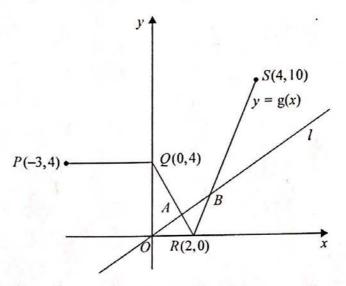


Figure 4

Figure 4 shows a sketch of the graph of y = g(x), $-3 \le x \le 4$ and part of the line l with equation $y = \frac{1}{2}x$

The graph of y = g(x) consists of three line segments, from P(-3,4) to Q(0,4), from Q(0,4) to R(2,0) and from R(2,0) to S(4,10).

The line *l* intersects y = g(x) at the points *A* and *B* as shown in Figure 4.

(a) Use algebra to find the x coordinate of the point A and the x coordinate of the point B.

Show each step of your working and give your answers as exact fractions.

(6)

(b) Sketch the graph with equation

$$y = \frac{3}{2}g(x), \quad -3 \leqslant x \leqslant 4$$

On your sketch show the coordinates of the points to which P, Q, R and S are transformed.

(2)

$$y = \frac{1}{2}\pi = \frac{1}{2}\pi = \frac{8}{5}$$

Consider SK: 81	adiat-5	
-= y = 5x =	+ 'k	
(2,0)=) 0=	(0+K =) K=-10	
y = 5n - 1	0	
タニシャラ	えれー5れー10	
	$\frac{9}{2}N = 10$	
	$n = \frac{20}{9}$	
· ·		
x _A = 8	$a_8 = \frac{20}{9}$	
)	9 9 (3	, S'(4, 15)
V ₂	16 2 2 3 1 1 1	
	0,(0,9)	
D ¹ (2 ()		
P'(-3,6).		
P'(-3,6).		
P'(-3,6).	£'(2,0)	

15.

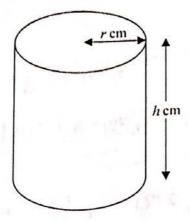


Figure 5

Figure 5 shows a design for a water barrel.

It is in the shape of a right circular cylinder with height h cm and radius r cm.

The barrel has a base but has no lid, is open at the top and is made of material of negligible thickness.

The barrel is designed to hold 60 000 cm³ of water when full.

(a) Show that the total external surface area, $S \text{ cm}^2$, of the barrel is given by the formula

$$S = \pi r^2 + \frac{120\,000}{r}$$

(3)

(b) Use calculus to find the minimum value of S, giving your answer to 3 significant figures.

(6)

(c) Justify that the value of S you found in part (b) is a minimum.

(a)
$$V = \pi r^2 h = 60000 =) h = \frac{60000}{\pi r^2}$$

$$S = \frac{1}{2\pi r^2 + 2\pi r} = \frac{1}{2\pi r^2 + 2\pi r} = \frac{1}{2\pi r^2 + 2\pi r} = \frac{60000}{\pi r^2}$$

Leav	e
	Leav

Question15 continued

(b)
$$\frac{\partial S}{\partial r} = 2\pi r - 120000 r^{-2}$$

$$\frac{\partial s}{\partial r} = 0 \Rightarrow \frac{2\pi r}{r} \left(-1\right)$$

$$: S = \pi (26.73...)^2 + \frac{120000}{26.73...}$$

$$\frac{(c)}{\partial r^2} = 2\pi + 240000 r^{-3}$$

$$\left(\frac{1^2S}{3c^2}\right)_{r=26.73...} = 2\pi + 240000 (26.73...)^3$$

$$\frac{1}{2} \left(\frac{\partial L_s}{\partial r^2} \right)_{26-73} > 0 = \frac{s}{6730} = \frac{s}{6730} = \frac{s}{6730}$$

(Total 5 marks)

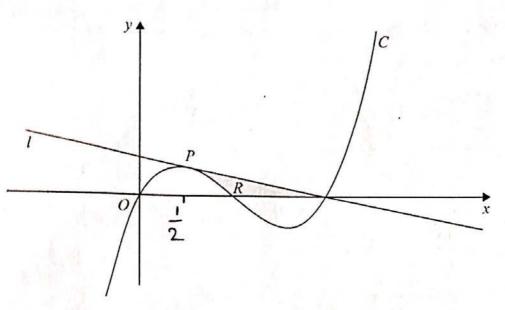


Figure 6

Figure 6 shows a sketch of part of the curve C with equation

$$y = x(x-1)(x-2)$$

The point P lies on C and has x coordinate $\frac{1}{2}$

The line l, as shown on Figure 6, is the tangent to C at P.

(a) Find $\frac{dy}{dx}$

(2)

(b) Use part (a) to find an equation for l in the form ax + by = c, where a, b and c are integers.

(4)

(6)

The finite region R, shown shaded in Figure 6, is bounded by the line l, the curve C and the x-axis.

The line I meets the curve again at the point (2, 0)

(c) Use integration to find the exact area of the shaded region R.

6).
$$y = n^3 - 3n^2 + 2n$$

 $\frac{\partial y}{\partial n} = 3x^2 - 6x + 2$

Question Continued

$$= -\frac{1}{4}(x - \frac{1}{2})$$

$$= 8y - 3 = -2n + 1$$

$$3 + 4y = 27$$
 $5 = 4$
 $6 = 2$

Ara
$$R = \sum_{n=1}^{\infty} -\int x^3 - 3n^2 + 2n \, dn$$

$$-\frac{1}{2} \times \frac{3}{2} \times \frac{3}{8} - \left[\frac{1}{4} \times - \chi^{3} + \chi^{2} \right]_{1/2}$$

$$=\frac{9}{3^2}-\left[\left(\frac{1}{4}\right)-\left(\frac{9}{64}\right)\right]$$

$$=\frac{9}{32}$$
 $\frac{7}{64}$ $=\frac{11}{64}$

Q#6

(Total(Zmarks)