

(12 Jan 2017 (MA))

Q1a)  $y = \frac{x^3}{3} - 2x^2 + 3x + 5$

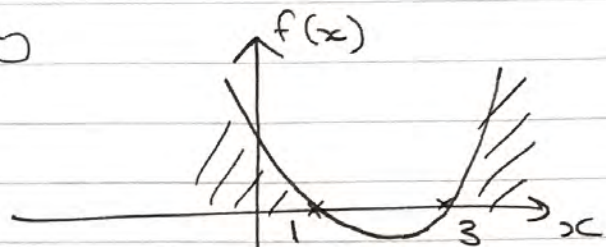
$$\frac{dy}{dx} = \boxed{x^2 - 4x + 3}$$

b)  $x^2 - 4x + 3 > 0$

Solving  $[x^2 - 4x + 3 = 0]$  to find critical values:

$$\Rightarrow (x-3)(x-1) = 0$$

$$\therefore \underline{x=3, x=1}$$



$\therefore$  set of values:

$$\boxed{x > 3 \text{ and } x < 1}$$

we want  
where  $\frac{dy}{dx} > 0$   
(ie where  $y > 0$  on  
the graph)

2a)  $x^2 + y^2 - 8x + 4y - 12 = 0$

$$(x-4)^2 - 16 + (y+2)^2 - 4 - 12 = 0$$

$$\therefore (x-4)^2 + (y+2)^2 = 32$$

$$\therefore C \boxed{(4, -2)}$$

b)  $r = \sqrt{32} = \boxed{4\sqrt{2}}$

b) at y-axis,  $x=0$  :  $(0-4)^2 + (y+2)^2 = 32$

$$(y+2)^2 = 16$$

$$y+2 = \pm\sqrt{16}$$

$$y+2 = \pm 4$$

$$\therefore y = -2 \pm 4$$

so  $y = -6$  and  $y = 2$

$\therefore A(0, 2)$  and  $B(0, -6)$

(It doesn't matter which point is labelled  
A or B)

3a)  $l = r\theta = 7 \times 0.8 = \boxed{5.6 \text{ cm}}$

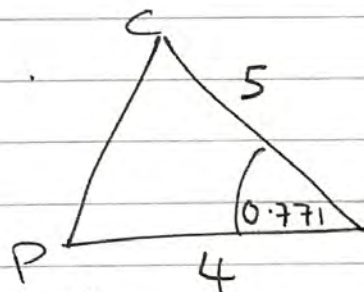
b)  $\angle AOC$  will be  $90^\circ$  as OABC is a rectangle.  
( $\frac{\pi}{2}$ )

$$\therefore \angle POC = 180^\circ - 90^\circ - \angle AOO$$

in radians:  $\angle POC = \pi - \frac{\pi}{2} - 0.8 = \boxed{0.771^\circ}$

c) We require the length CP.

cosine rule



$$CP^2 = 5^2 + 4^2 - 2(5)(4)\cos(0.771)$$

$$CP^2 = 12.311 \dots \therefore CP = 3.5088 \dots$$

$$\therefore \text{Perimeter} = 3.5088 + 4 + 7 + 5.6 + 5 + 7 = \boxed{32.11 \text{ cm}}$$

$$4a) \quad a + (a+d) + (a+2d) + \dots$$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$S_9 = \frac{9}{2} [2a + (8)d] = 54$$

$$12 = 2a + 8d$$

$$\div 2 : \underline{6 = a + 4d}$$

$$b) \quad 8^{\text{th}} \text{ term} = a + 7d$$

$$7^{\text{th}} \text{ term} = a + 6d.$$

8<sup>th</sup> term is half the 7<sup>th</sup> term

$$\therefore a + 7d = \frac{1}{2}(a + 6d).$$

$$2a + 14d = a + 6d$$

$$a = -8d \quad \text{--- (2)}$$

from (a),  $a = 6 - 4d$

$$\hookrightarrow \text{(2)} : 6 - 4d = -8d$$

$$-4d = -6$$

$$\boxed{d = -\frac{3}{2}}$$

$$\therefore a = -8\left(-\frac{3}{2}\right)$$

$$\boxed{a = 12}$$

$$5ai) y = \log_3 x$$

$$\log_3\left(\frac{x}{9}\right) = \log_3(x) - \log_3(9)$$
$$= \boxed{y - 2}$$

$$ii) \log_3 \sqrt{x} = \log_3(x^{\frac{1}{2}}) = \frac{1}{2} \log_3 x$$
$$= \boxed{\frac{1}{2} y}$$

$$b) 2(y - 2) - \left(\frac{1}{2} y\right) = 2$$

$$2y - 4 - \frac{y}{2} = 2$$

$$\frac{3y}{2} = 6 \quad \therefore y = \frac{6}{\frac{3}{2}} = \boxed{4}$$

substitute back into  $[y = \log_3 x]$  to find  $x$

$$4 = \log_3 x$$

$$3^4 = \boxed{x = 81}$$

Remember we are  
solving for  $x$   
not  $y$

6ai)  $2y = 3x + 5$

$$y = \frac{3}{2}x + \frac{5}{2}$$

$$\therefore \text{gradient } l_1 = \boxed{\frac{3}{2}}$$

ii)  $y=0: 3x+5=0 \quad \therefore \boxed{x = -\frac{5}{3}}$

b)  $l_2$  is perp. to  $l_1 \quad \therefore m_{l_2} = -\frac{1}{\frac{3}{2}} = -\frac{2}{3} //$

put  $x=1$  into  $l_1$  to find  $y$ :  $2y=8 \quad \therefore y=4 //$

so  $l_2$  passes through  $(1, 4)$ .

$$\Rightarrow y - 4 = -\frac{2}{3}(x - 1)$$

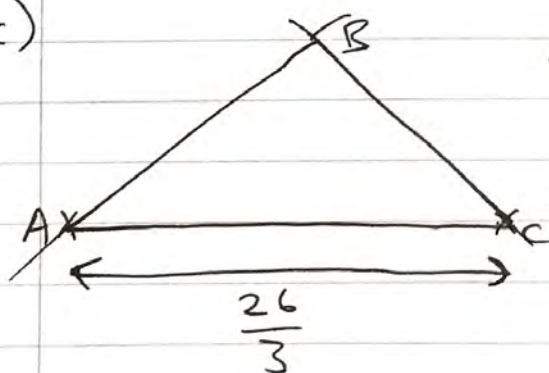
$$y = -\frac{2}{3}x + \frac{2}{3} + 4$$

$$y = -\frac{2}{3}x + \frac{14}{3}$$

(x3)  $3y = -2x + 14$

$$\boxed{2x + 3y - 14 = 0}$$

c)



$$\begin{aligned} y=0: x=7 // \\ \therefore C(7, 0) \\ \text{so } AC = 7 + \frac{5}{3} = \frac{26}{3} \end{aligned}$$

$$\begin{aligned} \text{Area} &= \frac{1}{2} \times \frac{26}{3} \times 4 \\ &= \boxed{\frac{52}{3}} \end{aligned}$$

$$7 \text{ i) } \int \left( \frac{2 + 4x^3}{x^2} \right) dx = \int [2x^{-2} + 4x] dx$$

$$= \left[ \frac{2x^{-1}}{-1} + \frac{4x^2}{2} \right] + C$$

$$= \boxed{-\frac{2}{x} + 2x^2 + C}$$

$$\text{ii) } \int_2^4 [4x^{-\frac{1}{2}} + k] dx = 30$$

$$\Rightarrow \left[ \frac{4x^{\frac{1}{2}}}{\frac{1}{2}} + kx \right]_2^4 = 30$$

$$\Rightarrow [8\sqrt{x} + kx]_2^4 = 30$$

$$\Rightarrow [16 + 4k] - [8\sqrt{2} + 2k] = 30$$

$$\Rightarrow 16 - 8\sqrt{2} + 2k = 30$$

$$\Rightarrow 2k = 14 + 8\sqrt{2}$$

$$\therefore \boxed{k = 7 + 4\sqrt{2}}$$

8a)  $f(x) = 2x^3 - 5x^2 - 23x - 10$

Long Division

$$\begin{array}{r}
 2x^2 + x - 20 \\
 x-3 \overline{) 2x^3 - 5x^2 - 23x - 10} \\
 \underline{2x^3 - 6x^2} \phantom{- 10} \\
 0 \phantom{0} x^2 - 23x \phantom{- 10} \\
 \phantom{0} \underline{x^2 - 3x} \phantom{- 10} \\
 \phantom{0} \phantom{0} -20x - 10 \\
 \phantom{0} \phantom{0} \underline{-20x + 60} \\
 \phantom{0} \phantom{0} \phantom{0} -70 \leftarrow \text{remainder} = \boxed{-70}
 \end{array}$$

b) If  $(x+2)$  is a factor then  $f(-2) = 0$ .

$$\begin{aligned}
 f(-2) &= 2(-2)^3 - 5(-2)^2 - 23(-2) - 10 \\
 &= -36 + 46 - 10 = 0
 \end{aligned}$$

$\therefore (x+2)$  is a factor.

c)

$$\begin{array}{r}
 2x^2 - 9x - 5 \\
 x+2 \overline{) 2x^3 - 5x^2 - 23x - 10} \\
 \underline{2x^3 + 4x^2} \phantom{- 10} \\
 0 \phantom{0} -9x^2 - 23x \phantom{- 10} \\
 \phantom{0} \underline{-9x^2 - 18x} \phantom{- 10} \\
 \phantom{0} \phantom{0} -5x - 10 \\
 \phantom{0} \phantom{0} \underline{-5x - 10} \\
 \phantom{0} \phantom{0} \phantom{0} 0 \phantom{0}
 \end{array}$$

$$\therefore f(x) = (x+2)(2x^2 - 9x - 5)$$

$$2x^2 - 9x - 5 = (2x + 1)(x - 5)$$

$$\therefore f(x) = \boxed{(x+2)(2x+1)(x-5)}$$

$$d) 2(3^{3t}) - 5(3^{2t}) - 23(3^t) = 10$$

$$\underline{\text{let } y = 3^t}$$

$$\Rightarrow 2y^3 - 5y^2 - 23y = 10$$

$$\Rightarrow 2y^3 - 5y^2 - 23y - 10 = 0 \quad \left. \begin{array}{l} \text{this is} \\ \text{exactly what} \\ \text{we factorised} \\ \text{in (c).} \end{array} \right\}$$

$$\Rightarrow (y+2)(2y+1)(y-5) = 0 //$$

$$\therefore \underline{y = -2} \rightarrow 3^t = -2 \quad \text{no valid solutions. (REJECT).}$$

$$\underline{2y+1=0} : y = -\frac{1}{2} \rightarrow 3^t = -\frac{1}{2} \quad \text{no valid solutions (REJECT).}$$

$$\underline{y-5=0} : y = 5 \rightarrow 3^t = 5$$

$$3^t = 5$$

$$\log(3^t) = \log(5) = t \log(3)$$

$$\therefore t = \frac{\log 5}{\log 3} = \boxed{1.465}$$



$$9a) f(x) = 8x^{-1} + \frac{1}{2}x - 5$$

$$f'(x) = -8x^{-2} + \frac{1}{2} = 0 \quad \left( \frac{dy}{dx} = 0 \text{ at } A \right)$$

$$\Rightarrow \frac{1}{2} = \frac{8}{x^2}$$

$$\Rightarrow x^2 = 16 \quad \therefore x = 4 \quad (\text{rej. } x = -4)$$

$(x > 0)$

$$f(4) = \frac{8}{4} + 2 - 5 = -1$$

$$\therefore A(4, -1)$$

b)  $x = 2, x = 8$  [no change to  $x$ ]

ii)  $(4, 1)$  [vertical translation of  $+2$ ]

ii)  $x$ -coordinates are multiplied by  $\frac{1}{4}$

$\therefore$  roots:  $x = \frac{1}{2}, x = 2$

$$\bullet 10a) (1+ax)^{20} = \binom{20}{1}(1)^{19}(ax)^1 + \binom{20}{2}(1)^{18}(ax)^2$$

$$= 1 + 20ax + 190a^2x^2 //$$

Compare with given terms to obtain:

$$20a = 4 \quad \text{--- (1)}$$

$$190a^2 = p \quad \text{--- (2)}$$

$$\bullet \text{ from (1) : } a = \frac{4}{20} = \boxed{\frac{1}{5}}$$

$$b) \text{ from (2) : } p = 190a^2 = 190\left(\frac{1}{5}\right)^2 = \boxed{\frac{38}{5}} = p$$

$$c) \begin{array}{l} 4^{\text{th}} \text{ term} \rightarrow x^3 \text{ term.} \\ 5^{\text{th}} \text{ term} \rightarrow x^4 \text{ term.} \end{array}$$

$$5^{\text{th}} \text{ term} = \binom{20}{4}(1)^{16}(ax)^4$$

$$= (4845a^4)x^4 //$$

$$\therefore q = 4845a^4 = 4845\left(\frac{1}{5}\right)^4 = \boxed{\frac{969}{125}}$$

ii)

$$\underline{0 \leq x < 2\pi}$$

$$3\cos^2 x + 1 = 4\sin^2 x$$

$$3 - 3\sin^2 x + 1 = 4\sin^2 x$$

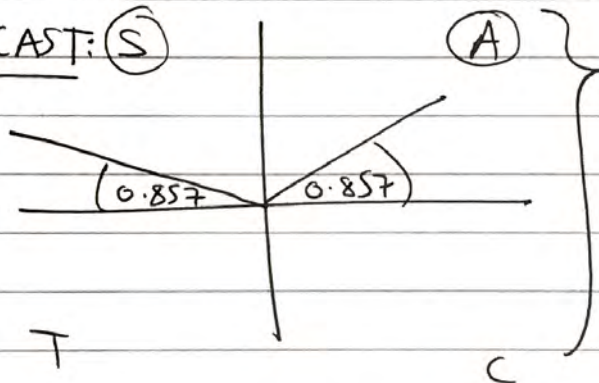
$$7\sin^2 x = 4$$

$$\sin^2 x = \frac{4}{7}$$

$$\therefore \sin x = \pm \frac{2}{\sqrt{7}}$$

$$x = \sin^{-1}\left(\frac{2}{\sqrt{7}}\right) = 0.857^\circ$$

By CAST: (S)

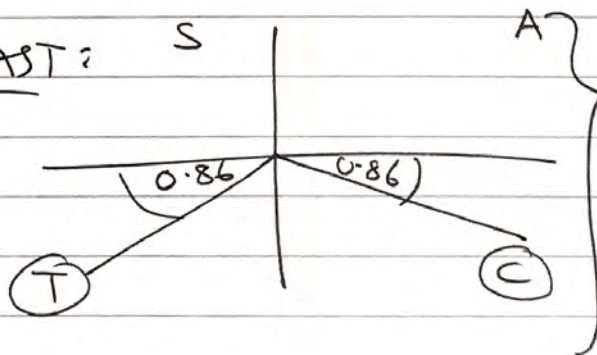


$$x = 0.86, \pi - 0.86$$

$$x = 0.86^\circ, 2.28^\circ$$

and  $x = \sin^{-1}\left(-\frac{2}{\sqrt{7}}\right) = -0.857^\circ$

By CAST: (S)



$$x = \pi + 0.86^\circ,$$

$$2\pi - 0.86^\circ$$

$$x = 4.00^\circ, 5.42^\circ$$

ii)

$$0 \leq \theta < 360$$

$$5 \sin(\theta + 10) = \cos(\theta + 10)$$

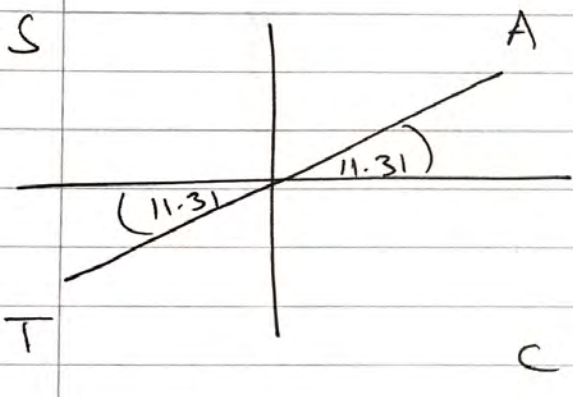
$$\boxed{\div \cos(\theta + 10)} : 5 \tan(\theta + 10) = 1$$

$$\tan(\theta + 10) = \frac{1}{5}$$

$$\theta + 10 = \tan^{-1}\left(\frac{1}{5}\right) = 11.31^\circ$$

new range :  $\boxed{10 \leq \theta + 10 < 370}$

By CAST



$$\theta + 10 = 11.31^\circ, 191.31^\circ$$

$$\therefore \theta = 1.3^\circ, 181.3^\circ$$

$$12a) y = \frac{3}{4}x^2 - 4x^{\frac{1}{2}} + 7 \quad P(4, 11)$$

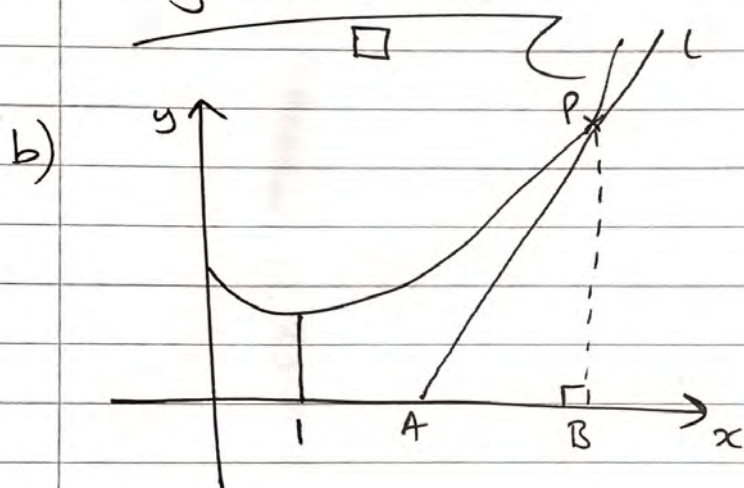
$$\frac{dy}{dx} = \frac{3}{2}x - 2x^{-\frac{1}{2}}$$

$$\text{at } P, \frac{dy}{dx} = \frac{3}{2}(4) - \frac{2}{\sqrt{4}} = 5 //$$

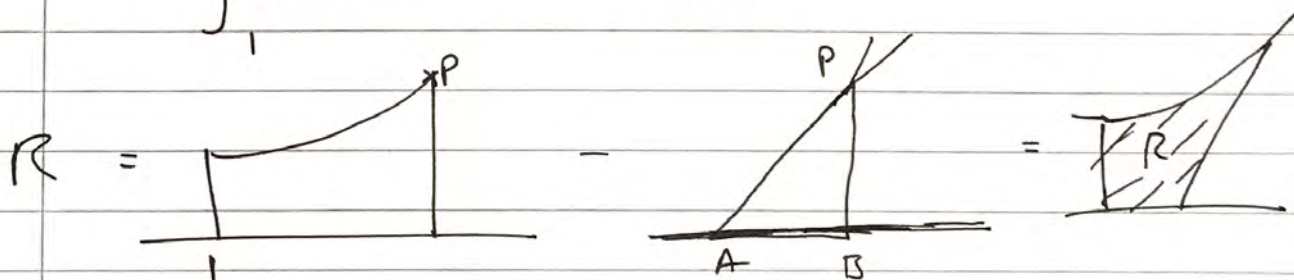
$$\therefore y - 11 = 5(x - 4) \quad \leftarrow \text{tangent line}$$

$$y = 5x - 20 + 11$$

$$y = 5x - 9$$



$$R = \int_1^4 [y] dx - \text{Area}_{ABP}$$



$$\text{Area } \Delta = \frac{1}{2} (AB) (11)$$

to find  
length AB...

put  $y=0$  into C:  $5x-9=0$   
 $x=9/5 //$

$$\therefore AB = 4 - \frac{9}{5} = \frac{11}{5} //$$

$$\therefore \text{Area } \Delta = \frac{1}{2} \times \frac{11}{5} \times 11 = \boxed{\frac{121}{10}}$$

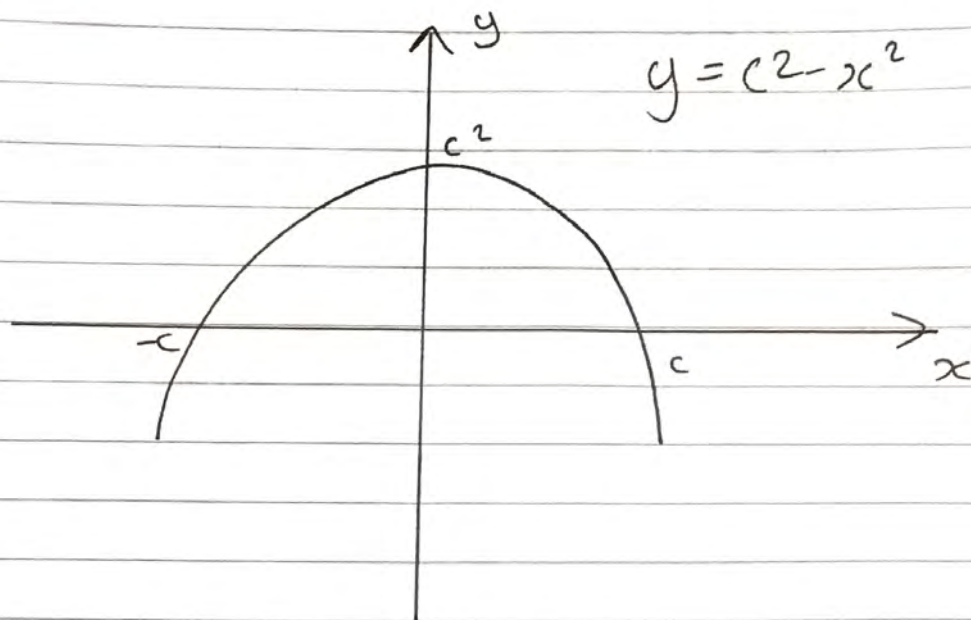
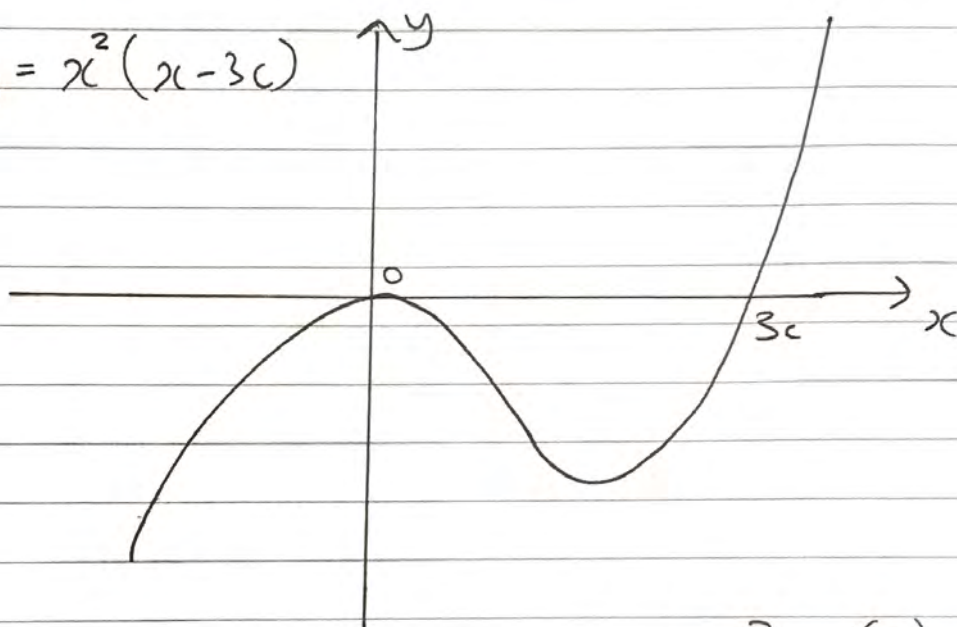
$$R = \int_1^4 \left[ \frac{3x^2}{4} - 4x^{\frac{1}{2}} + 7 \right] dx - \frac{121}{10}$$

$$= \left[ \frac{x^3}{4} - \frac{4x^{\frac{3}{2}}}{\frac{3}{2}} + 7x \right]_1^4 - \frac{121}{10}$$

$$= \left[ 16 - \frac{64}{3} + 28 \right] - \left[ \frac{55}{12} \right] - \frac{121}{10}$$

$$= \boxed{\frac{359}{60}} = \underline{5.98} \text{ (2 d.p.)}$$

13ai)

ii)  $y = x^2(x - 3c)$ 

$$y = x^3 - (3c)x^2$$

$$\underline{x=0}: y=0.$$

$$\underline{x=3c}: y=0.$$

general cubic shape.

$$b) \quad c^2 - x^2 = x^2(x - 3c).$$

$$c^2 - x^2 = x^3 - 3cx^2.$$

$$x^3 + x^2 - 3cx^2 - c^2 = 0$$

$$x^3 + (1 - 3c)x^2 - c^2 = 0.$$

□

c)  $x=2$  is a solution...  $\therefore$  substitute  $x=2$  into eqn. from (b).

$$\Rightarrow (2)^3 + (1 - 3c)(2^2) - c^2 = 0.$$

$$8 + 4(1 - 3c) - c^2 = 0.$$

$$12 - 12c - c^2 = 0$$

$$c^2 + 12c - 12 = 0$$

Quadratic formula

$$\left. \begin{array}{l} a=1 \\ b=12 \\ c=-12 \end{array} \right\} c = \frac{-12 \pm \sqrt{144 - 4(-12)}}{2} = \frac{-12 \pm \sqrt{192}}{2}$$

$$c = \frac{-12 + 8\sqrt{3}}{2} = \boxed{-6 + 4\sqrt{3}}$$

$c > 0$  so reject  $c = \frac{-12 - \sqrt{192}}{2}$



$$\bullet 14a) S_n = a + ar + ar^2 + \dots + ar^{n-1} \quad \text{--- (1)}$$

$$rS_n = ar + ar^2 + \dots + ar^n \quad \text{--- (2)}$$

$$\underline{\text{(1)} - \text{(2)}} : S_n - rS_n = a - ar^n$$

$$S_n(1-r) = a(1-r^n)$$

$$\therefore S_n = \frac{a(1-r^n)}{1-r}$$

$$b) \left. \begin{array}{l} a=180 \\ r=0.93 \end{array} \right\} \text{end of } S^{\text{th}} \text{ year} = ar^S = 180 \times (0.93)^S = \underline{\underline{125.2}}$$

$$c) \begin{array}{ccccccc} 180 & + & 180(0.93) & + & 180(0.93)^2 & + & \dots & + & 180(0.93)^{20} \\ a & & ar & & ar^2 & & & & ar^{20} \\ \text{start of } 1^{\text{st}} & & \text{end of } 1^{\text{st}} & & \text{end of } 2^{\text{nd}} & & & & \text{end of } 20^{\text{th}} \text{ year.} \\ \text{year} & & \text{year} & & & & & & \end{array}$$

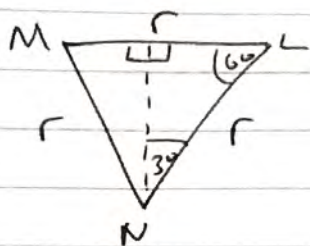
Because we want the total liquid at the end of 20 years, use:

$$\left. \begin{array}{l} a = 180(0.93) \\ r = 0.93 \end{array} \right\} S_{20} = \frac{180(0.93)(1-0.93^{20})}{1-0.93}$$

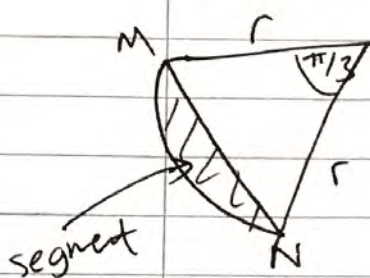
$$= \boxed{1831 \text{ litres}}$$

15)  $\triangle LMN$  is also equilateral.

$$\text{Area}_{LMN} = \frac{1}{2} \times r \times r \cos 30^\circ = \frac{r^2 \sqrt{3}}{4} //$$



Now to find the area of one of the segments outside the  $\triangle LMN$ :



$$\begin{aligned} \therefore \text{Area of sector MLN} &= \frac{1}{2} (r^2) \left( \frac{\pi}{3} \right) \\ &= \frac{\pi r^2}{6} // \end{aligned}$$

$$\begin{aligned} \text{So area}_{\text{segment}} &= -\frac{r^2 \sqrt{3}}{4} + \frac{r^2 \pi}{6} \\ &= r^2 \left( -\frac{\sqrt{3}}{4} + \frac{\pi}{6} \right) // \end{aligned}$$

$$\therefore R = 3 \times r^2 \left( -\frac{\sqrt{3}}{4} + \frac{\pi}{6} \right) + \frac{r^2 \sqrt{3}}{4}$$

$$= r^2 \left( \frac{3\sqrt{3}}{4} + \frac{\pi}{2} + \frac{\sqrt{3}}{4} \right) = \boxed{r^2 \left( \frac{\sqrt{3}}{2} + \frac{\pi}{2} \right)}$$

$$= \boxed{r^2 \left( \frac{\pi}{2} - \frac{\sqrt{3}}{2} \right)}$$