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1. Simplify the following expressions fully.

(a) $(x^6)^{\frac{1}{3}}$

$(x^a)^b = x^{ab}$

(1)

(b) $\sqrt{2}(x^3) \div \sqrt{\dfrac{32}{x^2}}$

(2)

a) $(x^6)^{1/3} = x^{6/3} = x^2$

b) $\sqrt{2}\,x^3 \div \sqrt{\dfrac{32}{x^2}} = \sqrt{2}\,x^3 \times \dfrac{\sqrt{x^2}}{\sqrt{32}}$

$= \sqrt{2}\,x^3 \times \dfrac{x}{\sqrt{16\times 2}}$

$= \sqrt{2}\,x^3 \times \dfrac{x}{4\sqrt{2}}$

$= \dfrac{x^4}{4}$

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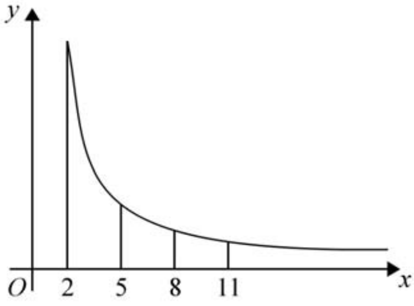


Figure 1

Figure 1 shows a sketch of part of the graph of $y = \dfrac{12}{\sqrt{(x^2 - 2)}}$, $x \geqslant 2$

The table below gives values of y rounded to 3 decimal places.

x	2	5	8	11
y	8.485	2.502	1.524	1.100

(a) Use the trapezium rule with all the values of y from the table to find an approximate value, to 2 decimal places, for

$$\int_2^{11} \frac{12}{\sqrt{(x^2 - 2)}} \, dx$$

$h = 5 - 2$
$= 3$

(4)

(b) Use your answer to part (a) to estimate a value for

$$\int_2^{11} \left(1 + \frac{6}{\sqrt{(x^2 - 2)}}\right) dx$$

(3)

a) $\displaystyle\int_2^{11} \frac{12}{\sqrt{x^2-2}} \, dx = \frac{1}{2} h \left[y_0 + y_3 + 2(y_1 + y_2)\right]$

$= \frac{1}{2}(3)\left[8.485 + 1.1 + 2(2.502 + 1.524)\right]$

$= 26.46$ (2dp)

b) $\displaystyle\int_2^{11} \left(1 + \frac{6}{\sqrt{x^2-2}}\right) dx = \int_2^{11} 1 \, dx + \frac{1}{2}\int_2^{11} \frac{12}{\sqrt{x^2-2}} \, dx$

$= [x]_2^{11} + \frac{1}{2}(26.46) = 9 + 13.23 = 22.23$ (2dp)

3.

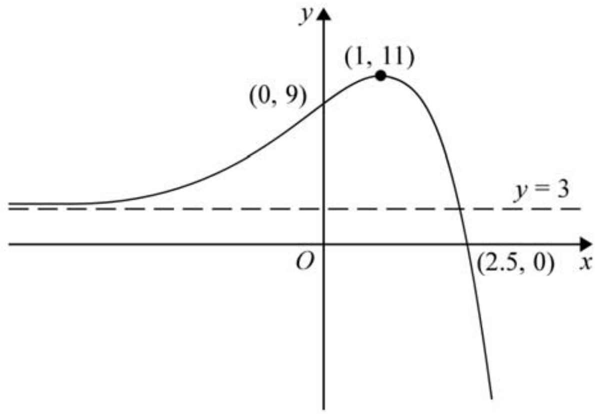


Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = \mathrm{f}(x)$.
The curve crosses the coordinate axes at the points (2.5, 0) and (0, 9), has a stationary point at (1, 11), and has an asymptote $y = 3$

On **separate** diagrams, sketch the curve with equation

(a) $y = 3\mathrm{f}(x)$

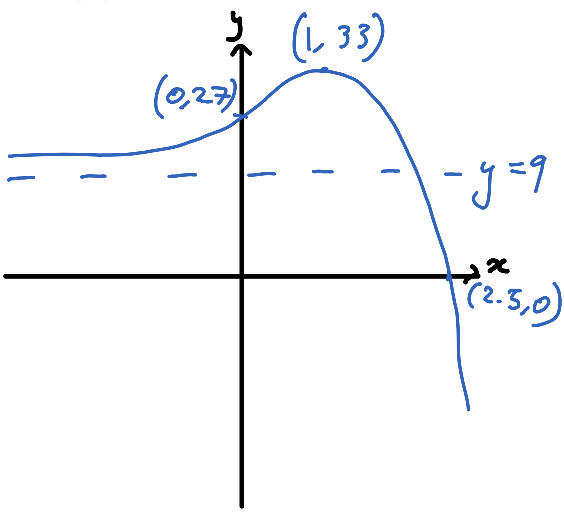
(3)

(b) $y = \mathrm{f}(-x)$

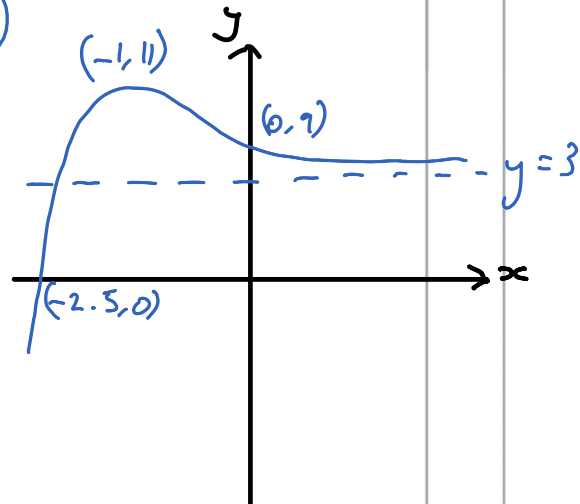
(3)

On each diagram show clearly the coordinates of the points of intersection of the curve with the two coordinate axes, the coordinates of the stationary point, and the equation of the asymptote.

a)



b)



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4. (a) Find the first 4 terms in ascending powers of x of the binomial expansion of

$$\left(2+\frac{x}{4}\right)^{10}$$

giving each term in its simplest form.

(4)

(b) Use your expansion to find an estimated value for 2.025^{10}, stating the value of x which you have used and showing your working.

(3)

a) $\left(2+\frac{x}{4}\right)^{10} \approx 2^{10} + {}^{10}C_1\, 2^9\left(\frac{x}{4}\right)^1 + {}^{10}C_2\, 2^8\left(\frac{x}{4}\right)^2 + {}^{10}C_3\, 2^7\left(\frac{x}{3}\right)^3$

$= 1024 + 1280x + 720x^2 + 240x^3$

b) When $x = 0.1$,

$\left(2+\frac{x}{4}\right)^{10} = 2.025^{10}$

$= 1024 + 1280(0.1) + 720(0.1)^2 + 240(0.1)^3$

$= 1159.44$

Leave blank

5. (a) Prove that the sum of the first n terms of an arithmetic series is given by the formula

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

where a is the first term of the series and d is the common difference between the terms.

(4)

(b) Find the sum of the integers which are divisible by 7 and lie between 1 and 500

(3)

a) $S_n = a + (a+d) + (a+2d) + \dots + [a+(n-2)d] + [a+(n-1)d]$

$S_n = [a+(n-1)d] + [a+(n-2)d] + \dots + (a+2d) + (a+d) + a$

$2S_n = [a+a+(n-1)d] + [a+d+a+(n-2)d] + \dots + [a+a+(n-1)d]$

$= [2a+(n-1)d]n$

$S_n = \frac{n}{2}[2a+(n-1)d]$

b) $S = 7 + 14 + \dots + 497$

$a = 7,\ l = 497,\ d = 7,\ n = \frac{497}{7} = 71$

$\Rightarrow S = \frac{71}{2}(7 + 497)$ $\qquad S = \frac{n}{2}(a+l)$

$= 17892$

Leave blank

6. Given that

$$2\log_4(2x+3) = 1 + \log_4 x + \log_4(2x-1), \quad x > \frac{1}{2}$$

(a) show that

$$4x^2 - 16x - 9 = 0$$

(5)

(b) Hence solve the equation

$$2\log_4(2x+3) = 1 + \log_4 x + \log_4(2x-1), \quad x > \frac{1}{2}$$

(2)

a) $2\log_4(2x+3) = 1 + \log_4 x + \log_4(2x-1)$

$\log_4(2x+3)^2 - \log_4 x - \log_4(2x-1) = 1$

$\log_4\left[\dfrac{(2x+3)^2}{x(2x-1)}\right] = 1$

$a\log b = \log b^a$

$\log a + \log b = \log(ab)$

$\log a - \log b = \log\left(\frac{a}{b}\right)$

$\dfrac{(2x+3)^2}{x(2x-1)} = 4^1$

$(2x+3)^2 = 4x(2x-1)$

$4x^2 + 12x + 9 = 8x^2 - 4x$

$4x^2 - 16x - 9 = 0$

b) $4x^2 - 16x - 9 = 0$

$(2x+1)(2x-9) = 0$

$x = -1/2$ or $9/2$

But given $x > \frac{1}{2}$, $x = \dfrac{9}{2}$

Leave blank

7. The circle C has equation

$$x^2 + y^2 + 10x - 6y + 18 = 0$$

Find

(a) the coordinates of the centre of C,

(2)

(b) the radius of C.

(2)

The circle C meets the line with equation $x = -3$ at two points.

(c) Find the exact values for the y coordinates of these two points, giving your answers as fully simplified surds.

(4)

$x^2 + y^2 + 10x - 6y + 18 = 0$

$(x+5)^2 - 25 + (y-3)^2 - 9 + 18 = 0$

$(x+5)^2 + (y-3)^2 = 16$

$(x-a)^2 + (y-b)^2 = r^2$

a) Centre : $(-5, 3)$

b) Radius $= \sqrt{16} = 4$

c) When $x = -3$,

$(-3+5)^2 + (y-3)^2 = 16$

$(y-3)^2 = 12$

$y - 3 = \pm\sqrt{12}$

$y = 3 \pm \sqrt{12}$

$= 3 \pm 2\sqrt{3}$

Leave blank

8. A sequence is defined by

$$u_1 = k$$
$$u_{n+1} = 3u_n - 12, \qquad n \geqslant 1$$

where k is a constant.

(a) Write down fully simplified expressions for u_2, u_3 and u_4 in terms of k.
(4)

Given that $u_4 = 15$

(b) find the value of k,
(2)

(c) find $\sum_{i=1}^{4} u_i$, giving an exact numerical answer.
(3)

a) $u_1 = k$

$u_2 = 3u_1 - 12$

$= 3k - 12$

$u_3 = 3u_2 - 12 = 3(3k-12) - 12$

$= 9k - 48$

$u_4 = 3u_3 - 12 = 3(9k-48) - 12$

$= 27k - 156$

b) $15 = 27k - 156$

$171 = 27k$

$k = \frac{19}{3}$

c) $\sum_{i=1}^{4} u_i = u_1 + u_2 + u_3 + u_4$

$= k + 3k - 12 + 9k - 48 + 27k - 156$

$= 40k - 216$

$= 40\left(\frac{19}{3}\right) - 216$

$= 112/3$

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9.

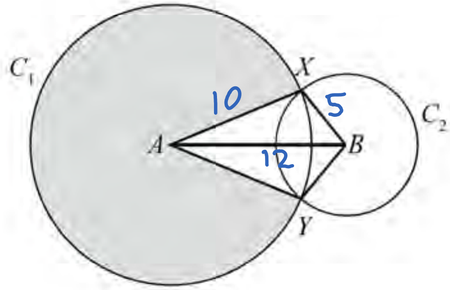

Figure 3

In Figure 3, the points A and B are the centres of the circles C_1 and C_2 respectively. The circle C_1 has radius 10 cm and the circle C_2 has radius 5 cm. The circles intersect at the points X and Y, as shown in the figure.

Given that the distance between the centres of the circles is 12 cm,

(a) calculate the size of the acute angle XAB, giving your answer in radians to 3 significant figures,

(2)

(b) find the area of the major sector of circle C_1, shown shaded in Figure 3,

(3)

(c) find the area of the kite $AYBX$.

(3)

a) $\cos X\hat{A}B = \dfrac{10^2 + 12^2 - 5^2}{2 \times 10 \times 12}$ cosine rule

$X\hat{A}B = 0.421^c$ (3 sf)

b) $X\hat{A}Y = 2\,X\hat{A}B = 2 \times 0.421 = 0.843^c$

Area of major sector $= \frac{1}{2}(10)^2(2\pi - 0.843)$ $A = \frac{1}{2}r^2\theta$

$= 272\text{ cm}^2$

c) Area of $\triangle XAB = \frac{1}{2} \times 10 \times 12 \sin(0.421) = 24.5$

Area of kite $= 2 \times 24.5$ $A = \frac{1}{2}ab\sin\theta$

$= 49.0\text{ cm}^2$

Leave blank

10.

$$f(x) = 6x^3 + ax^2 + bx - 5$$

where a and b are constants.

When $f(x)$ is divided by $(x + 1)$ there is no remainder.

When $f(x)$ is divided by $(2x - 1)$ the remainder is -15

(a) Find the value of a and the value of b.

(5)

(b) Factorise $f(x)$ completely.

(4)

a) $f(-1) = 6(-1)^3 + a(-1)^2 + b(-1) - 5 = 0$

$-6 + a - b - 5 = 0$

$a - b = 11 \quad (1)$

Remainder theorem

$f\left(\frac{1}{2}\right) = 6\left(\frac{1}{2}\right)^3 + a\left(\frac{1}{2}\right)^2 + b\left(\frac{1}{2}\right) - 5 = -15$

$\frac{3}{4} + \frac{a}{4} + \frac{b}{2} - 5 = -15$

$a + 2b = -43 \quad (2)$

$(1) - (2): \; -3b = 54$

$b = -18$

In (1): $a = 11 + (-18) = -7$

b) $f(x) = 6x^3 - 7x^2 - 18x - 5$

$= (x+1)(6x^2 - 13x - 5)$ by inspection

$= (x+1)(3x+1)(2x-5)$

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11.

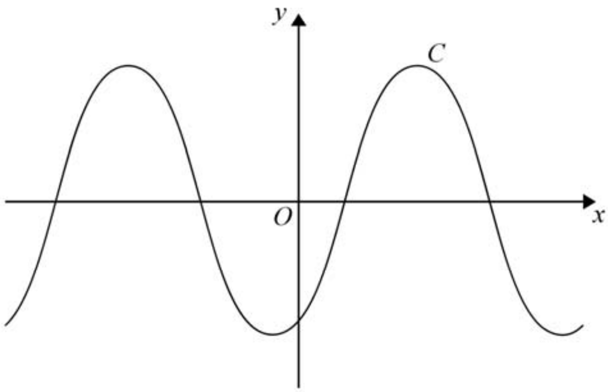


Figure 4

Figure 4 shows a sketch of the curve C with equation $y = \sin(x - 60°)$, $-360° \leqslant x \leqslant 360°$

(a) Write down the exact coordinates of the points at which C meets the two coordinate axes.
(3)

(b) Solve, for $-360° \leqslant x \leqslant 360°$,

$$4\sin(x - 60°) = \sqrt{6} - \sqrt{2}$$

showing each stage of your working.
(5)

a) Meets the x-axis at: $(-300, 0)$; $(-120, 0)$; $(60, 0)$; $(240, 0)$

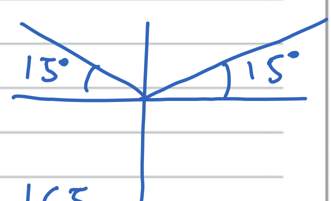
Transform $y = \sin x$

When $x = 0$, $y = \sin(-60) = -\sin 60 = \frac{-\sqrt{3}}{2}$

$\therefore$ Meets the y-axis at $(0, -\sqrt{3}/2)$

b) $4\sin(x - 60) = \sqrt{6} - \sqrt{2}$

$\sin(x - 60) = \frac{\sqrt{6} - \sqrt{2}}{4}$



$x - 60 = -345, -195, 15, 165$

$x = -285°, -135°, 75°, 225°$

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12. A business is expected to have a yearly profit of £275 000 for the year 2016. The profit is expected to increase by 10% per year, so that the expected yearly profits form a geometric sequence with common ratio 1.1

(a) Show that the difference between the expected profit for the year 2020 and the expected profit for the year 2021 is £40 300 to the nearest hundred pounds.

(3)

(b) Find the first year for which the expected yearly profit is more than one million pounds.

(4)

(c) Find the total expected profits for the years 2016 to 2026 inclusive, giving your answer to the nearest hundred pounds.

(3)

a) $u_n = ar^{(n-1)}$

$u_5 = 275000 \times 1.1^4 = 402628$

$u_6 = 275000 \times 1.1^5 = 442890$

$u_6 - u_5 = 442890 - 402628 = 40300$

b) $275000 \times 1.1^{(n-1)} = 1000000$

$1.1^{(n-1)} = 3.64$

$n - 1 = \dfrac{\ln 3.64}{\ln 1.1} = 13.6$

$\therefore$ Year is $2016 + 14 = 2030$

c) $S_n = \dfrac{a(r^n - 1)}{r - 1}$

$S_{11} = \dfrac{275000(1.1^{11} - 1)}{1.1 - 1}$

$= 5096100$

Leave blank

13. The curve C has equation

$$y = 3x^2 - 4x + 2$$

The line l_1 is the normal to the curve C at the point $P(1, 1)$

(a) Show that l_1 has equation

$$x + 2y - 3 = 0$$

(5)

The line l_1 meets curve C again at the point Q.

(b) By solving simultaneous equations, determine the coordinates of the point Q.

(4)

Another line l_2 has equation $kx + 2y - 3 = 0$, where k is a constant.

(c) Show that the line l_2 meets the curve C once only when

$$k^2 - 16k + 40 = 0$$

(4)

(d) Find the two exact values of k for which l_2 is a tangent to C.

(2)

a) $y = 3x^2 - 4x + 2$

$\frac{dy}{dx} = 6x - 4$

At $(1,1)$, $\frac{dy}{dx} = 6(1) - 4 = 2$

$\therefore$ Grad of l_1 is $-1/2$

$y - y_1 = m(x - x_1)$

$y - 1 = -1/2(x - 1)$

$2y - 2 = -x + 1$

$x + 2y - 3 = 0$

Leave blank

Question 13 continued

b) $y = 3x^2 - 4x + 2 \quad (1) \qquad x + 2y - 3 = 0 \quad (2)$

(1) in (2): $x + 6x^2 - 8x + 4 - 3 = 0$

$6x^2 - 7x + 1 = 0$

$(6x-1)(x-1) = 0$

$x = 1$ or $1/6$

When $x = \frac{1}{6}$, $y = 3(1/6)^2 - 4(1/6) + 2$

$= 1/12 - 2/3 + 2$

$= \frac{17}{12}$

$\therefore Q(1/6, 17/12)$

c) $kx + 2y - 3 = 0 \quad (3)$

(1) in (3): $kx + 6x^2 - 8x + 4 - 3 = 0$

$6x^2 + (k-8)x + 1 = 0$

For one solution, $\Delta = 0$

$(k-8)^2 - 4(6)(1) = 0$

$k^2 - 16k + 40 = 0$

d) $k = \frac{16 \pm \sqrt{16^2 - 4(40)}}{2} = \frac{8 \pm \sqrt{96}}{2} = 8 \pm$

$= 8 \pm \frac{1}{2}\sqrt{16 \times 6} = 8 \pm 2\sqrt{6}$

Leave blank

14. *In this question, solutions based entirely on graphical or numerical methods are not acceptable.*

(i) Solve, for $0 \leqslant x < 360°$,

$$3\sin x + 7\cos x = 0$$

Give each solution, in degrees, to one decimal place.

(4)

(ii) Solve, for $0 \leqslant \theta < 2\pi$,

$$10\cos^2\theta + \cos\theta = 11\sin^2\theta - 9$$

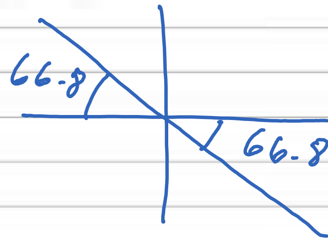
Give each solution, in radians, to 3 significant figures.

(6)

i) $3\sin x = -7\cos x$

$\tan x = \dfrac{-7}{3}$

$x = 113.2°,\ 293.2$



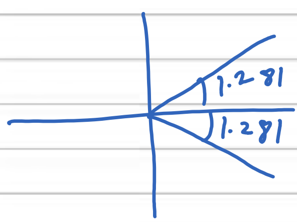
ii) $10\cos^2\theta + \cos\theta = 11(1-\cos^2\theta) - 9$

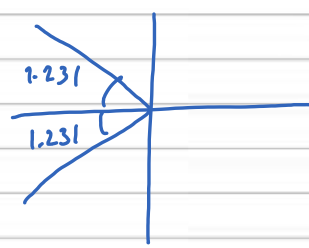
$= 11 - 11\cos^2\theta - 9$

$21\cos^2\theta + \cos\theta - 2 = 0$

$(7\cos\theta - 2)(3\cos\theta + 1)$

$\cos\theta = \dfrac{2}{7}$ or $-\dfrac{1}{3}$





$\theta = 1.28,\ 1.91,\ 4.37,\ 5.00$ (3 sf)

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15.

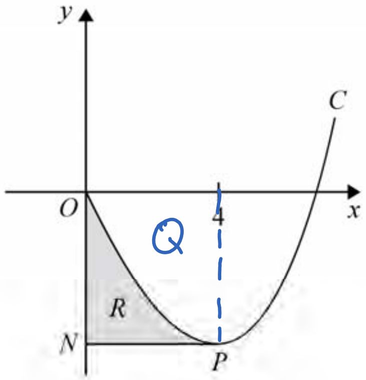

Figure 5

Figure 5 shows a sketch of part of the curve C with equation

$$y = x^3 + 10x^{\frac{3}{2}} + kx, \qquad x \geqslant 0$$

where k is a constant.

(a) Find $\frac{dy}{dx}$

(2)

The point P on the curve C is a minimum turning point.
Given that the x coordinate of P is 4

(b) show that $k = -78$

(2)

The line through P parallel to the x-axis cuts the y-axis at the point N.

The finite region R, shown shaded in Figure 5, is bounded by C, the y-axis and PN.

(c) Use integration to find the area of R.

(7)

a) $y = x^3 + 10x^{3/2} + kx$

$\frac{dy}{dx} = 3x^2 + 15x^{1/2} + k$

b) When $x = 4$, $\frac{dy}{dx} = 3(4)^2 + 15(4)^{1/2} + k = 0$

$48 + 30 + k = 0$

$k = -78$

Leave blank

Question 15 continued

c) When $x=4$, $y=(4)^3+10(4)^{3/2}-78(4)$

$=64+80-312$

$=-168$

$\therefore |ON|=168$

$R+Q=168\times 4=672$

$-Q=\int_0^4 \left(x^3+10x^{3/2}-78x\right)dx$

$=\left[\frac{x^4}{4}+4x^{5/2}-39x^2\right]_0^4$

$=\left[\frac{(4)^4}{4}+4(4)^{5/2}-39(4)^2\right]-0$

$=64+128-624$

$Q=432$

$\therefore R=672-432$

$=240$