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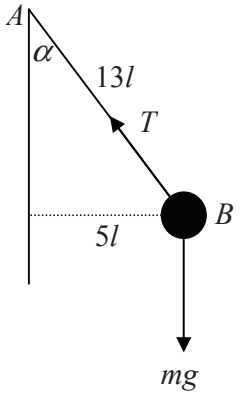
Question Number	Scheme	Marks
Q1		
(a)	$\cos\alpha = \frac{12}{13}$	B1
	R$(\uparrow)$ $T\cos\alpha = mg$	M1
	$T \times \frac{12}{13} = mg$	
	$T = \frac{13}{12}mg$ oe	A1 (3)
(b)	Eqn of motion $T\sin\alpha = m\frac{v^2}{5l}$	M1 A1
	$\frac{13mg}{12} \times \frac{5}{13} = m\frac{v^2}{5l}$	M1 dep
	$v^2 = \frac{25gl}{12}$	
	$v = \frac{5}{2}\sqrt{\frac{gl}{3}}$ [accept $5\sqrt{\frac{gl}{12}}$ or $\sqrt{\frac{25gl}{12}}$ or any other equiv]	A1 (4)
		[7]

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Q2 (a)	$F = (-)\dfrac{k}{x^2}$	M1
	$mg = (-)\dfrac{k}{R^2}$	M1
	$F = \dfrac{mgR^2}{x^2}$ *	A1 (3)
(b)	$m\ddot{x} = -\dfrac{mgR^2}{x^2}$	M1
	$v\dfrac{\mathrm{d}v}{\mathrm{d}x} = -\dfrac{gR^2}{x^2}$	M1
	$\dfrac{1}{2}v^2 = \int -\dfrac{gR^2}{x^2}\,\mathrm{d}x$	M1 dep on 1st M mark
	$\dfrac{1}{2}v^2 = \dfrac{gR^2}{x} \quad (+c)$	A1
$x = R,\ v = 3U$	$\dfrac{9U^2}{2} = gR + c$	M1 dep on 3rd M mark
	$\dfrac{1}{2}v^2 = \dfrac{gR^2}{x} + \dfrac{9U^2}{2} - gR$	
$x = 2R,\ v = U$	$\dfrac{1}{2}U^2 = \dfrac{gR^2}{2R} + \dfrac{9U^2}{2} - gR$	M1 dep on 3rd M mark
	$U^2 = \dfrac{gR}{8}$	
	$U = \sqrt{\dfrac{gR}{8}}$	A1 (7)
		[10]

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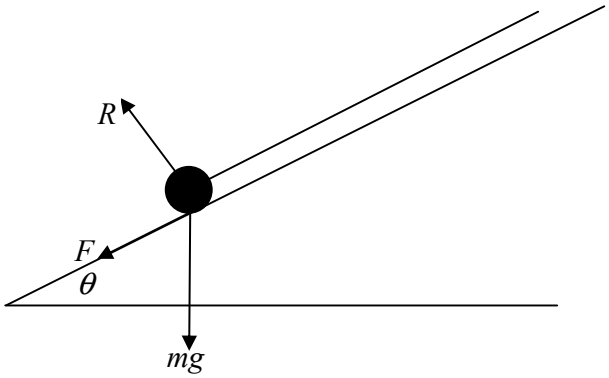
Q3		
	EPE lost $= \dfrac{\lambda\times0.6^2}{2\times0.9}-\dfrac{\lambda\times0.1^2}{2\times0.9}\left[=\dfrac{7}{36}\lambda\right]$	M1 A1
	$\mathrm{R}(\uparrow)\quad R = mg\cos\theta$	M1
	$= 0.5g\times\dfrac{4}{5} = 0.4g$	
	$F = \mu R = 0.15\times0.4g$	M1 A1
	P.E. gained = E.P.E. lost − work done against friction	
	$0.5\mathrm{g}\times0.7\sin\theta = \dfrac{\lambda\times0.6^2}{2\times0.9}-\dfrac{\lambda\times0.1^2}{2\times0.9}-0.15\times0.4g\times0.7$	M1 A1 A1
	$0.1944\lambda = 0.5\times9.8\times0.7\times\dfrac{3}{5}+0.15\times0.4\times9.8\times0.7$	
	$\lambda = 12.70.....$	
	$\lambda = 13$ N or 12.7	A1
		[9]

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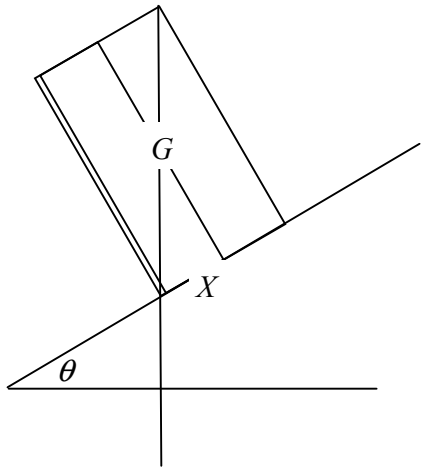
Question Number	Scheme	Marks
Q4 (a)	<table><tr><td></td><td>cone</td><td>container</td><td>cylinder</td></tr><tr><td>mass ratio</td><td>$\frac{4\pi l^3}{3}$</td><td>$\frac{68\pi l^3}{3}$</td><td>$24\pi l^3$</td></tr><tr><td></td><td>4</td><td>68</td><td>72</td></tr><tr><td>dist from O</td><td>l</td><td>$\bar{x}$</td><td>$3l$</td></tr></table>	M1 A1
		B1
	Moments: $4l + 68\bar{x} = 72\times 3l$	M1 A1ft
	$\bar{x} = \frac{212l}{68} = \frac{53}{17}l$ accept $3.12l$	A1 (6)
(b)		
	$GX = 6l - \bar{x}$ seen	M1
	$\tan\theta = \frac{2l}{6l-\bar{x}}$	M1 A1
	$= \frac{2\times 17}{49}$	
	$\theta = 34.75... = 34.8$ or 35	A1 (4)
		[10]

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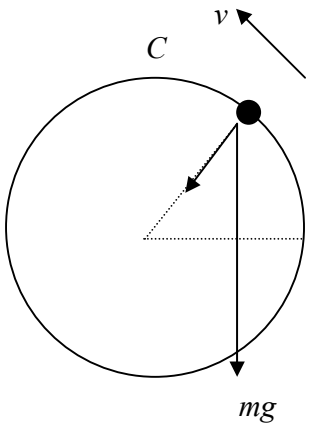
Q5		
(a)	Energy: $mga\sin\theta = \frac{1}{2}m\times 5ag - \frac{1}{2}mv^2$ $v^2 = 5ag - 2ag\sin\theta$	M1 A1 A1 (3)
(b)	Eqn of motion along radius: $T + mg\sin\theta = \frac{mv^2}{a}$ $T = \frac{m}{a}(5ag - 2ag\sin\theta) - mg\sin\theta$ $T = mg(5 - 3\sin\theta)$	M1 A1 M1 A1 (4)
(c)	At C, $\theta = 90°$ $T = mg(5-3) = 2mg$ $T > 0$ $\therefore P$ reaches C	M1 A1 A1 (3)
(d)	Max speed at lowest point ($\theta = 270°$; $v^2 = 5ag - 2ag\sin 270$) $v^2 = 5ag + 2ag$ $v = \sqrt{(7ag)}$	M1 A1 (2) **[12]**

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Question Number	Scheme	Marks
Q6 (a)	$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -\frac{3}{(t+1)^2}$	M1
	$\frac{\mathrm{d}x}{\mathrm{d}t} = \int -3(t+1)^{-2}\,\mathrm{d}t$	
	$= 3(t+1)^{-1}(+c)$	M1 A1
	$t=0,\ v=2 \quad 2=3+c \quad c=-1$	M1
	$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{3}{t+1} - 1$ *	A1 (5)
(b)	$x = \int \left[\frac{3}{t+1} - 1\right]\mathrm{d}t$	M1
	$= 3\ln(t+1) - t \quad (+c')$	A1
	$t=0,\ x=0 \quad \Diamond \quad c'=0$	B1
	$x = 3\ln(t+1) - t$	
	$v = 0 \Diamond \quad \frac{3}{t+1} = 1$	M1
	$t = 2$	A1
	$x = 3\ln 3 - 2$	M1
	$= 1.295...$	
	$= 1.30\ \text{m} \quad$ (Allow 1.3)	A1 (7)
		[12]

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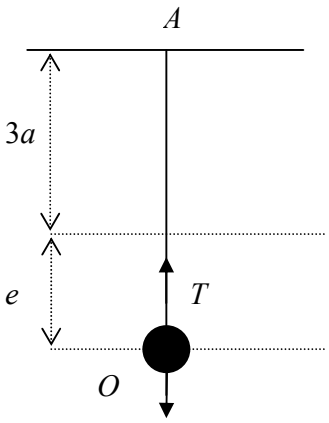
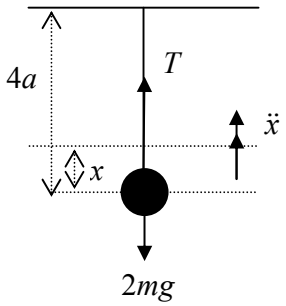
Question Number	Scheme	Marks
Q7		
(a)	R$(\uparrow)$ $T = 2mg$	B1
	Hooke's law: $T = \dfrac{6mge}{3a}$	
	$2mg = \dfrac{6mge}{3a}$	M1
	$e = a$	
	$AO = 4a$	A1 (3)
(b)		
	H.L. $T = \dfrac{6mg(a-x)}{3a} = \dfrac{2mg(a-x)}{a}$	B1ft
	Eqn. of motion $-2mg + T = 2m\ddot{x}$	M1
	$-2mg + \dfrac{2mg(a-x)}{a} = 2m\ddot{x}$	M1
	$-\dfrac{2mgx}{a} = 2m\ddot{x}$	
	$\ddot{x} = -\dfrac{g}{a}x$	A1
	period $2\pi\sqrt{\dfrac{a}{g}}$ *	A1 (5)

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Question Number	Scheme	Marks
(c)	$v^2 = \omega^2\left(a^2 - x^2\right)$ $v_{\max}{}^2 = \frac{g}{a}\left[\left(\frac{a}{4}\right)^2 - 0\right]$ $v_{\max} = \frac{1}{4}\sqrt{(ga)}$	M1 A1 A1 (3)
(d)	$x = -\frac{a}{8}$ $\quad v^2 = \frac{g}{a}\left[\frac{a^2}{16} - \frac{a^2}{64}\right]$ $= \frac{3ag}{64}$ $v^2 = u^2 + 2as$ $0 = \frac{3ag}{64} - 2gh$ $h = \frac{3a}{128}$ Total height above $O = \frac{a}{8} + \frac{3a}{128} = \frac{19a}{128}$	M1 M1 A1 A1 (4) **[15]**