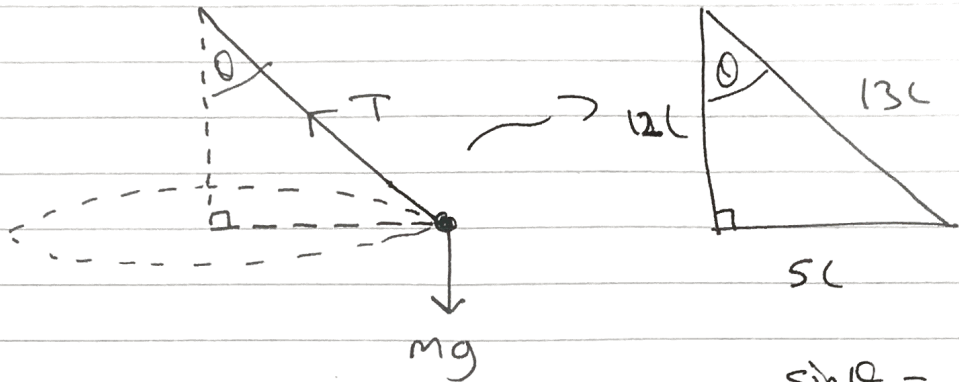


M3 Specimen IAL (MA)

Q1)



$$\sin \theta = \frac{5}{13}$$

$$\cos \theta = \frac{12}{13}$$

$$R(\uparrow\downarrow): T \cos \theta = mg$$

$$T = \frac{mg}{\cos \theta} = \boxed{\frac{13mg}{12}}$$

$$b) \overset{+}{\curvearrowleft} \underline{N2L(B)}: T \sin \theta = \frac{mv^2}{r}$$

$$\frac{13mg}{12} \left(\frac{5}{13} \right) = \frac{mv^2}{5L}$$

$$\frac{5mg}{12} \times 5L = mv^2$$

$$mv^2 = \frac{25mgL}{12}$$

$$\therefore v^2 = \frac{25gL}{12} \rightarrow v = \boxed{\frac{5}{6} \sqrt{3gL}}$$

$$(Q2a) \quad F \propto \frac{1}{x^2} : F = \frac{k}{x^2}$$

$$\underline{x=R, F=mg} : mg = \frac{k}{R^2}$$

$$\therefore k = mgR^2.$$

$$\text{hence } F = \frac{mgR^2}{x^2}$$

$$b) \quad -\frac{mgR^2}{x^2} = ma = m v \frac{dv}{dx}$$

$$-\frac{gR^2}{x^2} = v \frac{dv}{dx}$$

$$\int (v) dv = -gR^2 \int \left(\frac{1}{x^2}\right) dx$$

$$\frac{v^2}{2} = \frac{+gR^2}{x} + c.$$

$$\underline{v = 3u, x=R} : \frac{9u^2}{2} = \frac{gR^2}{R} + c$$

$$c = \frac{9u^2}{2} - gR$$

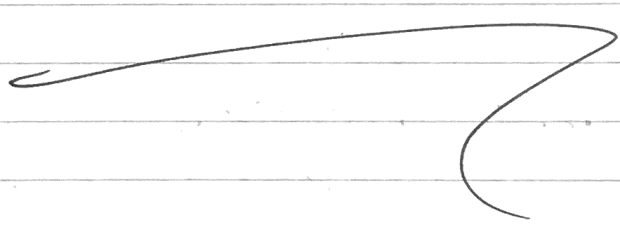
$$\therefore \frac{v^2}{2} = \frac{gR^2}{2c} + \frac{9u^2}{2} - gR$$

$$v = u, \quad c = 2R \quad : \quad \frac{u^2}{2} = \frac{gR^2}{2R} + \frac{9u^2}{2} - gR$$

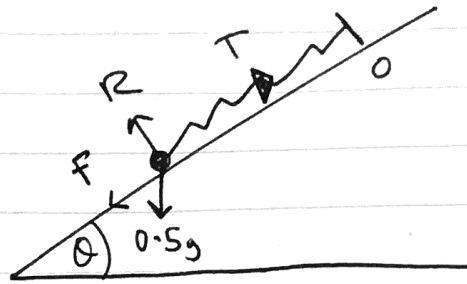
$$\frac{u^2}{2} = -\frac{gR}{2} + \frac{9u^2}{2}$$

$$4u^2 = \frac{gR}{2}$$

$$\therefore u^2 = \frac{gR}{8}$$

$$\text{hence } u = \sqrt{\frac{gR}{8}}$$


Q3)



$$R \downarrow: R = 0.5g \cos \theta$$

$$R = \frac{98}{25}$$

InitiallyAfter moving 0.7m up

KE = 0

EPE = $\frac{\lambda (1.5 - 0.9)^2}{1.8}$

KE = 0

EPE = $\frac{\lambda (0.9 - 0.8)^2}{2(0.9)}$

GPE = 0

GPE = $0.5g(0.7 \sin \theta)$

+ W.D by friction = $\left(\frac{98}{25}\right)(0.15)(0.7)$

Total energy at start
↓

↑ R ↑ H ↑ S

$$\text{So... : } \frac{\lambda (1.5 - 0.9)^2}{1.8} = \frac{\lambda (0.1)^2}{1.8} + 0.5(0.7)g(\sin \theta) + \text{W.D.}$$

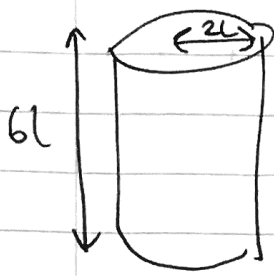
Total energy at end + work done

$$\Rightarrow \frac{\lambda (0.6)^2}{1.8} - \frac{\lambda (0.1)^2}{1.8} = 0.5(0.7g)\left(\frac{3}{5}\right) + \frac{98}{25}(0.15)(0.7)$$

$$\Rightarrow \lambda \frac{(0.6^2 - 0.1^2)}{1.8} = \frac{3087}{1250}$$

$$\Rightarrow \lambda = \frac{\frac{3087}{1250}(1.8)}{0.6^2 - 0.1^2} = \boxed{12.7 \text{ N}}$$

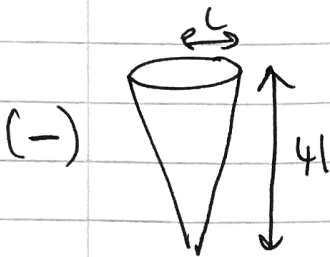
● (14a) Shape Mass (vol.) Distance of c.o.m from 0



$$\pi(2L)^2(6L)$$

$$= \boxed{24\pi L^3}$$

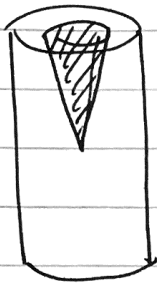
$$\boxed{3L}$$



$$\frac{1}{3}\pi(L)^2(4L)$$

$$= \boxed{\frac{4\pi L^3}{3}}$$

$$\frac{4L}{4} = \boxed{L}$$



$$\boxed{\frac{68\pi L^3}{3}}$$

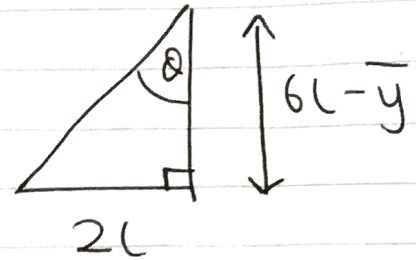
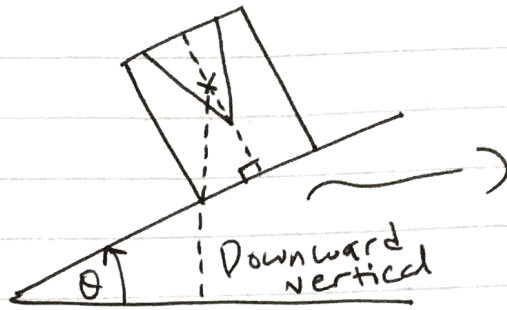
$$\boxed{\bar{y}}$$

● Moments about 0...

$$24(3L) = \frac{4}{3}(L) = \frac{68}{3}(\bar{y})$$

$$\frac{[24(3) - \frac{4}{3}]L}{\frac{68}{3}} = \bar{y} = \boxed{\frac{53L}{17}}$$

b)



On point of tipping
 \therefore downward vertical
 passes through lowest
 point of contact
 with plane

$$6l - \bar{y} = \frac{49l}{17} =$$

$$\therefore \tan \theta = \frac{2}{\frac{49}{17}} = \frac{34}{49}$$

$$\therefore \theta = \tan^{-1} \frac{34}{49} = \boxed{34.8^\circ}$$

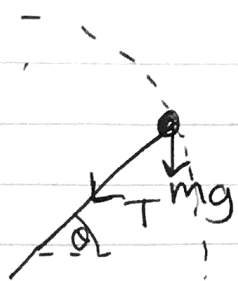
(Q5a) using conservation of energy:

$$\underbrace{\frac{mu^2}{2} - \frac{mv^2}{2}}_{\Delta KE} = \underbrace{mg \sin \theta}_{\Delta GPE}$$

$$\therefore \frac{u^2}{2} - ag \sin \theta = \frac{v^2}{2}$$

$$\therefore v^2 = u^2 - 2ag \sin \theta = \boxed{5ag - 2ag \sin \theta}$$

b)



$$+ \text{N2L(P)}: T + mg \sin \theta = \frac{mv^2}{a}$$

$$\therefore T = \frac{mv^2}{a} - mg \sin \theta$$

$$\Rightarrow T = \frac{m}{a} (5ag - 2ag \sin \theta) - mg \sin \theta$$

$$\Rightarrow T = 5mg - 2mg \sin \theta - mg \sin \theta$$

$$\Rightarrow T = 5mg - 3mg \sin \theta$$

c) at $\theta = 90^\circ$: $T = 5mg - 3mg$

$$T = 2mg > 0 \text{ at the top.}$$

so $T > 0$ at the top hence P moves in complete circles.

d) Max speed occurs at the lowest point.
(This is where GPE is lowest \therefore KE is max).

$$\underline{\theta = 270^\circ}: T = 5mg - 3mg \sin 270$$

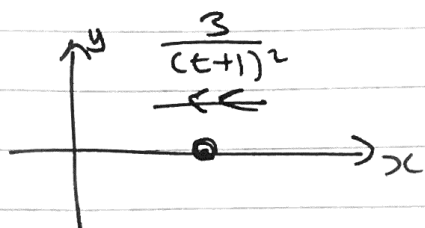
$$T = 5mg$$

$$\underline{\theta = 270^\circ}: v^2 = u^2 - 2ag (\sin 270)$$

$$v^2 = 5ag + 2ag = 7ag ; ;$$

$$\therefore v = \boxed{\sqrt{7ag}}$$

(16a)



$$a = \frac{-3}{(t+1)^2} = \frac{dv}{dt}$$

$$-3 \int \frac{1}{(t+1)^2} dt = \int (1) dv$$

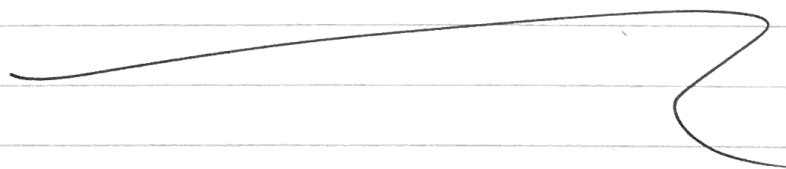
$$-3 \left[-(t+1)^{-1} \right] = v + c$$

$$\therefore v + c = \frac{3}{t+1}$$

$$\underline{t=0, v=2} : 2 + c = 3$$

$$\therefore c = 1$$

$$\text{hence } v = \frac{3}{t+1} - 1$$



$$b) \quad v = \frac{3}{t+1} - 1 = 0$$

$$\frac{3}{t+1} = 1$$

$$3 = t + 1$$

$$t = 2 //$$

$$v = \frac{dx}{dt} = \frac{3}{t+1} - 1$$

$$\int (1) dx = \int \left(\frac{3}{t+1} - 1 \right) dt$$

$$x = \int \frac{3}{t+1} - 1 dt = 3 \ln |t+1| - t + c //$$

$$\underline{x=0, t=0} : 0 = 3 \ln 1 - 0 + c$$

$$\therefore c = 0 //$$

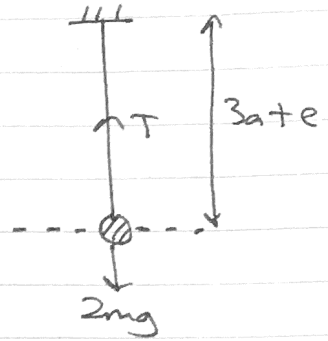
$$\text{So } x = 3 \ln |t+1| - t$$

$$\underline{t=2} : x = 3 \ln 3 - 2 = 1.296..$$

$$= \boxed{1.3 \text{ m}} \quad (2 \text{ s.f.})$$

• (Q7a) $T = 2mg$

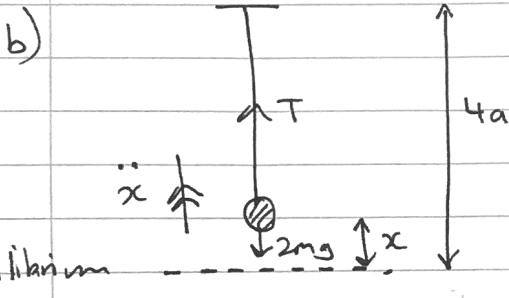
$$T = \frac{\lambda x}{L} = \frac{6mgx}{3a} = \frac{2mgx}{a}$$



$$\therefore 2mg = \frac{2mgx}{a}$$

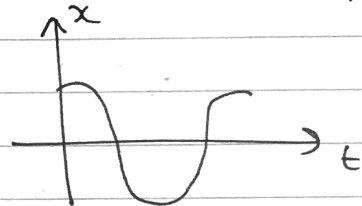
$$\therefore \frac{x}{a} = 1 \quad \therefore \boxed{x = a}$$

so $A_0 = \boxed{4a}$



finding where \ddot{x} is positive:

P starts at an endpoint so $x = a \cos \omega t$ applies.



as can be seen, x is max at $t=0$. this means that upwards is where x is increasing. so \ddot{x} is also increasing upwards hence \ddot{x} is positive upwards

• \uparrow N2L(P): $T - 2mg = 2m\ddot{x}$

$$T = \frac{6mg}{3a} (4a - x - 3a)$$

$$T = \frac{2mg}{a} (a - x)$$

$$T = 2mg - \frac{2mgx}{a}$$

$$\therefore 2mg - \frac{2mgx}{a} - 2mg = 2m\ddot{x}$$

$$\Rightarrow -\frac{gx}{a} = \ddot{x} \quad \text{hence P moves with S.H.M.}$$

$$\therefore \omega = \sqrt{\frac{g}{a}} \rightarrow T = 2\pi \times \frac{1}{\omega} = 2\pi \sqrt{\frac{a}{g}}$$

$$c) \quad a = \frac{a}{4} : v_{\max} = a\omega = \frac{a}{4} \times \sqrt{\frac{g}{a}} = \boxed{\frac{1}{4}\sqrt{ag}}$$

$$d) \quad x = \frac{a \cos \omega t}{4}$$

$$x = \frac{a}{4} \cos\left(t \sqrt{\frac{g}{a}}\right)$$

$$\frac{a}{8} = \frac{a}{4} \cos\left(t \sqrt{\frac{g}{a}}\right)$$

$$\frac{1}{2} = \cos\left(t \sqrt{\frac{g}{a}}\right)$$

$$\cos^{-1}\left(\frac{1}{2}\right) = t \sqrt{\frac{g}{a}} = \frac{\pi}{3}$$

$$t = \frac{\pi}{3} \sqrt{\frac{a}{g}}$$

$$\dot{x} = -\frac{a}{4} \sqrt{\frac{g}{a}} \sin\left(t \sqrt{\frac{g}{a}}\right)$$

$$\text{at } t = \frac{\pi}{3} \sqrt{\frac{a}{g}}, \quad \dot{x} = -\frac{a}{4} \sqrt{\frac{g}{a}} \times \frac{\sqrt{3}}{2} = \frac{-\frac{1}{4}\sqrt{ag} \times \sqrt{3}}{2}$$

$$= -\frac{1}{8}\sqrt{3ag}$$

$$\text{so } |\dot{x}| = \frac{\sqrt{3ag}}{8}$$

Swat to highest point :

$$\begin{array}{l}
 \uparrow \\
 s = s \\
 u = \frac{\sqrt{3ag}}{8} \\
 v = 0 \\
 a = -g \\
 t =
 \end{array}
 \left. \vphantom{\begin{array}{l} s \\ u \\ v \\ a \\ t \end{array}} \right\}
 \begin{array}{l}
 v^2 = u^2 + 2as \\
 0 = \frac{3ag}{64} - 2gs
 \end{array}$$

$$s = \frac{3ag}{64 \times 2g} = \frac{3a}{128} //$$

$$\therefore \text{greatest height above } \underline{0} = \frac{3a}{128} + \frac{a}{8}$$

$$= \boxed{\frac{19a}{128}}$$