Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel
International
Advanced Level

Centre Number ☐☐☐☐☐

Candidate Number ☐☐☐☐

Sample Assessment Materials for first teaching September 2018

(Time: 1 hour 30 minutes)

Paper Reference **WME03/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Mechanics M3

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 6 questions in this question paper. The total mark for this paper is 75.
- The marks for each question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over

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S 5 9 7 6 5 A 0 1 2 4

M3 specimen 2018.

Leave blank

Answer ALL questions. Write your answers in the spaces provided.

Unless otherwise indicated, whenever a numerical value of g is required, take $g = 9.8\,\text{m}\,\text{s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

1.

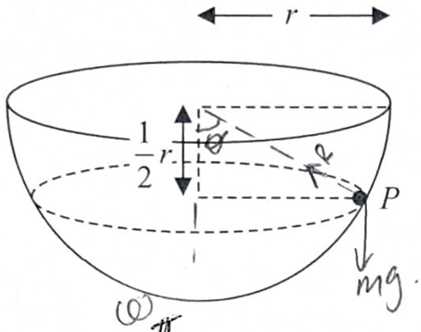


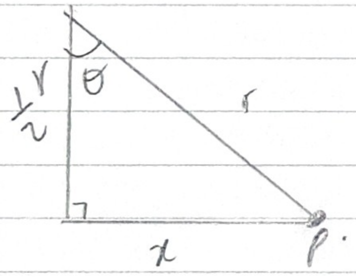
Figure 1

A hemispherical bowl, of internal radius r, is fixed with its circular rim upwards and horizontal. A particle P of mass m moves on the smooth inner surface of the bowl. The particle moves with constant angular speed in a horizontal circle. The centre of the circle is at a distance $\frac{1}{2}r$ vertically below the centre of the bowl, as shown in Figure 1.

The time taken by P to complete one revolution of its circular path is T.

Show that $T = \pi\sqrt{\frac{2r}{g}}$.

(8)



$\sqrt{r^2-(\frac{1}{2}r)^2} = \underline{\underline{\frac{r\sqrt{3}}{2}}} = x$

$R(\updownarrow)$ $R\cos\theta = mg.$

$\cos\theta = \frac{1}{2}$

$\sin\theta = \frac{\sqrt{3}}{2}$

$\frac{R}{2} = mg.$

$\underline{\underline{R = 2mg}}$

$\overset{+}{\leftarrow}$

N2L (P) : $R\sin\theta = mx\omega^2$

$R\left(\frac{\sqrt{3}}{2}\right) = mr\frac{\sqrt{3}}{2}\omega^2$

$R = mr\omega^2$

So $2mg = mr\omega^2$

$\omega^2 = \frac{2g}{r}$ $\therefore w = \sqrt{\frac{2g}{r}}$

$T = 2\pi \times \frac{1}{\omega} = 2\pi\sqrt{\frac{r}{2g.}}$

$2\pi\sqrt{\frac{r}{2g}} = \pi\sqrt{\frac{4r}{2g}} = \pi\sqrt{\frac{2r}{g}}$ //

2. A spacecraft S of mass m moves in a straight line towards the centre of the Earth. The Earth is modelled as a sphere of radius R and S is modelled as a particle. When S is at a distance x, $x \geqslant R$, from the centre of the Earth, the force exerted by the Earth on S is directed towards the centre of the Earth. The force has magnitude $\frac{K}{x^2}$, where K is a constant.

(a) Show that $K = mgR^2$

(2)

When S is at a distance $3R$ from the centre of the Earth, the speed of S is V. Assuming that air resistance can be ignored,

(b) find, in terms of g, R and V, the speed of S as it hits the surface of the Earth.

(7)

(a) $F = \frac{K}{x^2}$

at the surface of the earth, $x = R$ and $F = mg$.

$\therefore mg = \frac{K}{R^2}$

$K = mgR^2$

$F = -mgR^2/x^2$ (F is -ve b/c directed to centre of earth).

(b) $\frac{-mgR^2}{x^2} = ma$.

$\frac{-gR^2}{x^2} = v\frac{dv}{dx}$

$\int v\,dv = -gR^2\int x^{-2}\,dx$

$\frac{v^2}{2} = -gR^2\left(\frac{-1}{x}\right) + c$.

$x = 3R$ $v = V$ $\therefore \frac{V^2}{2} = \frac{gR^2}{3R} + c$

$\frac{V^2}{2} - \frac{gR}{3} = c$.

$\therefore \frac{v^2}{2} = \frac{gR^2}{x} + \frac{V^2}{2} - \frac{gR}{3}$

$v^2 = \frac{4gR}{3} + V^2$.

$v = \sqrt{V^2 + 4gR/3}$

3. At time $t = 0$, a particle P is at the origin O, moving with speed 8 m s^{-1} in the positive x direction. At time t seconds, $t \geqslant 0$, the acceleration of P has magnitude $2(t + 4)^{-\frac{1}{2}}$ m s^{-2} and is directed towards O.

(a) Show that, at time t seconds, the velocity of P is $16 - 4(t + 4)^{\frac{1}{2}}$ m s^{-1}

(5)

(b) Find the distance of P from O when P comes to instantaneous rest.

(7)

(a) $a = -2(t+4)^{-1/2}$.

$\frac{dv}{dt} = a = -2(t+4)^{-1/2}$.

$\therefore \int 1\,dv = -2\int (t+4)^{-\frac{1}{2}}\,dt$.

$v = -2\left[2(t+4)^{\frac{1}{2}}\right] + c$

$v = -4(t+4)^{1/2} + c$

$t=0 \quad v=8 : 8 = -4(2) + c$

$c = 16$

$\therefore v = -4(t+4)^{1/2} + 16.$

(b) $v=0 : 16 - 4\sqrt{t+4} = 0$

$\sqrt{t+4} = 4$.

$t+4 = 16$.

$t = 12$

$v = 16 - 4(t+4)^{1/2}$

$x = \int v\,dt$.

$x = \int \left[16 - 4(t+4)^{1/2}\right] dt$

$x = 16t - 4\frac{(t+4)^{3/2}}{3/2} + c$.

$x = 16t - \frac{8}{3}(t+4)^{3/2} + c$.

$t=0, \ x=0$:

$0 = -\frac{8}{3}(4)^{3/2} + c$.

$c = \frac{64}{3}$.

$\therefore x = 16t - \frac{8}{3}(t+4)^{3/2} + \frac{64}{3}$

at $t = 12$

$x = 16(12) - \frac{8}{3}(16)^{3/2} + \frac{64}{3}$

$= \frac{128}{3}$ m

4.

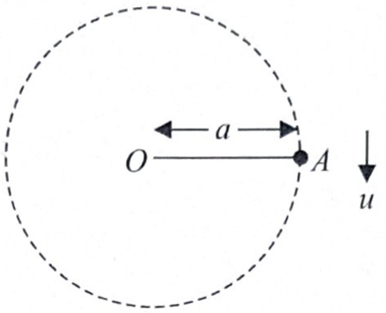


Figure 2

A particle of mass $3m$ is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. The particle is held at the point A, where OA is horizontal and $OA = a$. The particle is projected vertically downwards from A with speed u, as shown in Figure 2. The particle moves in complete vertical circles.

(a) Show that $u^2 \geqslant 3ag$.

(7)

Given that the greatest tension in the string is three times the least tension in the string,

(b) show that $u^2 = 6ag$.

(5)

(a) Particle moves in complete circles $\therefore T \geq 0$ at top.

Energy from A to highest Point

At A : $KE = \frac{1}{2}(3m)u^2$

$GPE = 0.$

At top : $KE = \frac{1}{2}(3m)v^2$

$GPE = 3mga.$

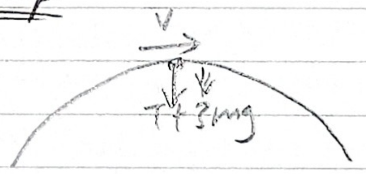
C.O.E : $\frac{\cancel{3m}u^2}{2} = \frac{3\cancel{m}v^2}{2} + \cancel{3}ag\cancel{m}$

$\frac{u^2}{2} = \frac{v^2}{2} + ag.$

$u^2 = v^2 + 2ag.$

$\therefore v^2 = u^2 - 2ag.$

At top



+↓ N2L (particle) : $T + 3mg = \frac{(3m)(v)^2}{a}.$

$T = \frac{3m}{a}(u^2 - 2ag) - 3mg.$

$T = \frac{3mu^2}{a} - 6mg - 3mg.$

$T = \frac{3mu^2}{a} - 9mg.$

Question 4 continued

$T \geq 0$ at top

$\frac{3mu^2}{a} - 9mg \geq 0.$

$\frac{3u^2}{a} \geq 9g$

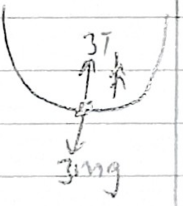
$u^2 \geq 3ag.$

(b) at top, T is minimum

$(T = \frac{3mu^2}{a} - 9mg).$

at bottom, T is max.

$+\uparrow$ N2L (particle)



$3T - 3mg = \frac{3mv^2}{a}$

Energy from A to Bottom to find v:

$\underbrace{\frac{3}{2}mv^2 - \frac{3}{2}mu^2}_{\Delta KE} = \underbrace{3mga}_{\Delta GPE}$ $(\Delta GPE = \Delta KE)$

$\therefore \frac{v^2}{2} - \frac{u^2}{2} = ga.$

$v^2 = u^2 + 2ag$

So: $3T - 3mg = \frac{3m}{a}(u^2 + 2ag)$

$3T - 3mg = \frac{3mu^2}{a} + 6mg$

$3T = \frac{3mu^2}{a} + 9mg$ — ②

and $T = \frac{3mu^2}{a} - 9mg$

$\boxed{\times 3.}$

$3T = \frac{9mu^2}{a} - 27mg$ — ①

equate ① and ②:

$\frac{9mu^2}{a} - 27mg = \frac{3mu^2}{a} + 9mg$

$\Rightarrow \frac{6\cancel{m}u^2}{a} = 36\cancel{m}g$

$\Rightarrow 6u^2 = 36ag.$

$u^2 = 6ag$ as req

5.

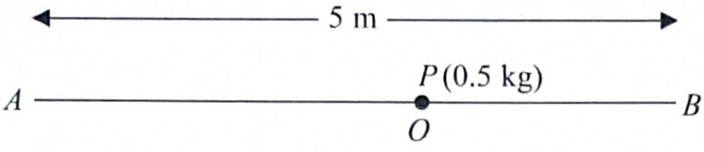


Figure 3

Two fixed points *A* and *B* are 5 m apart on a smooth horizontal floor. A particle *P* of mass 0.5 kg is attached to one end of a light elastic string, of natural length 2 m and modulus of elasticity 20 N. The other end of the string is attached to *A*. A second light elastic string, of natural length 1.2 m and modulus of elasticity 15 N, has one end attached to *P* and the other end attached to *B*.

Initially *P* rests in equilibrium at the point *O*, as shown in Figure 3.

(a) Show that *AO* = 3 m.
(4)

The particle is now pulled towards *A* and released from rest at the point *C*, where *ACB* is a straight line and *OC* = 1 m.

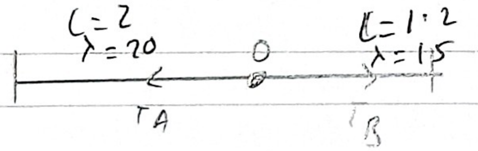
(b) Show that, while both strings are taut, *P* moves with simple harmonic motion.
(4)

(c) Find the speed of *P* at the instant when the string *PB* becomes slack.
(4)

The particle first comes to instantaneous rest at the point *D*.

(d) Find the distance *DB*.
(5)

a)



$T_A = T_B$

$\frac{20}{2}(AO-2) = \frac{15}{1.2}(5-AO-1.2)$

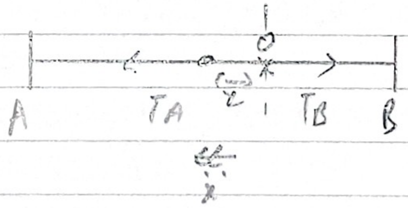
$10(AO-2) = \frac{25}{2}(3.8-AO)$

$10AO - 20 = 47.5 - 17.5AO$

$22.5AO = 67.5$

$\therefore AO = \frac{67.5}{22.5} = \underline{\underline{3m}}$

(b)



$\overset{+}{\leftarrow}$ N2L : $T_A - T_B = 0.5\ddot{x}$

$T_A = \frac{20}{2}((3-x)-2) = 10(1-x)$

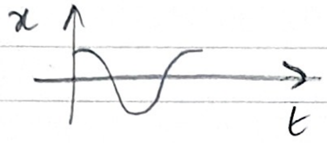
$T_B = \frac{15}{1.2}(5-3+x-1.2)$

$T_B = 25/2(0.8+x)$

Question 5 continued

Why $\ddot{x}$ is positive

P starts at an endpoint (at $t=0$) so $x = a\cos\omega t$ applies.



This means x is maximum at $t=0$ as can be seen from the graph. So x is inc. in the direction OC hence $\ddot{x}$ is also increasing in this direction ($\ddot{x}$ is +ve in the direction OC)

Continuation $\Rightarrow 10(1-x) - 12.5(0.8+x) = 0.5\ddot{x}$

$\Rightarrow 10 - 10x - 12.5x - 10 = 0.5\ddot{x}$

$-22.5x = \frac{1}{2}\ddot{x}$

$-45x = \ddot{x}$ ∴ SHM

(c) $OB = 2m$

this is +ve in direction

so ($x = -0.8$) when PB is slack.

$x = a\cos\omega t$

$x = 1\cos(t\sqrt{45})$

$-0.8 = \cos(t\sqrt{45})$

$\cos^{-1}(-0.8) = t\sqrt{5} = 2.498\ldots$

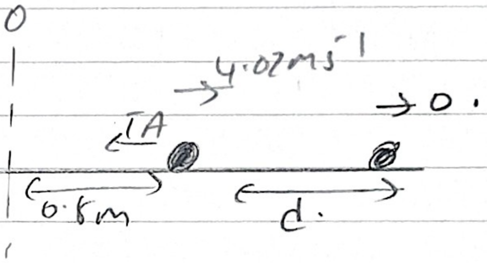
$t = \frac{2.498\ldots}{\sqrt{45}} = \underline{0.3724}$

$\dot{x} = -\sqrt{45}\sin(t\sqrt{45})$

$t = 0.3724\ldots : \dot{x} = -\sqrt{45}\left(\frac{3}{5}\right)$

∴ speed $= \underline{4.02ms^{-1}}$

(d)



P is now under the influence of just TA as PB is slack.

Consider Energy

Initially: $KE = \frac{1}{2}(0.5)\left(\frac{9}{25}\right)(45)$
$= \frac{81}{20}$

$EPE = \frac{20}{4}(3+0.8-2)^2 = \frac{81}{5}$

At rest $KE = 0$.
$EPE = \frac{20}{4}(3+0.8+d-2)^2$

$C.O.E: \frac{81}{20} + \frac{81}{5} = 5(1.8+d)^2$

$\frac{81}{4} = 5(d+1.8)^2$

$(d+1.8)^2 = \frac{81}{20}$

Leave blank

Question 5 continued

$d + 1.8 = \pm\sqrt{\frac{81}{20}}$

$d = -1.8 \pm \sqrt{\frac{81}{20}}$

$d > 0 \therefore d = -1.8 + \sqrt{\frac{81}{20}}$

So $AD = 3.8 + d = 4.012$

$\therefore BD = 5 - 4.012$

$= 0.99\text{m}$

6.

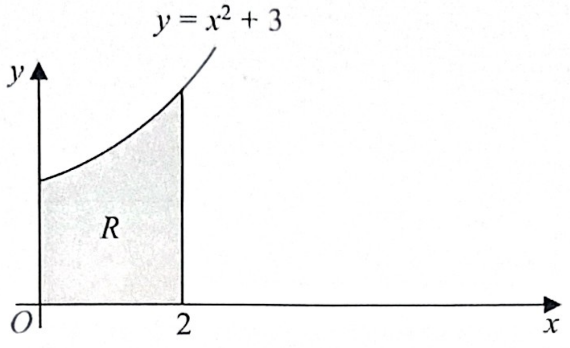


Figure 4

The shaded region R is bounded by part of the curve with equation $y = x^2 + 3$, the x-axis, the y-axis and the line with equation $x = 2$, as shown in Figure 4. The unit of length on each axis is one centimetre. The region R is rotated through 2π radians about the x-axis to form a uniform solid S.
Using algebraic integration,

(a) show that the volume of S is $\dfrac{202}{5}\pi\,\text{cm}^3$,

(4)

(b) show that, to 2 decimal places, the centre of mass of S is 1.30 cm from O.

(5)

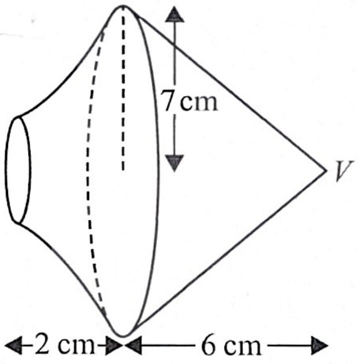


Figure 5

A uniform right circular solid cone, of base radius 7 cm and height 6 cm, is joined to S to form a solid T. The base of the cone coincides with the larger plane face of S, as shown in Figure 5. The vertex of the cone is V.
The mass per unit volume of S is twice the mass per unit volume of the cone.

(c) Find the distance from V to the centre of mass of T.

(5)

The point A lies on the circumference of the base of the cone. The solid T is suspended from A and hangs freely in equilibrium.

(d) Find the size of the angle between VA and the vertical.

(3)

Question 6 continued

$$V = \pi\int_0^2 y^2\,dx$$

$$= \pi\int_0^2 (x^2+3)^2\,dx$$

$$= \pi\int_6^2 (x^4+6x^2+9)\,dx$$

$$= \pi\left[\frac{x^5}{5}+\frac{6x^3}{3}+9x\right]_0^2$$

$$= \pi\left[\frac{202}{5}\right]-\pi(0) = \frac{202\pi}{5}$$

(b) $$M\bar{x} = \int_0^2 \pi y^2 x\,dx = \pi\int_0^2 x(x^2+3)^2\,dx$$

By pattern

$$= \pi\left[\frac{\frac{1}{2}(x^2+3)^2}{3}\right]_6^2 = \pi\left[\frac{1}{6}(x^2+3)^3\right]_6^2$$

$$= \pi\left[\frac{1}{6}(7)^3\right] - \pi\left[\frac{1}{6}(3)^3\right]$$

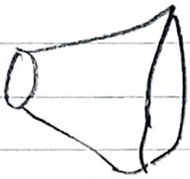
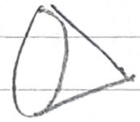
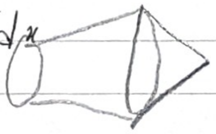
$$= \frac{\pi}{6}\left[7^3-3^3\right] = \frac{158\pi}{3}$$

$$\frac{M\bar{x}}{M} = \bar{x} = \frac{\frac{158\pi}{3}}{\frac{202\pi}{5}} = \frac{395}{303} = \underline{\underline{1.30}}$$

(c) $\frac{\text{Mass}}{\text{volume}} = \rho$ so mass of $S = 2\rho\times$ volume

mass of cone $= \rho\times$ volume

where ρ is constant

Shape	Mass(Vol.)	Distance of C.o.m from V.
	$2\rho\times\frac{202\pi}{5}$ $=\frac{404}{5}\pi$	$6+\left(2-\frac{395}{303}\right)$ $=\boxed{\frac{2029}{303}}$
	$\frac{1}{3}\rho\pi(7)^2(6)$ $=98\rho\pi$	$\frac{3}{4}\times 6 = \boxed{\frac{9}{2}}$
	$\frac{894\rho\pi}{5}$	$\bar{x}$

Taking Moments about V

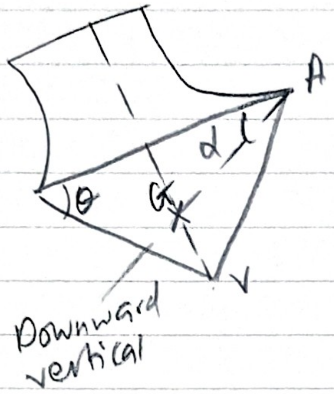
$$\frac{404}{5}\left(\frac{2029}{303}\right)+98\left(\frac{9}{2}\right) = \frac{894}{5}(\bar{x})$$

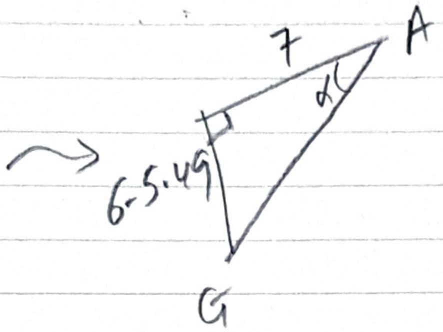
$$\bar{x} = \frac{\frac{404}{5}\left(\frac{2029}{303}\right)+98(9/2)}{\frac{894}{5}}$$

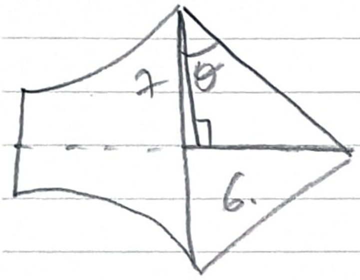
$$= 5.94$$

Question 6 continued

(d)







$$\tan\alpha = \frac{6-5.49}{7}$$

$$\alpha = \tan^{-1}\left(\frac{6-5.49}{7}\right) = \underline{\underline{4.17^\circ}}$$

$$\tan\theta = 6/7 \therefore \theta = \tan^{-1}(6/7)$$

$$\text{angle required} = \theta - \alpha = \tan^{-1}\left(\frac{6}{7}\right) - 4.17$$

$$= \underline{\underline{36^\circ}}$$