Write your name here

Surname	Other names

Pearson Edexcel
International
Advanced Level

Centre Number	Candidate Number

Mechanics M3

Advanced/Advanced Subsidiary

Wednesday 16 May 2018 – Morning
Time: 1 hour 30 minutes

Paper Reference
WME03/01

You must have:
Mathematical Formulae and Statistical Tables (Blue)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}, and give your answer to either two significant figures or three significant figures.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

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1. A light elastic string of modulus of elasticity 29.4 N has one end attached to a fixed point A. A particle P of mass 1.5 kg is attached to the other end of the string and P hangs freely in equilibrium 0.5 m vertically below A. Find the natural length of the string.

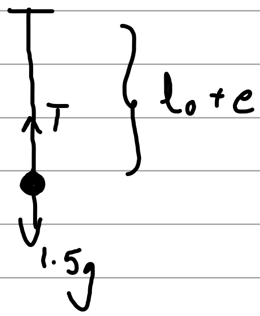
(4)

$\lambda = 29.4\text{N}$

$m = 1.5\text{kg} \Rightarrow W = 1.5g$

$e + l_0 = 0.5\text{m} \qquad l_0 = ?$

$\Rightarrow e = 0.5 - l_0$



$T = \dfrac{\lambda e}{l_0}$ (Hooke's law)

$\Rightarrow 1.5g = \dfrac{29.4e}{l_0}$

$\Rightarrow 1.5g = \dfrac{29.4(0.5 - l_0)}{l_0}$

$\Rightarrow 1.5g\, l_0 = 14.7 - 29.4\, l_0$

$\Rightarrow l_0(1.5g + 29.4) = 14.7$

$\Rightarrow l_0 = \dfrac{14.7}{1.5g + 29.4}$

$g = 9.8 \Rightarrow l_0 = \underline{\underline{0.33\text{m}}}$ (2 sf)

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2. A particle P is moving in a straight line with simple harmonic motion about the fixed point O as centre. When P is a distance 0.02 m from O, the speed of P is 0.3 m s^{-1} and the magnitude of the acceleration of P is 0.5 m s^{-2}

(a) Find the period of the motion.

(4)

The amplitude of the motion is a metres.

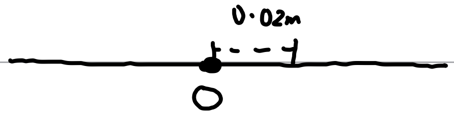
Find

(b) the value of a,

(3)

(c) the total length of time during each complete oscillation for which P is within $\frac{1}{2}a$ metres of O.

(4)



$x = 0.02\text{m}$

$\dot{x} = 0.3\text{ms}^{-1}$

$\ddot{x} = 0.5\text{ms}^{-2}$

$\dot{x} = \frac{dx}{dt} = $ disp.

$\ddot{x} = \frac{d^2x}{dt^2} = $ acc.

(a) $T = ?$

$\ddot{x} = -\omega^2 x$

$\Rightarrow\ 0.5 = -\omega^2(0.02)$

$\Rightarrow\ 25 = -\omega^2$

'$-$' can be ignored because we don't need direction.

$\therefore\ \omega = \sqrt{25} = 5$ rad s^{-1}

$\omega = \frac{2\pi}{T} = 5$

$\Rightarrow\ T = \frac{2\pi}{5} = \underline{\underline{1.26\text{ s}}}$ (2dp)

(b) $a = ?$

$v^2 = \omega^2(a^2 - x^2)$

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Question 2 continued

Based on info. in question and $\omega = 5$

$\Rightarrow (0.3)^2 = 25(a^2 - 0.02^2)$

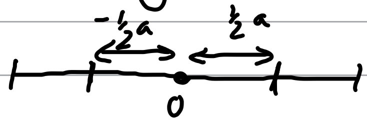
$\Rightarrow \frac{0.09}{25} = a^2 - 0.02^2$

$\Rightarrow \frac{0.09}{25} + 0.02^2 = a^2$

$\Rightarrow 0.004 = a^2$

$\Rightarrow a = \underline{\underline{0.063\,s}}$ (2sf.)

(c) Time during which $-\frac{1}{2}a < x < \frac{1}{2}a$



$x = a\sin\omega t$

based on question:

$\frac{1}{2}\cancel{a} = \cancel{a}\sin(5t)$

$\frac{1}{2} = \sin(5t)$

$\Rightarrow \sin^{-1}\left(\frac{1}{2}\right) = 5t$

$\frac{\pi}{6} = 5t \Rightarrow t = \frac{\pi}{30}$

$\therefore$ Total time during 1 full oscillation $= 4 \times \frac{\pi}{30}$

$= \frac{4\pi}{30}\,s = \underline{\underline{0.42\,s}}$ (2sf)

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3.

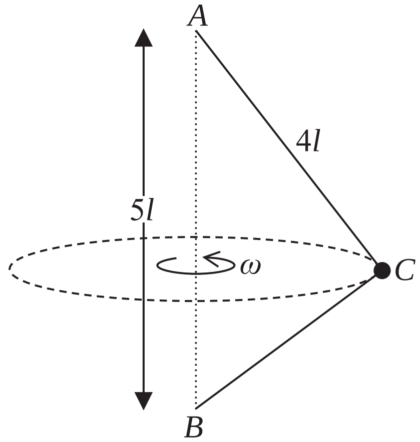


Figure 1

A light inextensible string of length $7l$ has one end attached to a fixed point A and the other end attached to a fixed point B, where A is vertically above B and $AB = 5l$. A particle of mass m is attached to the string at the point C where $AC = 4l$, as shown in Figure 1. The particle moves in a horizontal circle with constant angular speed ω. Both parts of the string are taut.

(a) Find, in terms of m, g, l and ω,

(i) the tension in AC,

(ii) the tension in BC.

(9)

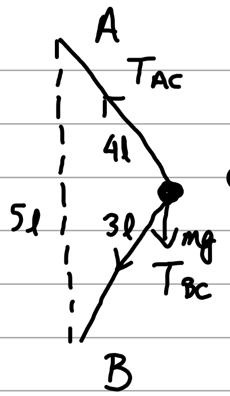
The time taken by the particle to complete one revolution is R.

Given that $R \leqslant k\pi\sqrt{\dfrac{l}{5g}}$

(b) find the least possible value of k.

(3)

(a)



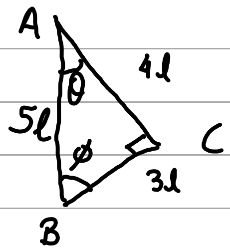
$AC = 4l$

$BC = 7l - 4l = 3l$

Angular speed is ω

(i) $T_{AC} = ?$

ABC is right angled triangle



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Question 3 continued

Resolving forces vertically:

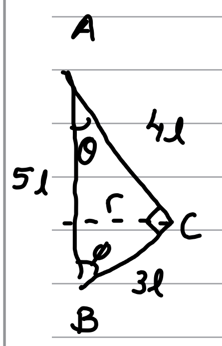
Let $\angle BAC = \theta$ and $\angle ABC = \phi$

$\therefore \quad T_{AC} \cos\theta = mg + T_{BC} \cos\phi$

Using $\triangle ABC$ $\quad \cos\theta = \frac{4}{5} \quad \cos\phi = \frac{3}{5}$

$\Rightarrow \quad \frac{4}{5} T_{AC} = mg + \frac{3}{5} T_{BC} \quad — (I)$

Resolving forces horizontally:



$r = 4l\sin\theta$

$T_{AC} \sin\theta + T_{BC} \sin\phi = m\omega^2 r$ ← radius of horizonal circle.

$\frac{3}{5} T_{AC} + \frac{4}{5} T_{BC} = m\omega^2 4l\sin\theta$ – (II)

$\therefore$ (I) – $\frac{4}{5} T_{AC} - \frac{3}{5} T_{BC} = mg$

(II) – $\frac{3}{\cancel{5}} T_{AC} + \frac{4}{\cancel{5}} T_{BC} = m\omega^2 4l \times \frac{3}{\cancel{5}}$

$$4\left\{ \begin{array}{l} 4T_{AC} - 3T_{BC} = 5mg \\ 3T_{AC} + 4T_{BC} = 12m\omega^2 l \end{array} \right. 3$$

$$\begin{array}{l} 16T_{AC} - \cancel{12T_{BC}} = 20mg \\ 9T_{AC} + \cancel{12T_{BC}} = 36m\omega^2 l \\ \hline 25T_{AC} \qquad = 4m(5g + 9\omega^2 l) \end{array}$$

$\Rightarrow T_{AC} = \frac{4m}{25}(5g + 9\omega^2 l)$

(ii) $4\left(\frac{4m}{25}(5g + 9\omega^2 l)\right) - 3T_{BC} = 5mg$ (Subs. in I)

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Question 3 continued

$\Rightarrow \quad \frac{80}{25}mg + \frac{144}{25}m\omega^2 l - 5mg = 3T_{BC}$

$\Rightarrow \quad \frac{144}{25}m\omega^2 l - \frac{9}{5}mg = 3T_{BC}$

$\Rightarrow \quad \frac{48}{25}m\omega^2 l - \frac{3}{5}mg = T_{BC}$

$\Rightarrow \quad \frac{3}{25}m\left(16\omega^2 l - 5g\right) = T_{BC}$

(b) R which is time period $\leq k\pi\sqrt{\frac{l}{5g}}$

$T_{BC} \geq 0$

$\Rightarrow \frac{3}{25}m\left(16\omega^2 l - 5g\right) \geq 0$

$\Rightarrow \quad 16\omega^2 l \geq 5g$

$\Rightarrow \quad \omega \geq \sqrt{\frac{5g}{16l}}$

$\Rightarrow \quad \frac{2\pi}{R} \geq \frac{1}{4}\sqrt{\frac{5g}{l}}$

$\Rightarrow \quad 8\pi\sqrt{\frac{l}{5g}} \geq R \quad \Rightarrow \quad \underline{\underline{K \geq 8}}$

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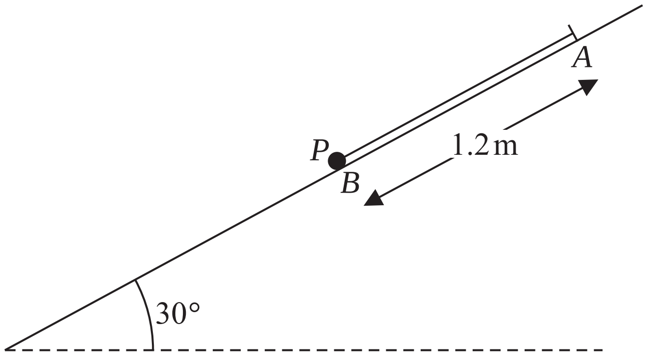
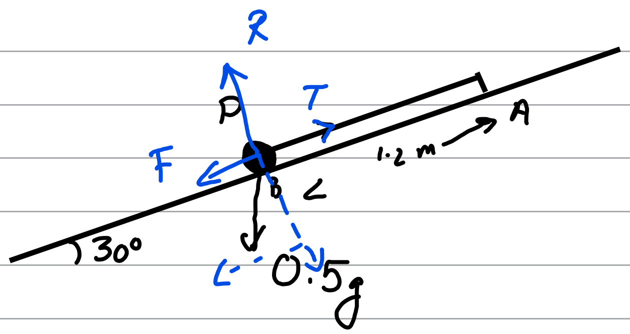


Figure 2

Figure 2 shows a light elastic string, of modulus of elasticity λ newtons and natural length 0.6 m. One end of the string is attached to a fixed point A on a rough plane which is inclined at 30° to the horizontal. The other end of the string is attached to a particle P of mass 0.5 kg. The string lies along a line of greatest slope of the plane. The particle is held at rest on the plane at the point B, where B is lower than A and $AB = 1.2$ m. The particle then receives an impulse of magnitude 1.5 N s in the direction parallel to the string, causing P to move up the plane towards A. The coefficient of friction between P and the plane is 0.7. Given that P comes to rest at the instant when the string becomes slack, find the value of λ.

(8)

$\Rightarrow$



$\lambda = \lambda N$

$l_0 = 0.6 m$

Impulse $\overset{+}{\rightarrow}$ = 1.5 Ns

$\mu = 0.7$

velocity v, i.e. initial velocity for motion after impulse:

$0.5u = 1.5$

$\Rightarrow u = \underline{\underline{3ms^{-1}}}$

Friction down the slope $= \mu R$

$= 0.7 \times 0.5g \cos 30$

P 5 1 3 0 3 A 0 1 2 2 8

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Question 4 continued

$$= \frac{7g\sqrt{3}}{40} \text{ N}$$

Conservation of energy:

Initial KE + Initial GPE + Initial EPE + Work

=

Final KE + Final GPE + Final EPE + Work against friction

$$\Rightarrow \frac{1}{2}mu^2 + mgh_0 + \frac{\lambda x^2}{2l} + \underbrace{Fs}_{\text{force}} = \frac{1}{2}mv^2 + mgh_1 + \frac{\lambda x^2}{2l} + \underbrace{F_r}_{\text{Friction}} s$$

$$\Rightarrow \frac{1}{2} \times 0.5 \times 9 + 0 + \frac{\lambda(0.6)^2}{2(0.6)} + 0 = 0 + 0.5g(0.6\sin 30) + 0 + \frac{7g\sqrt{3}}{40} \times 0.6$$

$$\Rightarrow 2.25 + 0.3\lambda = 0.15g + \frac{21g\sqrt{3}}{200}$$

$$\Rightarrow \lambda = 3.34 \text{ N} \quad (3\text{sf.})$$

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5. A particle P of mass 0.8 kg moves along the x-axis in the positive x direction under the action of a resultant force. This force acts in the direction of x increasing. At time t seconds, $t \geqslant 0$, P is x metres from the origin O, P is moving with speed v m s^{-1} and the force has magnitude $\dfrac{4}{(x+1)^3}$ N.

When $t = 0$, P is at rest at O.

(a) Show that $v^2 = 5\left(\dfrac{(x+1)^2 - 1}{(x+1)^2}\right)$

(6)

When $t = 2$, P is at the point A. When $t = 4$, P is at the point B.

(b) Using algebraic integration, find the distance AB.

(7)

$O \longrightarrow x$ $\qquad m = 0.8\text{ kg}$

time $= t$ s

$S_x = x$ m

$V_x = v$ ms^{-1}

$F = \dfrac{4}{(x+1)^3}$ N

$t = 0 \quad v = 0$

(a) $F = ma \qquad (N2L)$

$\therefore \dfrac{4}{(x+1)^3} = 0.8a$

$a = \dfrac{dv}{dt} \qquad v = \dfrac{dx}{dt}$

$\Rightarrow dt = \dfrac{dv}{a} \qquad dt = \dfrac{dx}{v}$

$\Rightarrow \dfrac{dv}{a} = \dfrac{dx}{v} \Rightarrow v\dfrac{dv}{dx} = a$

$\therefore \dfrac{4}{(x+1)^3} = 0.8v\dfrac{dv}{dx}$

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Question 5 continued

$$\Rightarrow \quad v\frac{dv}{dx} = \frac{5}{(x+1)^3}$$

$$\Rightarrow \int v \, dv = \int \frac{5}{(x+1)^3} \, dx$$

$$\Rightarrow \frac{v^2}{2} = -\frac{5}{2}(x+1)^{-2} + c$$

$$\Rightarrow \frac{v^2}{2} = \frac{-5}{2(x+1)^2} + c$$

$$\Rightarrow \frac{v^2}{2} = \frac{-5}{2(x+1)^2} + c$$

acc. to question $t=0$, $x=0 \Rightarrow v=0$

$$\therefore \quad c = \frac{5}{2}$$

$$\Rightarrow \frac{v^2}{\cancel{2}} = \frac{-5}{\cancel{2}(x+1)^2} + \frac{5}{\cancel{2}}$$

$$\Rightarrow v^2 = \frac{-5}{(x+1)^2} + 5$$

$$\Rightarrow v^2 = \frac{-5+5(x+1)^2}{(x+1)^2} = 5\left(\frac{(x+1)^2-1}{(x+1)^2}\right)$$

shown

(b) $t=2$ P is at A

$t=4$ P is at B

$$v = \frac{dx}{dt} = \sqrt{5\left(\frac{(x+1)^2-1}{(x+1)^2}\right)}$$

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Question 5 continued

$$\Rightarrow \frac{dx}{dt} = \frac{\sqrt{5}\sqrt{(x+1)^2 - 1}}{x+1}$$

$$\Rightarrow \int \frac{x+1}{\sqrt{(x+1)^2 - 1}}\,dx = \int \sqrt{5}\,dt$$

$$\Rightarrow \sqrt{(x+1)^2 - 1} = \sqrt{5}t + K$$ → because we used c before.

$t = 0 \quad x = 0$

$\therefore \quad K = 0$

$$\Rightarrow \sqrt{(x+1)^2 - 1} = \sqrt{5}\,t$$

$$\Rightarrow (x+1)^2 - 1 = 5t^2$$

$$\Rightarrow (x+1)^2 = 5t^2 + 1$$

$$\Rightarrow x+1 = \sqrt{5t^2+1}$$

$$\Rightarrow x = \sqrt{5t^2+1} - 1$$

$t = 2$ $\qquad\qquad$ $t = 4$

$\Rightarrow x = \sqrt{21} - 1$ $\qquad\qquad$ $\Rightarrow x = \sqrt{81} - 1 = 8$

$AB = 8 - (\sqrt{21} - 1)$

$= \underline{\underline{4.42\text{m}}}$ (3 s.f.)

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6. A uniform solid right circular cone has base radius r and height h.

(a) Use algebraic integration to show that the distance of the centre of mass of the cone from its vertex is $\frac{3}{4}h$.

(5)

[*You may assume that the volume of a cone of base radius r and height h is* $\frac{1}{3}\pi r^2 h$]

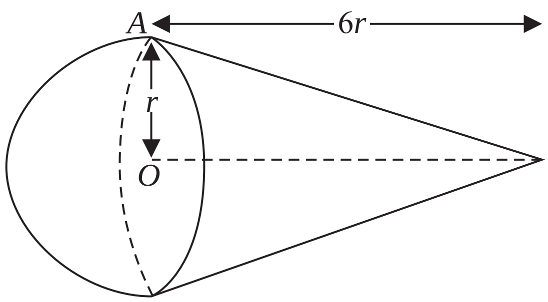

Figure 3

A solid S is formed by joining a uniform right circular solid cone of mass $5m$ to a uniform solid hemisphere, of radius r and mass km where $k < 20$. The cone has base radius r and height $6r$. The plane face of the cone coincides with the plane face of the hemisphere. The centre of the plane face of the cone is O and the point A is on the circular edge of this plane face, as shown in Figure 3.

(b) Find the distance from O to the centre of mass of S.

(4)

The solid is suspended from A and hangs freely in equilibrium. The angle between the axis of the cone and the horizontal is 30°.

(c) Find, to the nearest whole number, the value of k.

(4)

(a) $\pi\int_a^b y^2 x \, dx = \pi\int_0^h \left(\frac{r}{h}\right)^2 x^3 \, dx$

$= \left[\pi\left(\frac{r}{h}\right)^2 \frac{x^4}{4}\right]_0^h = \left(\pi \frac{r^2}{h^2}\frac{h^4}{4}\right) - 0$

$= \frac{\pi r^2 h^2}{4}$

$\Rightarrow V\bar{y} = \frac{\pi r^2 h^2}{4}$

$\Rightarrow \frac{1}{3}\cancel{\pi}\cancel{r^2}\cancel{h}\,\bar{y} = \frac{\cancel{\pi}\cancel{r^2}h^{\cancel{2}}}{4}$

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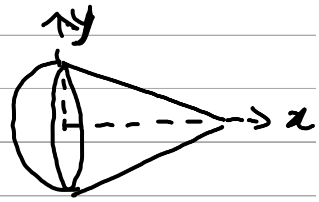
Question 6 continued

$\Rightarrow \quad \bar{y} = \frac{3}{4}h$ shown.

(b)



	σ	x
Cone	$5m$	$\frac{3}{2}r$
Hemisphere	Km	$-\frac{3}{8}r$
Σ	$(5+k)m$	$\bar{x}$

$$M\bar{x} = \Sigma mx$$

$$\Rightarrow (5+k)\cancel{m}\bar{x} = \frac{15}{2}\cancel{m}r - \frac{3}{8}K\cancel{m}r$$

$$\Rightarrow (5+k)\bar{x} = \frac{60r - 3kr}{8}$$

$$\therefore \bar{x} = \frac{3r(20-k)}{8(5+k)}$$

c) $\angle$ b/w AO and $\updownarrow = 30°$

$$\Rightarrow \tan 30 = \frac{\bar{x}}{r} = \frac{3(20-k)}{8(5+k)}$$

$$\Rightarrow \frac{\sqrt{3}}{3} = \frac{3(20-k)}{8(5+k)}$$

$$\Rightarrow 40\sqrt{3} + 8\sqrt{3}k = 180 - 9k$$

$$\Rightarrow (9 + 8\sqrt{3})k = 180 - 40\sqrt{3}$$

$$\Rightarrow K = \underline{\underline{4.84}} \quad (3sf)$$

~ 5 to nearest integer.

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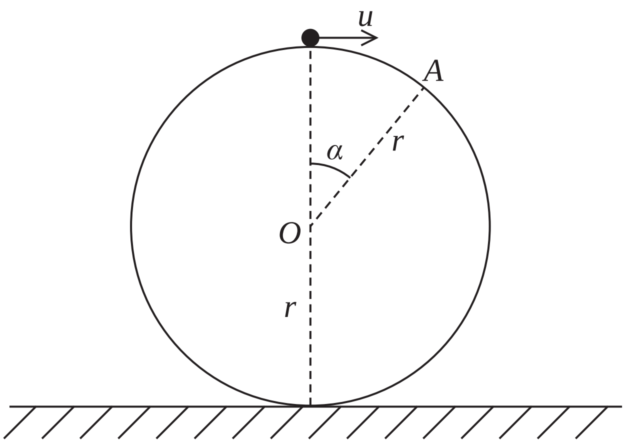


Figure 4

A smooth solid sphere, with centre O and radius r, is fixed with its lowest point on a horizontal plane. A particle is placed on the surface of the sphere at the highest point of the sphere. The particle is then projected horizontally with speed u and starts to move on the surface of the sphere. The particle leaves the surface of the sphere at the point A where OA makes an angle α, $\alpha > 0$, with the upward vertical, as shown in Figure 4.

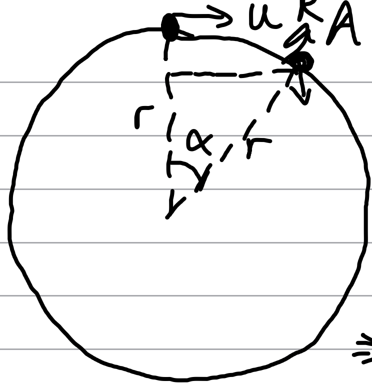
(a) Show that $\cos\alpha = \frac{1}{3gr}\left(u^2 + 2gr\right)$ **(7)**

(b) Show that $u < \sqrt{gr}$ **(2)**

After leaving the surface of the sphere, the particle strikes the plane with speed $3\sqrt{\frac{gr}{2}}$

(c) Find the value of $\cos\alpha$. **(5)**

(a)



Conservation of energy:

$\frac{1}{2}mu^2 + mgh_0 = \frac{1}{2}mv^2 + mgh_1$

$\Rightarrow \frac{1}{2}mu^2 + mg(r) = \frac{1}{2}mv^2 + mgr\cos\alpha$

$\Rightarrow mgr - mgr\cos\alpha = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

$\Rightarrow 2gr(1-\cos\alpha) = v^2 - u^2$ — (i)

N2L radially to the centre:

$mg\cos\alpha - R = \frac{mv^2}{r}$ — (ii)

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Question 7 continued

Subs (i) in (ii) :

$$mg\cos\alpha - R = \frac{m}{r}\left(2gr - 2gr\cos\alpha + u^2\right)$$

$R = 0$ as mass is at point of leaving sphere.

$$\therefore g\cos\alpha = 2g - 2g\cos\alpha + \frac{u^2}{r}$$

$$3g\cos\alpha = 2g + \frac{u^2}{r}$$

$$\Rightarrow \cos\alpha = \frac{2}{3} + \frac{u^2}{3gr}$$

$$\Rightarrow \cos\alpha = \frac{1}{3gr}\left(u^2 + 2gr\right) \text{ shown.}$$

(b) T.P. $u < \sqrt{gr}$

$\cos\alpha < 1$

$$\Rightarrow \frac{1}{3gr}\left(u^2 + 2gr\right) < 1$$

$$\Rightarrow u^2 + 2gr < 3gr$$

$$\Rightarrow u < \sqrt{gr} \text{ shown.}$$

(c) $\therefore$ Loss of PE = Gain in KE

$$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = mg(2r)$$

$$\Rightarrow \frac{1}{2}\cancel{m}\left(3\sqrt{\frac{gr}{2}}\right)^2 - \frac{1}{2}\cancel{m}u^2 = 2\cancel{m}gr$$

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Question 7 continued

$\Rightarrow \frac{1}{2} \times \frac{9}{2}gr - \frac{1}{2}u^2 = 2gr$

$\Rightarrow \frac{1}{2}u^2 = \frac{1}{4}gr$

$\Rightarrow u^2 = \frac{1}{2}gr$

Subs in $\cos\alpha$:

$\cos\alpha = \frac{1}{3gr}\left(\frac{1}{2}gr + 2gr\right)$

$\cos\alpha = \frac{1}{6} + \frac{2}{3}$

$\cos\alpha = \frac{5}{6}$

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