M3 June 2017 IAL (MA)

Q1) $V = \pi \int_2^4 y^2 \, dx = \pi \int_2^4 \frac{x}{4} dx = \pi \left[\frac{x^2}{8} \right]_2^4 = \pi[2] - \pi\left[\frac{1}{2}\right]$

$= \frac{3\pi}{2}$ //

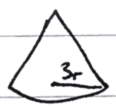
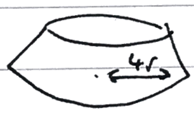
$\therefore M = \frac{3p\pi}{2}$

$M\bar{x} = p\pi \int_2^4 y^2 x \, dx = p\pi \int_2^4 \left[\frac{x^2}{4}\right] dx = p\pi \left[\frac{x^3}{12} \right]_2^4$

$= p\pi\left[\frac{16}{3}\right] - p\pi\left[\frac{2}{3}\right] = \frac{14p\pi}{3}$ //

$\therefore \bar{x} = \dfrac{\frac{14p\pi}{3}}{\frac{3p\pi}{2}} = \boxed{\frac{28}{9}}$

Q2a)

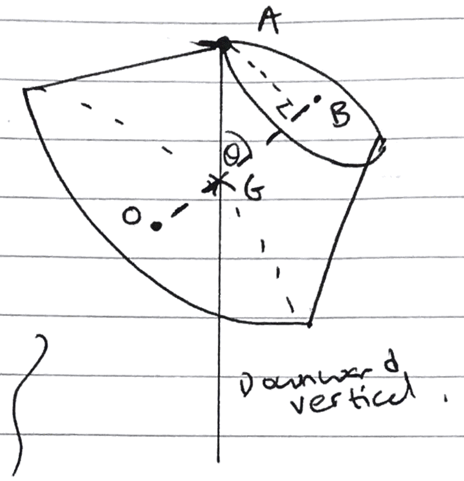
Shape	Mass ratio (Vol.)	Distance of c.o.m from O
 4h	$\frac{1}{3}\pi(4r)^2(4h) = \boxed{\frac{64\pi r^2 h}{3}}$	h
(−)  3h	$\frac{1}{3}\pi(3r)^2(3h) = \boxed{9\pi r^2 h}$	$h + \frac{3h}{4} = \frac{7h}{4}$
 h	$= \boxed{\frac{37}{3}\pi r^2 h}$	$\bar{y}$

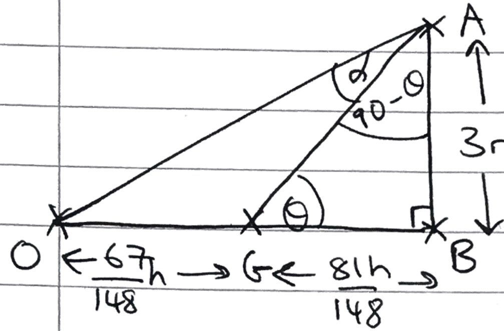
taking moments about diameter through O:

$\frac{64}{3}(h) - 9\left(\frac{7h}{4}\right) = \frac{37}{3}(\bar{y})$

$\therefore \bar{y} = \left(\dfrac{\frac{64}{3} - \frac{63}{4}}{37/3}\right) h = \boxed{\frac{67}{148} h}$

b)





angle required $= \alpha$.

$\tan\theta = \dfrac{3r}{\frac{81h}{148}} = 148/27$ $(h=r)$.

$\therefore \theta = \tan^{-1}\left(\frac{148}{27}\right) = 79.66°$

$\therefore 90 - \theta = 10.34°$

<u>finding $\angle OAB$</u>: $\tan(OAB) = \dfrac{h}{3r} = \dfrac{1}{3}$ $(h=r)$

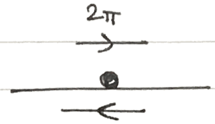
$\therefore \angle OAB = \tan^{-1}\left(\frac{1}{3}\right) = 18.43°$

$\therefore \alpha = 18.43° - 10.34° = \boxed{8.10°}$

3a) 4 oscillations in 1 second $\longrightarrow T = \frac{1}{4}$. $\therefore \omega = \dfrac{2\pi}{\frac{1}{4}} = 8\pi$

amplitude $= 0.25m$.

$\therefore a_{max} = a\omega^2 = 0.25 \times (8\pi)^2 = \boxed{158 \text{ ms}^{-2}}$

b) 

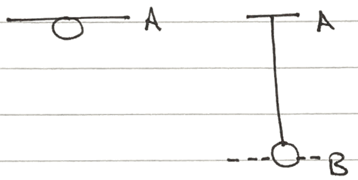
$V_{max} = a\omega = 0.25 \times 8\pi = (2\pi) m/s$

For new S.H.M $\begin{cases} T = \frac{1}{4} \text{ (unchanged)} & \therefore \omega = 8\pi \\ a = 0.125m \end{cases}$

new max speed = speed right after impulse

$= 0.125 \times 8\pi = \pi$

$I = m(v-u) = 0.5(\pi - -2\pi) = \boxed{\frac{3\pi}{2} Ns}$

4a) 

$\underline{\Delta GPE = \Delta EPE}$

let $AB = d$

$\Delta GPE = 0.3gd$

$\Delta EPE = \frac{\lambda x^2}{2l} = \frac{49}{0.8}(x-0.4)^2$

$\Rightarrow 0.3gd = \frac{245}{4}(d^2 - 0.8d + 0.16)$

$\Rightarrow 0.048d = d^2 - 0.8d + 0.16$

$\Rightarrow d^2 - 0.848d + 0.16 = 0$

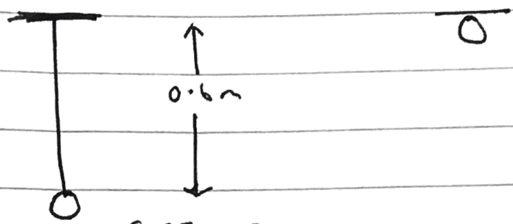
Quadratic Formula: $d = 0.5646...$

$d = 0.28337...$

but $AB > 0.4$

$\therefore AB = \boxed{0.56m}$

b)



$KE = 0.15v^2$
$GPE = 0.3g(0.6)$
$EPE = 0$

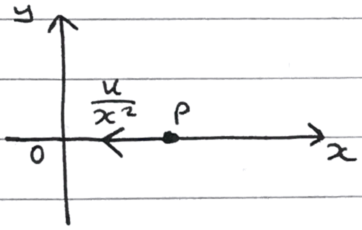
$GPE = 0$
$KE = 0$
$EPE = \frac{49}{0.8}(0.2)^2$

Conservation of Energy

$$\frac{49}{0.8}(0.2)^2 = 0.15v^2 + 1.764$$

$$0.15v^2 = 0.686$$
$$\therefore v^2 = 4.57\dot{3}$$
$$\therefore \boxed{v = 2.14 m/s}$$

5a)



$\xrightarrow{}$ N2L (P) : $-\frac{k}{x^2} = 0.4v\frac{dv}{dx}$ $\quad \left(a = v\frac{dv}{dx}\right)$

$$\Rightarrow 0.4\int (v)\,dv = -k\int (x^{-2})\,dx$$

$$\Rightarrow \frac{0.4v^2}{2} = -k\left[-\frac{1}{x}\right] + c$$

$$\Rightarrow 0.2v^2 = \frac{k}{x} + c$$

$x=2,\ v=5$: $0.2(25) = \frac{k}{2} + c$

$$\therefore 5 = \frac{k}{2} + c \quad —①$$

$\underline{x=5, v=2}$: $0.2(4) = \frac{u}{5} + c$

$0.8 = \frac{u}{5} + c \quad \text{—} \; 2$

$1 - 2$: $5 - 0.8 = \frac{u}{2} - \frac{u}{5} + c - c$

$\Rightarrow \quad 4.2 = \frac{3}{10} u \qquad \therefore \boxed{u = 14}$

and $c = 5 - \frac{u}{2} = 5 - 7 = -2$

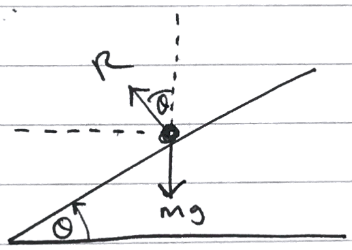
b) $0.2v^2 = \frac{14}{x} + (-2)$

$\underline{v=0}$: $\frac{14}{x} - 2 = 0$

$\frac{14}{x} = 2$

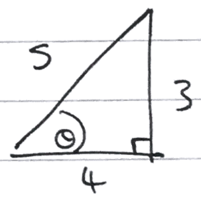
$\therefore x = \frac{14}{2} = \boxed{7m} = OA.$

6a)



$\sin\theta = 3/5$

$r = 50$



$R(\updownarrow)$: $R\cos\theta = mg \quad \sim 1$

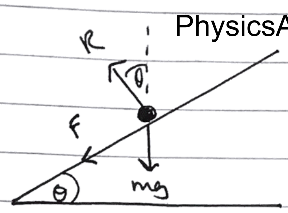
$R(\leftrightarrow^{+})$: $R\sin\theta = \frac{mv^2}{r} = \frac{mv^2}{50} \quad \text{—} \; 2$

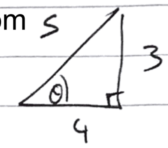
$\frac{2}{1}$: $\tan\theta = \frac{\cancel{m}v^2}{50\cancel{m}g} = \frac{v^2}{50g}$

$\therefore v^2 = 50g\tan\theta = 50g\left(\frac{3}{4}\right) = 367.5$

$\therefore v = \sqrt{367.5} = \boxed{19.2m/s}$

b)





$R(\updownarrow)$: $R\cos\theta = mg + F\sin\theta$ — ①

$R(\overset{+}{\leftrightarrow})$: $R\sin\theta + F\cos\theta = \dfrac{mv^2}{50}$ — ②

① : $R\cos\theta - F\sin\theta = mg$ //

now $\dfrac{②}{①}$: $\dfrac{R\sin\theta + F\cos\theta}{R\cos\theta - F\sin 30} = \dfrac{mv^2}{50mg}$

$\Rightarrow \dfrac{R\sin\theta + F\cos\theta}{R\cos\theta - F\sin 30} = \dfrac{v^2}{50g}$ //

A larger frictional force means v will be greater
∴ We want F to be max (ie $\underline{\underline{F_{max} = \mu R}}$)

$(\mu = 1/4)$

$\Rightarrow \dfrac{R\sin\theta + \frac{1}{4}R\cos\theta}{R\cos\theta - \frac{1}{4}R\sin(\theta)} = \dfrac{v^2}{50g}$.

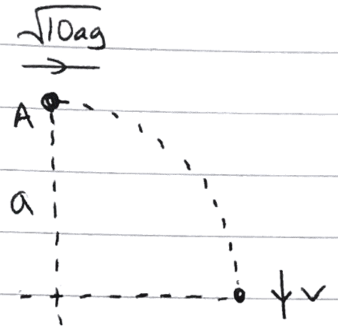
cancelling 'R':
$\Rightarrow \dfrac{\sin\theta + \frac{1}{4}\cos\theta}{\cos\theta - \frac{1}{4}\sin(\theta)} = \dfrac{v^2}{50g}$ //

$\sin\theta = 3/5$ $\quad\cos\theta = \dfrac{4}{5}$

$\therefore V^2_{max} = 50g\left[\dfrac{\frac{3}{5} + \frac{1}{4}\left(\frac{4}{5}\right)}{\frac{4}{5} - \frac{1}{4}\left(\frac{3}{5}\right)}\right] = 603.077\ldots$

$\therefore \boxed{V_{Max} = 24.6 \text{ m/s}}$

7c)



$\underline{C.O.E} \Rightarrow (KE + GPE) = (KE)$

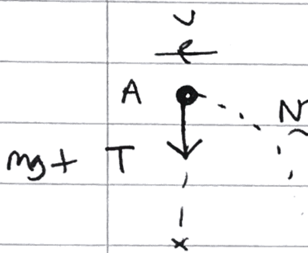
at top — at bottom

$\Rightarrow \frac{1}{2}m(10ag) + mga = \frac{1}{2}m(v^2)$

$\Rightarrow \frac{6ag}{\frac{1}{2}} = v^2 = 12ag \quad \therefore v = \sqrt{12ag}$ //

so speed after rebounding $= \left[e\sqrt{12ag}\right]$ by N.L.L.

we are told the particle reaches A after rebounding, so $T \geq 0$ at the top.



$\underline{N2L}\,(\downarrow)$: $T + mg = \frac{mv^2}{a}$

$T = \frac{mv^2}{a} - mg \geq 0$ //

<u>using C.O.E to find speed at A after rebounding:</u>

(KE) = (GPE + KE)

at bottom — at top.

$\frac{1}{2}\cancel{m}(12age^2) = \cancel{m}ga + \frac{1}{2}\cancel{m}v^2$

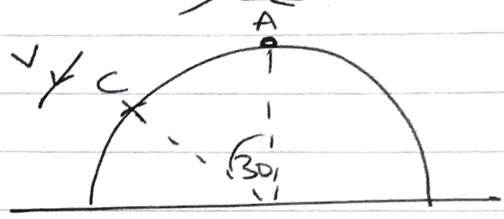
$6age^2 - ag = \frac{1}{2}v^2 \quad \therefore v^2 = 12age^2 - 2ag$

<u>subbing into inequality with T:</u> $\frac{m}{a}(12age^2 - 2ag) - mg \geq 0$

$\Rightarrow 12ge^2 - 2g - g \geq 0$

$\Rightarrow e^2 \geq \frac{3g}{12g} \rightarrow e^2 \geq \frac{1}{4} \quad \therefore \boxed{e \geq \frac{1}{2}}$

b) at top, $v = \sqrt{ag\left(12\left(\frac{\sqrt{3}}{2}\right)^2 - 2\right)} = \sqrt{7ag}$ // $(\sqrt{7ag})$



using c.o.e to find speed at C :

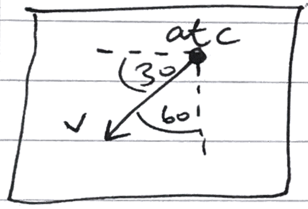
(KE + GPE) = (KE + GPE)
↑ at A ↑ at C

$\Rightarrow \frac{1}{2}\cancel{m}(7ag) + \cancel{m}ga = \frac{1}{2}\cancel{m}v^2 + \cancel{m}ga\cos 30$

$\Rightarrow \frac{9}{2}ag - ag\cos 30 = \frac{1}{2}v^2 \rightarrow v^2 = 9ag - 2ag\cos 30$ //

$= ag(9 - 2\cos 30)$.

$\therefore \overleftarrow{v}$ (horizontal component) $= \left[\sqrt{ag(9-2\cos 30)}\right]\cos 30$



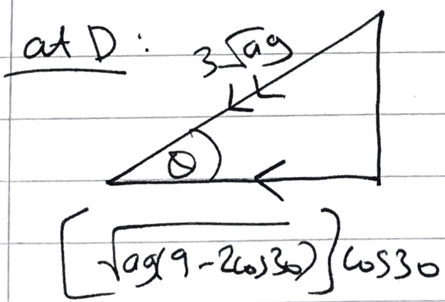
We can find the final speed at D using c.o.e :

(KE + GPE) = (KE)
↑ at A ↑ at D

$\Rightarrow \frac{1}{2}\cancel{m}(7ag) + \cancel{m}ga = \frac{1}{2}\cancel{m}v^2$

$\Rightarrow v^2 = 9ag \quad \therefore \boxed{v = 3\sqrt{ag}}$

at D :



$\therefore \cos\theta = \dfrac{\cos 30\sqrt{9-2\cos 30} \times \cancel{\sqrt{ag}}}{3 \times \cancel{\sqrt{ag}}}$

$= 0.7782\ldots$

$\therefore \boxed{\theta = 38.9°}$ //