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Pearson Edexcel
International Advanced Level

Centre Number

Candidate Number

Mechanics M3

Advanced/Advanced Subsidiary

Monday 19 May 2014 – Morning
Time: 1 hour 30 minutes

Paper Reference
WME03/01

You must have:
Mathematical Formulae and Statistical Tables (Blue)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}, and give your answer to either two significant figures or three significant figures.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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1. A particle P moves in a straight line with simple harmonic motion. The period of the motion is $\frac{\pi}{4}$ seconds. At time $t = 0$, P is at rest at the point A and the acceleration of P has magnitude 20 m s^{-2}.

Find

(a) the amplitude of the motion,

(3)

(b) the greatest speed of P during the motion,

(1)

(c) the time P takes to travel a total distance of 1.5 m after it has first left A.

(4)

$T = \frac{\pi}{4}s \implies \omega = \frac{2\pi}{T} = 8 \text{ rads}^{-1}$; $a_{max} = 20 \text{ ms}^{-2}$

a) $a_{max} = \omega^2 A$

$A = \frac{a_{max}}{\omega^2} = \frac{20}{8^2} = 0.3125 \text{ m}$

b) $V_{max} = \omega A = 8 \times 0.3125 = 2.5 \text{ ms}^{-1}$

c) Distance after 1 period $= 0.3125 \times 4$
$= 1.25 \text{ m}$

$\therefore$ 0.25 m left.

$x = A\cos\omega t$

$0.25 = 0.3125 \cos(8t)$

$8t = \cos^{-1}(0.8) = 0.644$

$t = 0.080 \text{ s}$

$\therefore$ Total time is $0.080 + \frac{\pi}{4} = 0.87 \text{ s}$

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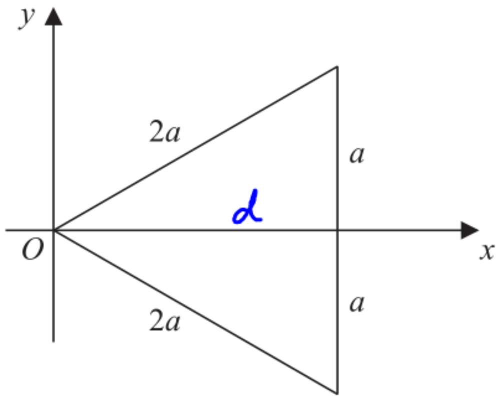

Figure 1

A uniform lamina L is in the shape of an equilateral triangle of side $2a$. The lamina is placed in the xy-plane with one vertex at the origin O and an axis of symmetry along the x-axis, as shown in Figure 1.

Use algebraic integration to find the x coordinate of the centre of mass of L.

(6)

$d^2 = (2a)^2 - a^2 = 3a^2$

$d = \sqrt{3}\,a$

$A = \frac{1}{2} \times 2a \times \sqrt{3}\,a = \sqrt{3}\,a^2$

Gradient of one side : $m = \frac{a}{2a} = \frac{1}{2}$

$\Rightarrow$ Eqn. of side : $y = \frac{x}{2}$

$A\bar{x} = \int_0^{\sqrt{3}a} 2yx\,dx = \int_0^{\sqrt{3}a} x^2\,dx = \left[\frac{x^3}{3}\right]_0^{\sqrt{3}a}$

$= \frac{3\sqrt{3}\,a^3}{3} = \sqrt{3}\,a^3$

$\bar{x} = \frac{\sqrt{3}\,a^3}{\sqrt{3}\,a^2} = a$

P 4 4 5 2 1 A 0 4 2 8

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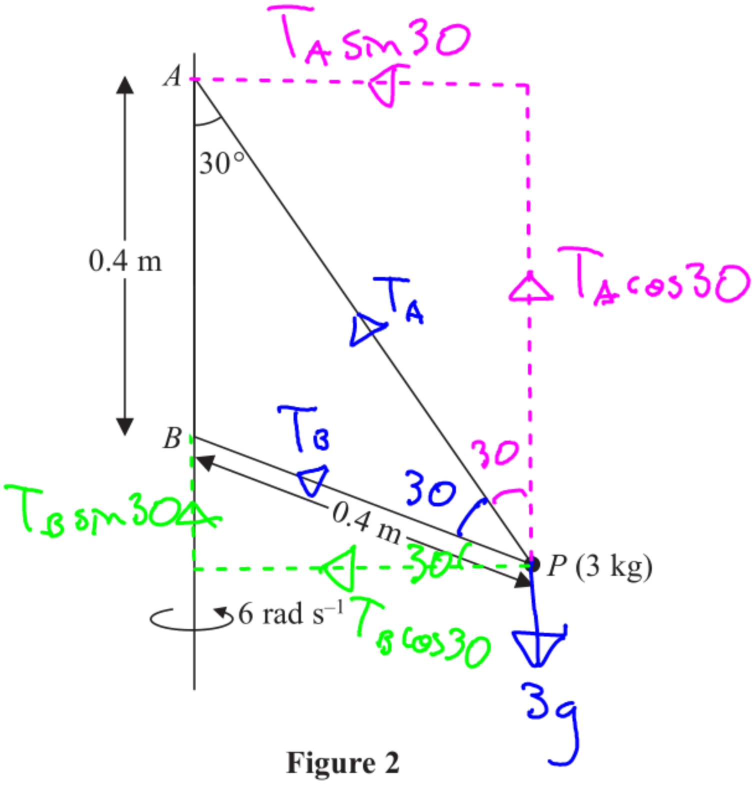

Figure 2

A particle P of mass 3 kg is attached by two light inextensible strings to two fixed points A and B on a fixed vertical pole. Both strings are taut and P is moving in a horizontal circle with constant angular speed 6 rad s^{-1}. String AP is inclined at 30° to the vertical. String BP has length 0.4 m and A is 0.4 m vertically above B, as shown in Figure 2.

Find the tension in

(i) AP,

(ii) BP.

(9)

$\downarrow$ $\exists$ equilibrium

$$3g - T_B \sin 30 - T_A \cos 30 = 0$$

$$6g - \sqrt{3}\,T_A = T_B \quad (1)$$

$\leftarrow$ [F=ma] $T_B \cos 30 + T_A \sin 30 = m\omega^2 r$

$$\frac{\sqrt{3}\,T_B}{2} + \frac{T_A}{2} = 3(6)^2\, 0.4\cos 30$$

$$\sqrt{3}\,T_B + T_A = 43.2\sqrt{3} \quad (2)$$

P 4 4 5 2 1 A 0 8 2 8

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Question 3 continued

① in ②: $\sqrt{3}\left(6g-\sqrt{3}T_A\right)+T_A=43.2\sqrt{3}$

$2T_A=6\sqrt{3}g-43.2\sqrt{3}$

$T_A=13.5\text{ N}$

In ①: $T_B=6g-\sqrt{3}(13.5)$

$=35.4\text{ N}$

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4. At time $t = 0$, a particle P of mass 0.4 kg is at the origin O moving with speed 4 m s^{-1} along the x-axis in the positive x direction. At time t seconds, $t \geqslant 0$, the resultant force acting on P has magnitude $\dfrac{4}{(t+5)^2}$ N and is directed away from O.

(a) Show that the speed of P cannot exceed 6 m s^{-1}.

(5)

The particle passes through the point A when $t = 2$ and passes through the point B when $t = 7$

(b) Find the distance AB.

(4)

(c) Find the gain in kinetic energy of P as it moves from A to B.

(3)

a) $[F = ma] \rightarrow$

$$\frac{4}{(t+5)^2} = 0.4\frac{dv}{dt}$$

$$\int_0^t \frac{dt}{(t+5)^2} = \int_4^v 0.1\,dv$$

$$\left[\frac{-1}{t+5}\right]_0^t = 0.1v - 0.4$$

$$0.1v = \frac{1}{5} - \frac{1}{t+5} + 0.4$$

$$v = 6 - \frac{10}{t+5}$$

As $t \to \infty$, $\dfrac{10}{t+5} \to 0$

$\therefore V_{max} = 6\text{ms}^{-1}$

Leave blank

Question 4 continued

b) $\frac{dx}{dt} = 6 - \frac{10}{t+5}$

$$\int_0^x dx = \int_0^t \left(6 - \frac{10}{t+5}\right) dt$$

$$x = 6t - 10\ln|t+5| + 10\ln 5$$

When $t = 2$, $x_A = 6(2) - 10\ln 7 + 10\ln 5$

$t = 7$, $x_B = 6(7) - 10\ln 12 + 10\ln 5$

$AB = x_2 - x_1 = 42 - 12 - 10\ln 12 + 10\ln 7$

$= 30 + \ln\left(\frac{7^{10}}{12^{10}}\right) = 24.6\text{m}$ $\quad v = 6 - \frac{10}{t+5}$

c) $\Delta KE = \frac{1}{2}m\left(v_B^2 - v_A^2\right) = 0.2[$

$= 0.2\left[\left(6 - \frac{10}{12}\right)^2 - \left(6 - \frac{10}{7}\right)^2\right]$

$= 1.16\text{ J}$

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5.

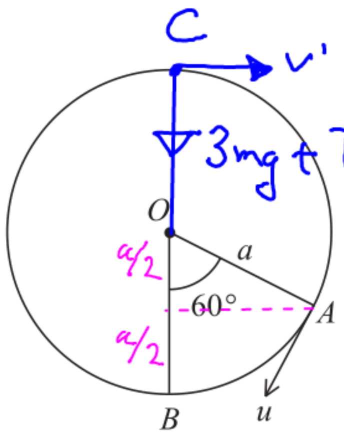


Figure 3

A particle P of mass $2m$ is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. Initially the particle is at the point A where $OA = a$ and OA makes an angle 60° with the downward vertical. The particle is projected downwards from A with speed u in a direction perpendicular to the string, as shown in Figure 3. The point B is vertically below O and $OB = a$. As P passes through B it strikes and adheres to another particle Q of mass m which is at rest at B.

(a) Show that the speed of the combined particle immediately after the impact is $\frac{2}{3}\sqrt{u^2 + ag}$.

(6)

(b) Find, in terms of a, g, m and u, the tension in the string immediately after the impact.

(3)

The combined particle moves in a complete circle.

(c) Show that $u^2 \geqslant \frac{41ag}{4}$.

(6)

a) B m u' ← A $2m$ $\Rightarrow$ V ← $3m$

Find u' using CoE

$$\frac{1}{\cancel{2}}(\cancel{2m})u^2 + \cancel{2m}g\frac{a}{\cancel{2}} = \frac{1}{\cancel{2}}(\cancel{2m})(u')^2$$

$$u' = \sqrt{u^2 + ga}$$

CoM: $2mu' = 3mv$

$$v = \frac{2}{3}u' = \frac{2}{3}\sqrt{u^2 + ag}$$

P 4 4 5 2 1 A 0 1 6 2 8

Leave blank

Question 5 continued

b) $\uparrow [F = ma]$

$$T - 3mg = \frac{3mv^2}{a}$$

$$T = 3mg + \frac{\cancel{3}m}{a} \cdot \frac{4}{\cancel{9}_3}(u^2 + ag)$$

$$= 3mg + \frac{4mu^2}{a} + 4mg$$

$$= 7mg + \frac{4mu^2}{a}$$

c) Find v' using CoE:

$$\frac{1}{\cancel{2}}(\cancel{3m})\frac{4}{9}(u^2 + ag) = \frac{1}{\cancel{2}}(\cancel{3m})(v')^2 + \cancel{3m}g(\cancel{2}^{4}a)$$

$$(v')^2 = \frac{4u^2}{9} + ag\left(\frac{4}{9} - 4\right)$$

For complete circle, $T \geqslant 0$ at C.

$\downarrow [F = ma]$: $T + 3mg = \frac{3m(v')^2}{a}$

$$0 \leqslant \frac{3m(v')^2}{a} - 3mg$$

$$\frac{\cancel{3m}}{a}\left(\frac{4u^2}{9} - \frac{32ag}{9}\right) - \cancel{3m}g \geqslant 0$$

$$4u^2 - 32ag \geqslant 9ag$$

$$u^2 \geqslant \frac{41ag}{4}$$

6. A particle of mass m is attached to one end of a light elastic string, of natural length $6a$ and modulus of elasticity $9mg$. The other end of the string is attached to a fixed point A on a ceiling. The particle hangs in equilibrium at the point B, where B is vertically below A and $AB = (6 + p)a$.

(a) Show that $p = \dfrac{2}{3}$ **(2)**

The particle is now released from rest at a point C vertically below B, where $AC < \dfrac{22}{3}a$.

(b) Show that the particle moves with simple harmonic motion. **(4)**

(c) Find the period of this motion. **(2)**

(d) Explain briefly the significance of the condition $AC < \dfrac{22}{3}a$. **(1)**

The point D is vertically below A and $AD = 8a$. The particle is now released from rest at D. The particle first comes to instantaneous rest at the point E.

(e) Find, in terms of a, the distance AE. **(4)**

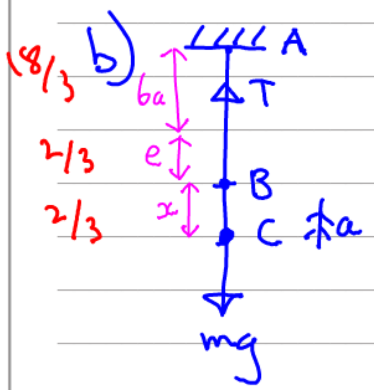
a)

$$T = \frac{\lambda e}{l} = mg$$

$$e = \frac{mg \cdot 6a}{9mg} = \frac{2a}{3}$$

$$AB = l + e = 6a + \frac{2a}{3} = a\left(6 + \frac{2}{3}\right)$$

b)



$$\downarrow \left[F = ma\right]$$

$$-T + mg = m\ddot{x}$$

$$-\frac{9\cancel{m}g(e+x)}{6a} + \cancel{m}g = \cancel{m}\ddot{x}$$

$$-\cancel{9}g \cdot \frac{\cancel{2a}}{\cancel{3}} \cdot \frac{1}{\cancel{6a}} - \frac{\cancel{9}^{3}gx}{{}_{2}\cancel{6}a} + g = \ddot{x}$$

$$\ddot{x} = -\frac{3g}{2a}x$$

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Question 6 continued

c) $\omega^2 = \left(\frac{2\pi}{T}\right)^2 = \frac{3g}{2a}$

$$T = 2\pi\sqrt{\frac{2a}{3g}}$$

d) So that when the mass moves up it doesn't go too high to make the string slack

e) Use CoE:

$$\frac{1}{\cancel{2}}\,\frac{\cancel{9}^{3}\cancel{mg}(\cancel{8a-6a}^{a})^2}{{}_{2}\cancel{6a}} = \cancel{mg}(8a - AE)$$

$$AE = 8a - 3a$$

$$= 5a$$

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7.

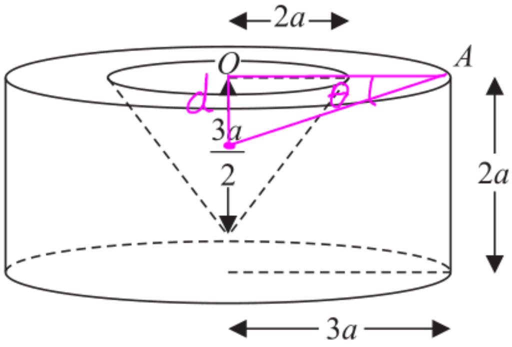

Diagram not drawn to scale

Figure 4

A uniform right circular solid cylinder has radius $3a$ and height $2a$. A right circular cone of height $\frac{3a}{2}$ and base radius $2a$ is removed from the cylinder to form a solid S, as shown in Figure 4. The plane face of the cone coincides with the upper plane face of the cylinder and the centre O of the plane face of the cone is also the centre of the upper plane face of the cylinder.

(a) Show that the distance of the centre of mass of S from O is $\frac{69a}{64}$.

(5)

The point A is on the open face of S such that $OA = 3a$, as shown in Figure 4. The solid is now suspended from A and hangs freely in equilibrium.

(b) Find the angle between OA and the horizontal.

(3)

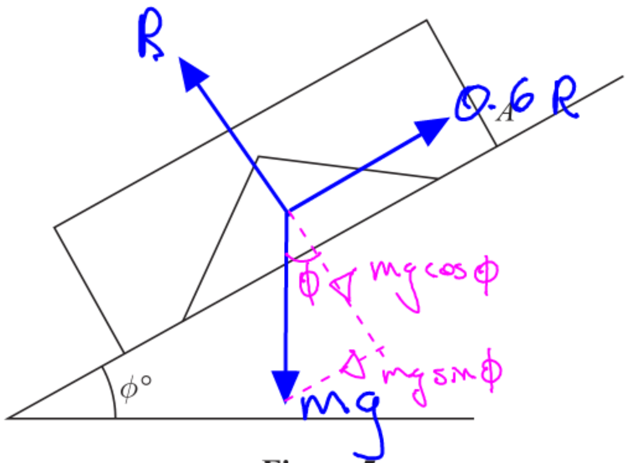

Figure 5

The solid is now placed on a rough inclined plane with the face through A in contact with the inclined plane, as shown in Figure 5. The solid rests in equilibrium on this plane. The coefficient of friction between the plane and S is 0.6 and the plane is inclined at an angle $\phi°$ to the horizontal. Given that S is on the point of sliding down the plane,

(c) show that $\phi = 31$ to 2 significant figures.

(4)

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Question 7 continued

a) Distance of centre of mass of cylinder from O, $d_o = a$

Distance of centre of mass of cone from O, $d_\Delta = \frac{1}{4} \cdot \frac{3a}{2} = \frac{3a}{8}$

Volume of cylinder, $V_o = \pi(3a)^2 \cdot 2a = 18\pi a^3$

" " cone, $V_\Delta = \frac{1}{3}\pi(2a)^2 \cdot \frac{3a}{2}$
$= 2\pi a^3$

Let the distance between centre of mass of S and O be d.

$$d(V_o - V_\Delta) = d_o V_o - d_\Delta V_\Delta$$

$$d = \frac{18\pi a^4 - \frac{3a}{8} \cdot 2\pi a^3}{18\pi a^3 - 2\pi a^3}$$

$$= \frac{69a}{64}$$

b) $\tan\theta = \frac{d}{3a} = \frac{69a}{64} \cdot \frac{1}{3a} = \frac{23}{64}$
$\theta = 19.8°$

$\therefore$ Angle between OA and horizontal is
$90 - 19.8 = 70.2°$

Leave blank

Question 7 continued

c) $\nwarrow \quad R = mg\cos\phi \quad (1)$

$\nearrow \quad 0.6R = mg\sin\phi \quad (2)$

$(2) \div (1): \quad \frac{0.6R}{R} = \frac{mg\sin\phi}{mg\cos\phi}$

$0.6 = \tan\phi$

$\phi = 31°$