Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel International Advanced Level

Centre Number | Candidate Number

Wednesday 9 January 2019

Afternoon (Time: 1 hour 30 minutes) | Paper Reference **WME03/01**

Mechanics M3

Advanced/Advanced Subsidiary

You must have:
Mathematical Formulae and Statistical Tables (Blue)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}, and give your answer to either two significant figures or three significant figures.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

Pearson

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1. A particle P moves on the x-axis. At time t seconds, $t \geqslant 0$, the displacement of P from the origin O is x metres and the acceleration of P is $\left(\frac{7}{2} - 2x\right)$ m s^{-2}, measured in the positive x direction. At time $t = 0$, P passes through O moving with speed $3\,\text{m s}^{-1}$ in the positive x direction. Find the distance of P from O when P first comes to instantaneous rest.

(6)

$a = \left(\frac{7}{2} - 2x\right) \text{ms}^{-2}$

$t = 0 \qquad v = 3\,\text{ms}^{-1}$

$a = \frac{dv}{dt} \qquad v = \frac{dx}{dt}$

$\Rightarrow dt = \frac{dv}{a} \qquad dt = \frac{dx}{v}$

$\Rightarrow \frac{dv}{a} = \frac{dx}{v} \Rightarrow a = v\frac{dv}{dx}$

$\Rightarrow \frac{7}{2} - 2x = v\frac{dv}{dx}$

$\Rightarrow \int \frac{7}{2} - 2x \; dx = \int v \; dv$

$\Rightarrow \frac{7}{2}x - x^2 = \frac{v^2}{2} + c$

$t = 0 \quad x = 0 \quad v = 3$ (given)

$\Rightarrow 0 = \frac{9}{2} + c \Rightarrow c = -\frac{9}{2}$

$\Rightarrow \frac{7}{2}x - x^2 = \frac{v^2 - 9}{2}$

Acc. to question $x = ?$ when $v = 0$

Leave blank

Question 1 continued

Subs. $v = 0$ in equation:

$\frac{7}{2}x - x^2 = -\frac{9}{2}$

$\Rightarrow 7x - 2x^2 + 9 = 0$

$\Rightarrow x = 4.5$ or $x = -1 \rightarrow$ Invalid because moving in '+'ve x.

$\therefore x = \underline{\underline{4.5}}$

Q1

(Total 6 marks)

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2.

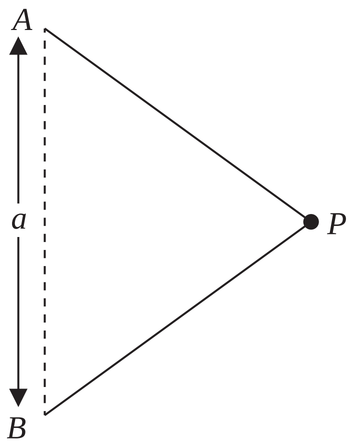


Figure 1

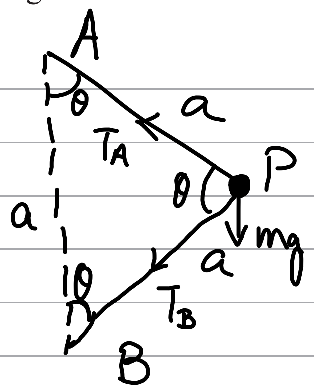
A small ball P of mass m is attached to the midpoint of a light inextensible string of length $2a$. The ends of the string are attached to fixed points A and B, where A is vertically above B and $AB = a$, as shown in Figure 1. The system rotates about the line AB with constant angular speed ω. The ball moves in a horizontal circle with both parts of the string taut. The tension in the string must be less than $3mg$ otherwise the string will break.

Given that the time taken by the ball to complete one revolution is S, show that

$$\pi\sqrt{\frac{a}{g}} < S < \pi\sqrt{\frac{ka}{g}}$$

stating the value of the constant k.

(12)



Angular speed $= \omega$

$\theta = 60^\circ$ $\quad \sin\theta = \frac{\sqrt{3}}{2}$

$\cos\theta = \frac{1}{2}$

Resolving $\updownarrow$:

$T_A \cos\theta = mg + T_B \cos\theta$

$(T_A - T_B)\cos\theta = mg$

$T_A - T_B = 2mg \quad$ — (i)

Resolving $\leftrightarrow$: $\quad T_A \sin\theta + T_B \sin\theta = m\omega^2 r$

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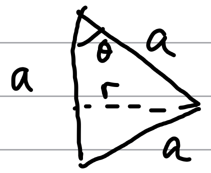
Question 2 continued



$\sin\theta = \frac{r}{a}$

$\Rightarrow \quad a\sin\theta = r$

$\therefore \quad (T_A + T_B)\cancel{\sin\theta} = m\omega^2 a\cancel{\sin\theta}$

$\Rightarrow \quad T_A + T_B = m\omega^2 a$ — (ii)

(i) + (ii) : $\quad T_A - T_B = 2mg$

$T_A + T_B = m\omega^2 a$

$T_A = \frac{m}{2}(2g + \omega^2 a)$

$\frac{m}{2}(2g + \omega^2 a) - T_B = 2mg$

$mg + \frac{m\omega^2 a}{2} - T_B = 2mg$

$-mg + \frac{m\omega^2 a}{2} = T_B$

$\Rightarrow T_B = \frac{m}{2}(\omega^2 a - 2g)$

Based on info. in question:

$T_A < 3mg$

$\Rightarrow \quad \frac{\cancel{m}}{2}(2g + \omega^2 a) < 3\cancel{m}g$

$\Rightarrow \quad g + \frac{\omega^2 a}{2} < 3g$

$\Rightarrow \quad \frac{\omega^2 a}{2} < 2g$

$\Rightarrow \quad \omega^2 < \frac{4g}{a} \rightarrow (I)$

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Question 2 continued

If that is the case, then $T_B > 0$ because the total tension in the string must be less than $3mg$ and if $T_A < 3mg$ then $T_B > 0$.

$$\therefore \frac{\cancel{m}}{\cancel{2}}(\omega^2 a - 2g) > 0$$

$$\omega^2 a > 2g$$

$$\omega^2 > \frac{2g}{a} \rightarrow (\text{II})$$

$$\omega = \frac{2\pi}{T} \Rightarrow T = S = \frac{2\pi}{\omega}$$

$$(\text{I}): \quad \omega < \sqrt{\frac{4g}{a}} \qquad \sqrt{\frac{4g}{a}} > \frac{2\pi}{S}$$

$$S > 2\pi\sqrt{\frac{a}{4g}}$$

$$S > \pi\sqrt{\frac{a}{g}}$$

$$(\text{II}): \quad \omega > \sqrt{\frac{2g}{a}} \qquad \sqrt{\frac{2g}{a}} < \frac{2\pi}{S}$$

$$S < 2\pi\sqrt{\frac{a}{2g}}$$

$$S < \pi\sqrt{\frac{2a}{g}}$$

$$\Rightarrow \quad \pi\sqrt{\frac{a}{g}} < S < \pi\sqrt{\frac{2a}{g}} \quad \text{shown.}$$

$$\therefore k = 2$$

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Question 2 continued

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Q2

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3. A particle P is moving in a straight line with simple harmonic motion between two points A and B, where AB is $2a$ metres. The point C lies on the line AB and $AC = \frac{1}{2}a$ metres. The particle passes through C with speed $\frac{3a\sqrt{3}}{2}\,\text{m s}^{-1}$.

(a) Find the period of the motion.

(3)

The maximum magnitude of the acceleration of P is $45\,\text{m s}^{-2}$. Find

(b) the value of a,

(2)

(c) the maximum speed of P.

(2)

The point D lies on AB and P takes a quarter of one period to travel directly from C to D.

(d) Find the distance CD.

(5)

(a) $v^2 = \omega^2 (a^2 - x^2)$

$\left(\frac{3a\sqrt{3}}{2}\right)^2 = \left(\frac{2\pi}{T}\right)^2 \left(a^2 - \frac{a^2}{4}\right)$

$\frac{\cancel{27}^{9}\cancel{a^2}}{\cancel{4}} = \frac{4\pi^2}{T^2}\left(\frac{\cancel{3}\cancel{a^2}}{\cancel{4}}\right)$

$9 = \frac{4\pi^2}{T^2}$

$T = \frac{2\pi}{3}$ s

(b) Max $\ddot{x} = 45\,\text{ms}^{-2}$

$\Rightarrow \omega^2 a = \ddot{x}$

$\Rightarrow \left(2\pi \times \frac{3}{2\pi}\right)^2 (a) = \ddot{x}$

$\Rightarrow 9a = 45$

$\Rightarrow a = 5\text{m}$

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Question 3 continued

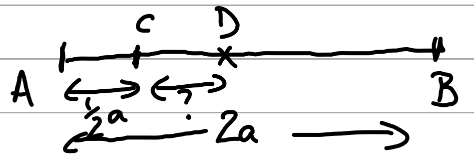
(c) $x = a\sin\omega t$

$\dot{x} = a\omega\cos\omega t$

at max $\dot{x}$ $\quad \cos\omega t = \underline{1}$

$\therefore \dot{x}_{max} = 5 \times 3 = \underline{\underline{15\,ms^{-1}}}$

(d)



t for C to D $= \frac{1}{4}T = \frac{1}{\cancel{4}\,2} \times \frac{\cancel{2}\pi}{3}$

$= \pi/6 \; \cancel{8}$

time from A to C = ?

$\frac{1}{2}a = a\cos(\omega t)$

$\omega t = \pi/3$

$t = \pi/9$

time from A to D $=$ time A to C + time C to D

$= \pi/9 + \pi/6$

$= 5\pi/18$

Displacement of D $= a\cos\omega t$

$= 5\cos\left(3 \times \frac{5\pi}{18}\right)$

$= 5\cos\left(\frac{5\pi}{6}\right)$

$= -4.33\ m$

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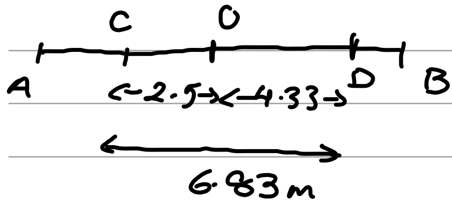
Question 3 continued

$\therefore$ C to D $= \frac{1}{2}a + 4.33\ldots$



$= 2.5 + 4.33\ldots$

$= \underline{\underline{6.83\text{m}}}$

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Question 3 continued

Q3

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4.

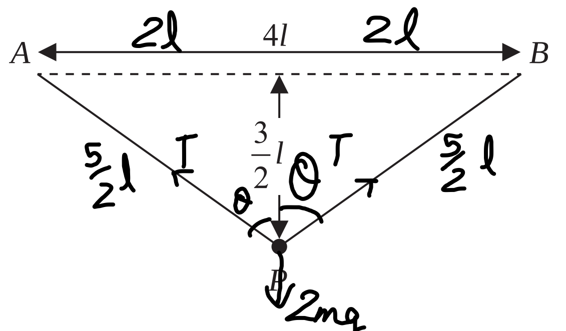


Figure 2

The ends of a light elastic string, of natural length $4l$ and modulus of elasticity λ, are attached to two fixed points A and B, where AB is horizontal and $AB = 4l$. A particle P of mass $2m$ is attached to the midpoint of the string. The particle hangs freely in equilibrium at a distance $\frac{3}{2}l$ vertically below the midpoint of AB, as shown in Figure 2.

(a) Show that $\lambda = \frac{20}{3}mg$.

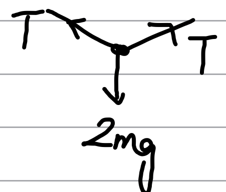
(7)

The particle is pulled vertically downwards from its equilibrium position until the total length of the string is $6l$. The particle is then released from rest.

(b) Show that P comes to instantaneous rest before reaching the line AB.

(6)

(a)



Diagonal lengths are

$2l + \Delta x$

Tensions equal.

Using pythagoras:

$l_0 = 4l$

$\text{diagonal}^2 = \left(\frac{3}{2}l\right)^2 + (2l)^2$

$\Rightarrow \text{diagonal} = \frac{5}{2}l$

$\therefore \Delta x$ on each side of Δ

$= 0.5l$

Total $\Delta x = l$

Resolving $\updownarrow$: $2mg = 2T\cos\theta$

$mg = T\cos\theta$

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Question 4 continued

Using triangle: $\cos\theta = \frac{3}{2} \div \frac{5}{2} = \frac{3}{5}$

$\Rightarrow \quad mg = \frac{3}{5}T$

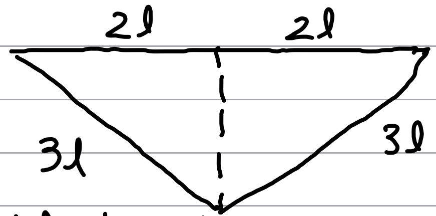
$\Rightarrow \quad \frac{5}{3}mg = T$

$\therefore \quad T = \frac{\lambda x}{l_0}$ (Hooke's law)

$\frac{5}{3}mg = \frac{\lambda(\cancel{l})}{4\cancel{l}}$

$\frac{20}{3}mg = \lambda$ shown.

b) If total length $= 6l$ then total extension is $= 2l$



Height pulled down by

$= \sqrt{9l^2 - 4l^2}$

$= 2.23\ldots l$

At bottom, EPE $= \frac{1}{2} \times \lambda \times \frac{x^2}{l_0}$

$= \frac{1}{\cancel{2}} \times \frac{\cancel{20}^{10}}{3} mg \times \frac{4l^2}{4l}$

$= \left(\frac{10}{3} mgl\right) J = 3.33\, mgl\ J$

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Question 4 continued

At top all energy will have become GPE,

if mass reaches AB then

$GPE = 2mg(2.23..l)$

$= 4.46mgl$ J

$\Rightarrow 3.33 < 4.46$

$\therefore$ The max GPE is more than the available EPE at the bottom.

Therefore, P does not reach AB.

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Question 4 continued

Q4

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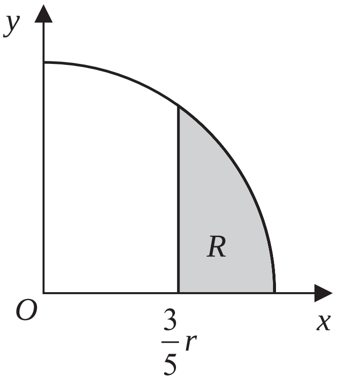

Figure 3

The region R, shown shaded in Figure 3, is bounded by the circle with centre O and radius r, the line with equation $x = \frac{3}{5}r$ and the x-axis. The region is rotated through one complete revolution about the x-axis to form a uniform solid S.

(a) Use algebraic integration to show that the x coordinate of the centre of mass of S is $\frac{48}{65}r$.

(8)

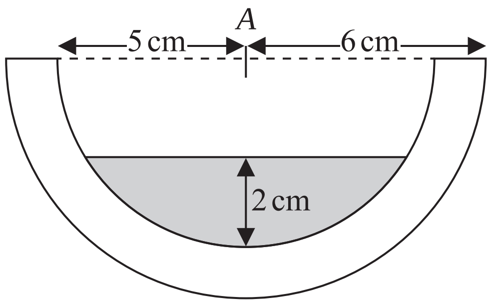

Figure 4

A bowl is made from a uniform solid hemisphere of radius 6 cm by removing a hemisphere of radius 5 cm. Both hemispheres have the same centre A and the same axis of symmetry. The bowl is fixed with its open plane face uppermost and horizontal. Liquid is poured into the bowl. The depth of the liquid is 2 cm, as shown in Figure 4. The mass of the empty bowl is $5M$ kg and the mass of the liquid is $2M$ kg.

(b) Find, to 3 significant figures, the distance from A to the centre of mass of the bowl with its liquid.

(8)

(a) Vol: $\pi \int_a^b y^2 \, dx$ $\qquad x^2 + y^2 = r^2$

$= \pi \int_{\frac{3}{5}r}^{r} r^2 - x^2 \, dx = \left[\pi\left(r^2 x - \frac{x^3}{3}\right)\right]_{\frac{3}{5}r}^{r}$

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Question 5 continued

$$= \pi r^3 - \frac{\pi r^3}{3} - \frac{3}{5}\pi r^3 + \frac{9\pi r^3}{125}$$

$$= \frac{52}{375}\pi r^3$$

$$V\bar{x} = \int_a^b \pi y^2 x \, dx$$

$$\frac{52}{375}\pi r^3 \bar{x} = \pi \int_{\frac{3}{5}r}^{r} (r^2 - x^2)x \, dx$$

$$\frac{52}{375}\pi r^3 \bar{x} = \pi \int_{\frac{3}{5}r}^{r} r^2 x - x^3 \, dx$$

$$\frac{52}{375}\cancel{\pi} r^3 \bar{x} = \cancel{\pi}\left[\frac{1}{2}r^2x^2 - \frac{1}{4}x^4\right]_{\frac{3}{5}r}^{r}$$

$$\frac{52}{375} r^3 \bar{x} = \frac{1}{2}r^4 - \frac{1}{4}r^4 - \frac{9}{50}r^4 + \frac{81}{2500}r^4$$

$$\frac{52}{375}\cancel{r^3}\bar{x} = \frac{64}{625}\cancel{r^4}$$

$$\bar{x} = \frac{64r}{625} \div \frac{52}{375}$$

$$\bar{x} = \frac{48}{65}r \quad \text{shown.}$$

(b) Bowl:

Because $\frac{4\pi}{6}$ in vol. will cancel

	Mass σ	y
	216	18/8
	−125	15/8
Σ	91	$\bar{y}$

$$\Rightarrow 216 \times \frac{18}{8} - 125 \times \frac{15}{8} = 91\bar{y} \Rightarrow \bar{y} = 2.765$$

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Question 5 continued

Bowl + Liquid :

	Mass σ	z
	5	2.765...
	2	48/13
Σ	7	$\bar{z}$

$\Rightarrow \quad 7\bar{z} = 2 \times \frac{48}{13} + 5 \times \frac{2013}{728}$

$\Rightarrow \quad \bar{z} = 3.03\text{cm}$

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Question 5 continued

Q5

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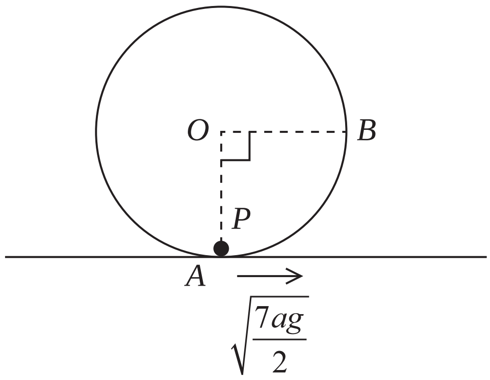


Figure 5

Figure 5 shows a hollow sphere, with centre O and internal radius a, which is fixed to a horizontal surface. A particle P of mass m is projected horizontally with speed $\sqrt{\frac{7ag}{2}}$ from the lowest point A of the inner surface of the sphere. The particle moves in a vertical circle with centre O on the smooth inner surface of the sphere. The particle passes through the point B, on the inner surface of the sphere, where OB is horizontal.

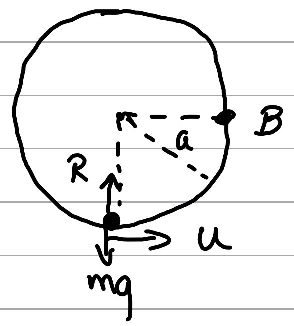
(a) Find, in terms of m and g, the normal reaction exerted on P by the surface of the sphere when P is at B.

(5)

The particle leaves the inner surface of the sphere at the point C, where OC makes an angle θ, $\theta > 0$, with the upward vertical.

(b) Show that, after leaving the surface of the sphere at C, the particle is next in contact with the surface at A.

(11)



$u = \sqrt{\frac{7ag}{2}}$ ms^{-1}

$r = a$

(a) at B: $R = \frac{mv^2}{r}$ (N2L radially ←)

Cons. of energy between bottom and B:

$\frac{1}{2}mu^2 + mg(0) = \frac{1}{2}mv^2 + mg(a)$

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Question 6 continued

$\Rightarrow \frac{1}{2} \times \cancel{m} \times \frac{7ag}{2} + 0 = \frac{1}{2}\cancel{m}v^2 + \cancel{m}g a$

$\Rightarrow \frac{7}{4}ag = \frac{1}{2}v^2 + ag$

$\Rightarrow \frac{3}{4}ag = \frac{1}{2}v^2$

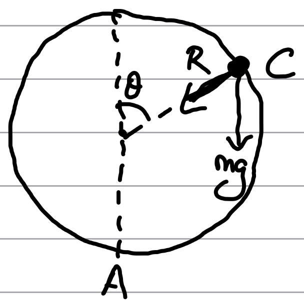
$\Rightarrow \frac{3}{2}ag = v^2$

Subs in N2L equation:

$R = m \times \frac{3}{2}\cancel{a}g \times \frac{1}{\cancel{a}}$

$R = \frac{3}{2}mg$

(b)



N2L radially at C ↙:

$R + mg\cos\theta = \frac{mv^2}{a}$

Because it leaves at C

$R = 0$

$\therefore\ mg\cos\theta = \frac{mv^2}{a}$

Cons. of energy between A and C:

$\Rightarrow \frac{1}{2}\cancel{m}u^2 + \cancel{m}g(0) = \frac{1}{2}\cancel{m}v^2 + \cancel{m}g(h)$

$\Rightarrow \frac{7ag}{4} = \frac{1}{2}v^2 + g(a + a\cos\theta)$

$\Rightarrow 2\left(\frac{3}{4}ag - ag\cos\theta\right) = v^2$

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Question 6 continued

Subs in second radial N2L eqn. :

$$\cancel{m}g\cos\theta = \frac{\cancel{m}}{a} \times 2 \times \left(\frac{3}{4}ag - ag\cos\theta\right)$$

$$\cancel{g}\cos\theta = \frac{3\cancel{g}}{2} - 2g\cos\theta$$

$$3\cos\theta = \frac{3}{2}$$

$$\cos\theta = \frac{1}{2}$$

$$\theta = \underline{\underline{60}}^\circ$$

After dropping down, P follows projectile motion.

Horizontal disp. $= a\sin\theta = s_x$

" velocity $= v\cos\theta = v_x$

$s_x = v_x t$ as acc. $= 0$.

$$\therefore t = \frac{s_x}{v_x} = \frac{a}{v}\tan\theta$$

$$= \sqrt{\frac{2}{ag}} \times a\sqrt{3} = \sqrt{\frac{6a}{g}}\ \cancel{s}$$

Vertical disp.

$$= s_y = -v\sin\theta\, t + \frac{1}{2}gt^2$$

$$s_y = \left(-\sqrt{\frac{ag}{2}} \times \frac{\sqrt{3}}{2} \times \sqrt{\frac{6a}{g}}\right) + \left(\frac{g}{2} \times \frac{6a}{g}\right)$$

$$= -\frac{3a}{2} + 3a = \frac{3}{2}a$$

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Question 6 continued

The vertical distance from A to C

is : $a + a\cos\theta = \frac{3}{2}a$

$\therefore$ P strikes the surfacl at A.

shown

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Question 6 continued

Q6

(Total 16 marks)

TOTAL FOR PAPER: 75 MARKS

END