

M3 January 2018 (IAL) (MA)

Q1) Shape Mass (vol.) Displacement of c.o.m from O

(+)



$$\frac{1}{3} \pi (r)^2 (4h)$$

$$= \boxed{\frac{4\pi r^2 h}{3}}$$

$$\frac{4h}{4} = \boxed{h}$$

(+)



$$\pi (r)^2 (3h)$$

$$= \boxed{3\pi r^2 h}$$

$$\boxed{-\frac{3h}{2}}$$

(=)



$$\boxed{\frac{13\pi r^2 h}{3}}$$

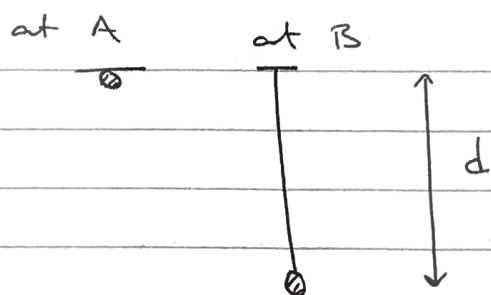
$$\boxed{\bar{y}}$$

taking moments about O . . .

$$\frac{4}{3}(h) + 3\left(-\frac{3h}{2}\right) = \frac{13}{3}(\bar{y})$$

$$\frac{\frac{4h}{3} - \frac{9h}{2}}{\frac{13}{3}} = \bar{y} = \boxed{\frac{19h}{26}}$$

Q2)



at A : $GPE = 0.9gd$
 $KE = EPE = 0$

at B : $GPE = 0$
 $KE = 0$

$$EPE = \frac{\lambda x^2}{2L} = \frac{29.4(d-1.2)^2}{2.4}$$

C.O.E : $0.9gd = \frac{49}{4}(d-1.2)^2$

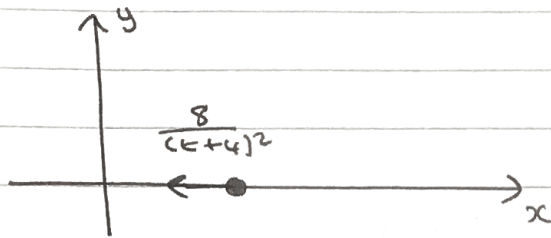
$$\frac{18}{245}gd = d^2 - 2.4d + 1.44$$

$$d^2 - 3.12d + 1.44 = 0$$

By Quadratic Formula : $d = 2.56$
 $d = 0.56$

$$d > 1.2 \text{ so } d = \boxed{2.56 = AB}$$

Q3a)



$$\underline{F=ma} : \quad \frac{-8}{(t+4)^2} = 0.4a$$

$$\frac{-8}{(t+4)^2} = 0.4 \frac{dv}{dt}$$

$$\frac{-20}{(t+4)^2} = \frac{dv}{dt}$$

$$\int (1) dv = -20 \int (t+4)^{-2} dt$$

$$v = -20 \left[-(t+4)^{-1} \right] + c$$

$$v = \frac{20}{t+4} + c$$

$$\underline{t=0, v=10} : \quad 10 = \frac{20}{4} + c$$

$$c = 10 - 5 = 5 //$$

$$v = \frac{20}{t+4} + 5$$

$$b) \quad v = \frac{20}{t+4} + 5$$

$$x = \int (v) dt = \int \left[\frac{20}{t+4} + 5 \right] dt$$

$$x = 20 \ln |t+4| + 5t + c //$$

$$t=0, x=0 : 0 = 20 \ln 4 + c$$

$$c = -20 \ln 4 //$$

$$x = 20 \ln |t+4| + 5t - 20 \ln 4$$

$$x = 20 \ln \left| \frac{t+4}{4} \right| + 5t$$

$$\text{at } v=6 \dots \quad 6 = \frac{20}{t+4} + 5$$

$$1 = \frac{20}{t+4}$$

$$t+4 = 20 \quad \therefore t = 16 // \text{ at } v=6$$

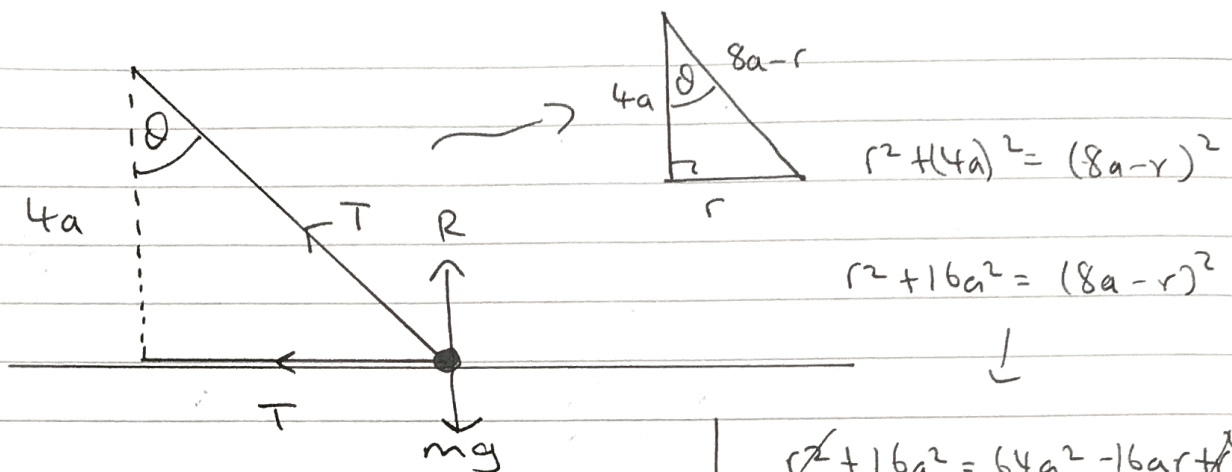
$$\text{So at } v=6 \dots \quad x = 20 \ln \left| \frac{20}{4} \right| + 80$$

$$x = 20 \ln 5 + 80$$

$$b = 20$$

$$a = 80$$

Q4)



$$r^2 + (4a)^2 = (8a - r)^2$$

$$r^2 + 16a^2 = (8a - r)^2$$

$$R(\uparrow): T \cos \theta + R = mg$$

$$R = mg - \frac{4}{5}T$$

$$\underline{N2L(P)}: T + T \sin \theta = m(3a)\omega^2$$

$$r^2 + 16a^2 = 64a^2 - 16ar + r^2$$

$$48a^2 = 16ar$$

$$\therefore r = \frac{48a}{16}$$

$$= 3a$$

$$\text{so } \cos \theta = \frac{4}{5}$$

$$\sin \theta = \frac{3}{5}$$

$$T(1 + \sin \theta) = 3mace^2$$

$$T = \frac{3mace^2}{1 + \frac{3}{5}} = \frac{15}{8}mace^2$$

$$\therefore R = mg - \frac{4}{5} \left(\frac{15}{8} \right) mace^2$$

$$R = mg - \frac{3}{2}mace^2$$

but $R \geq 0$ for this circular motion to continue.

$$\therefore mg - \frac{3}{2}mace^2 \geq 0$$

$$g \geq \frac{3}{2}ace^2$$

$$\omega^2 \leq \frac{2g}{3a}$$

$$\text{So } \omega \leq \sqrt{\frac{2g}{3a}}$$

$$\text{now } T = \frac{2\pi}{\omega} \rightarrow \omega = \frac{2\pi}{T}$$

$$\omega \leq \sqrt{\frac{2g}{3a}}$$

$$\text{so } \frac{2\pi}{S} \leq \sqrt{\frac{2g}{3a}}$$

take reciprocal from each side ...

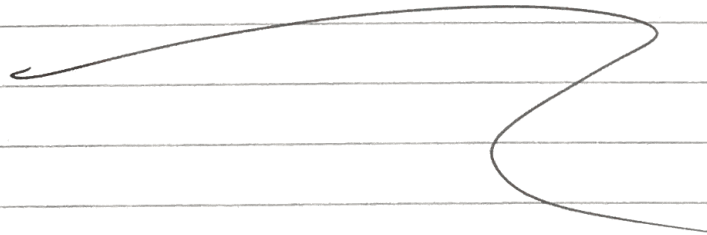
so sign will flip :

$$\frac{S}{2\pi} \geq \sqrt{\frac{3a}{2g}}$$

$$S \geq 2\pi \sqrt{\frac{3a}{2g}}$$

$$S \geq \pi \sqrt{\frac{4 \times 3a}{2g}}$$

$$S \geq \pi \sqrt{\frac{6a}{g}}$$



$$\bullet \text{ (Q5a)} \quad V = \pi \int_0^{\pi/2} y^2 dx = \pi \int_0^{\pi/2} [\sin^2 2x] dx$$

$$\cos 2x = 1 - 2\sin^2 x$$

$$\therefore \sin^2 x = \frac{1 - \cos 2x}{2}$$

$$V = \frac{1}{2} \pi \int_0^{\pi/2} [1 - \cos 2x] dx$$

$$= \frac{\pi}{2} \left[x - \frac{1}{2} \sin 2x \right]_0^{\pi/2}$$

$$= \frac{\pi}{2} \left[\frac{\pi}{2} - \frac{1}{2} \sin \pi \right] = \boxed{\frac{\pi^2}{4}}$$

$$\text{b) } M\bar{x} = \pi \int_a^b y^2 x dx = \pi \int_0^{\pi/2} [x \sin^2 2x] dx$$

By Parts : $\frac{dv}{dx} = \sin^2 2x$ ↖ from a

$$v = \frac{1}{2} x - \frac{1}{4} \sin 2x$$

$$u = x$$

$$u' = 1$$

$$\Rightarrow M\bar{x} = \pi \left[\left[\frac{1}{2} x^2 - \frac{1}{4} x \sin 2x \right]_0^{\pi/2} - \int_0^{\pi/2} \left[\frac{1}{2} x - \frac{1}{4} \sin 2x \right] dx \right]$$

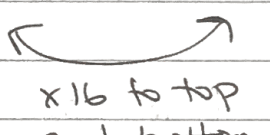
$$= \pi \left[\frac{\pi^2}{8} \right] - \pi \left[\frac{x^2}{4} + \frac{1}{8} \cos 2x \right]_0^{\pi/2}$$

$$= \frac{\pi^3}{8} - \pi \left[\frac{\pi^2}{16} - \frac{1}{8} \right] + \pi \left[\frac{1}{8} \right]$$

$$= \frac{\pi^3}{8} - \frac{\pi^3}{16} + \frac{\pi}{8} + \frac{\pi}{8} = \frac{\pi^3}{16} + \frac{\pi}{4} = M\bar{x}$$

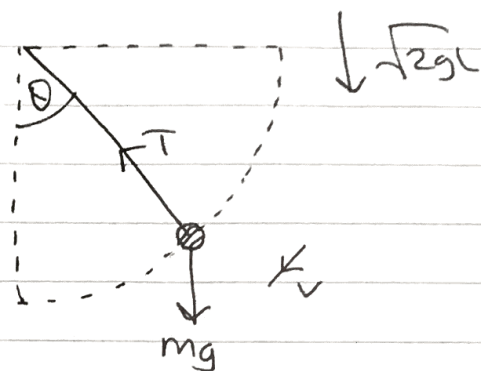
$$\frac{M\bar{x}}{M} = \bar{x} = \frac{\pi \left(\frac{\pi^2}{16} + \frac{1}{4} \right)}{\frac{\pi^2}{4}}$$

$$= \frac{\frac{\pi^2}{16} + \frac{1}{4}}{\frac{\pi^2}{4}} = \frac{\pi^2 + 4}{4\pi}$$



 x16 to top
and bottom

● (Q6a)



at A : $KE = \frac{1}{2} m(2gl) = mgl$

GPE = mgl

at angle θ to d.v : $KE = \frac{1}{2} mv^2$

GPE = $mgl(1 - \cos\theta)$

C.O.E : $mgl + mgl = \frac{1}{2} mv^2 + mgl(1 - \cos\theta)$

$2mgl = \frac{1}{2} mv^2 + mgl - mgl\cos\theta$

x2 : $2mgl + 2mgl\cos\theta = mv^2$

$v^2 = 2gl(1 + \cos\theta)$

N2L (P) : $T - mg\cos\theta = \frac{mv^2}{l}$



$T = mg\cos\theta + \frac{m}{l} (2gl)(1 + \cos\theta)$

$T = mg\cos\theta + 2mgl + 2mg\cos\theta$

$$\text{hence } T = mg(3\cos\theta + 2)$$

$$\text{b) } \underline{T=0} : 3\cos\theta + 2 = 0$$

$$\cos\theta = -\frac{2}{3}$$

$$\text{from (a), } v^2 = 2gl(H\cos\theta)$$

$$v^2 = 2gl\left(1 - \frac{2}{3}\right) = \frac{2gl}{3}$$

$$\text{hence } \boxed{v = \sqrt{\frac{2gl}{3}}}$$

$$\text{c) vertical speed at B} = \uparrow u$$

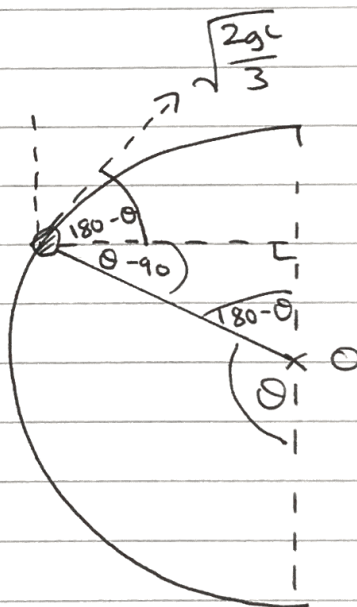
$$\uparrow u = \sqrt{\frac{2gl}{3}} \sin(180-\theta)$$

$$\sin\theta = \sqrt{1 - \cos^2\theta}$$

$$\sin\theta = \sqrt{1 - \left(-\frac{2}{3}\right)^2} = \frac{\sqrt{5}}{3}$$

$$\therefore \sin(180-\theta) = \sin(\theta) = \frac{\sqrt{5}}{3}$$

$$\text{so } \uparrow u = \frac{\sqrt{5}}{3} \times \sqrt{\frac{2gl}{3}} = \frac{1}{3} \sqrt{\frac{10gl}{3}}$$



$$\left. \begin{array}{l} \uparrow \\ S = h \\ u = \frac{1}{3} \sqrt{\frac{10gl}{3}} \\ v = 0 \\ a = -g \\ t = \end{array} \right\} \begin{array}{l} v^2 = u^2 + 2as \\ 0^2 = \frac{1}{9} \left(\frac{10gl}{3} \right) - 2gh \end{array}$$

$$\frac{10gl}{27} = 2gh$$

$$\therefore h = \frac{5L}{27} //$$

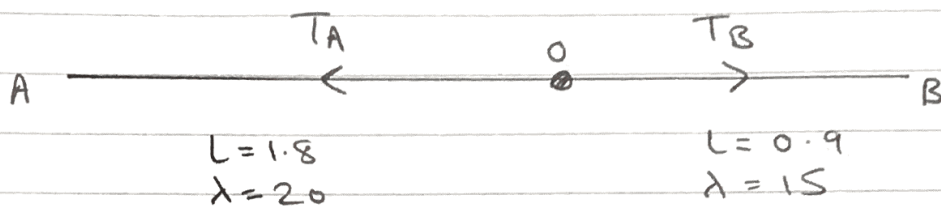
so height above 0 = $\frac{5L}{27} + L \cos(180^\circ - \theta)$

$$\sin(180^\circ - \theta) = \frac{\sqrt{5}}{3} \quad \therefore \cos \theta = \sqrt{1 - \left(\frac{\sqrt{5}}{3}\right)^2} = \frac{2}{3}$$

$$\text{so height above 0} = \frac{5L}{27} + \frac{2L}{3}$$

$$= \boxed{\frac{23L}{27}}$$

(7a)



$$T_A = T_B$$

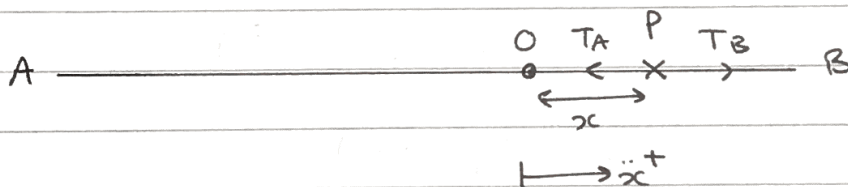
$$\frac{20}{1.8} (A_0 - 1.8) = \frac{15}{0.9} (4.2 - A_0 - 0.9)$$

$$\frac{100}{9} (A_0) - 20 = 70 - 15 - \frac{50}{3} (A_0)$$

$$\left(\frac{100}{9} + \frac{50}{3} \right) A_0 = 75 = \frac{250}{9} A_0$$

$$\therefore A_0 = \frac{75}{\frac{250}{9}} = \boxed{2.7\text{m}}$$

b)



$$\underline{N2L(P)}^{\rightarrow} : T_B - T_A = m\ddot{x}$$

$$\frac{15}{0.9} (0.6 - x) - \frac{20}{1.8} (0.9 + x) = m\ddot{x}$$

$$\Rightarrow 10 - \frac{50}{3}x - 10 - \frac{100}{9}x = m\ddot{x}$$

$$\Rightarrow \underline{\underline{\frac{-250}{9m} = \ddot{x}}} \quad \text{hence P moves with S.H.M} \\ \text{centre O.}$$

$$c) \quad AC = 2.9 \quad \therefore \quad a = 0.2m //$$

$$\omega = \sqrt{\frac{250}{9 \times 10}} = \frac{5}{3} //$$

$$J = 10(v-u)$$

$$v_{max} = a\omega = \frac{5}{3} \times 0.2 = \frac{1}{3} = \text{speed right after impulse}$$

$$\text{so } J = 10\left(\frac{1}{3} - 0\right) = \boxed{\frac{10}{3} //$$

$x = a \sin \omega t$ as P starts at the centre of oscillation.

$$x = 0.2 \sin\left(\frac{5t}{3}\right)$$

when total distance travelled = 0.5m, $x = -0.1 //$

$$-0.1 = 0.2 \sin\left(\frac{5t}{3}\right)$$

$$-\frac{1}{2} = \sin\left(\frac{5t}{3}\right)$$

$$t = \frac{3}{5} \sin^{-1}\left(-\frac{1}{2}\right) = \boxed{2.2s}$$