

M3 January 2017 IAL (MA)

$$Q1) V = \pi \int (y^2) dx = 9\pi \int_0^4 (4-x) dx = 9\pi \left[4x - \frac{x^2}{2} \right]_0^4$$

$$y=0: x=4 //$$

$$= 9\pi [16 - 8] = 8(9\pi) = 72\pi //$$

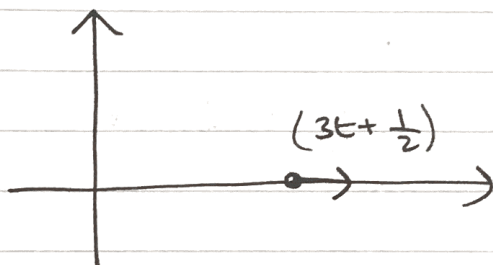
$$M\bar{x} = \pi \rho \int_0^4 y^2 x dx = 9\pi \rho \int_0^4 (4x - x^2) dx$$

$$= 9\pi \rho \left[2x^2 - \frac{x^3}{3} \right]_0^4 = 9\pi \rho \left[32 - \frac{64}{3} \right]$$

$$= 96\pi \rho //$$

$$\bar{x} = \frac{M\bar{x}}{M} = \frac{96\pi \rho}{72\pi \rho} = \boxed{\frac{4}{3}}$$

Q2a)



$$F = 3t + \frac{1}{2} = ma$$

$$3t + \frac{1}{2} = 0.6 \frac{dv}{dt}$$

$$\int (0.6) dv = \int (3t + \frac{1}{2}) dt$$

$$0.6v = \frac{3t^2}{2} + \frac{1}{2}t + c$$

$$\underline{t=0, v=0} : c=0 //$$

$$\text{so } 0.6v = \frac{3}{2}t^2 + \frac{1}{2}t$$

$$\underline{\div 0.6} : \boxed{v = \frac{5}{2}t^2 + \frac{5}{6}t}$$

$$\text{b) } v = \frac{10}{3} : \frac{10}{3} = \frac{5}{2}t^2 + \frac{5}{6}t$$

$$\underline{\times 6} : 20 = 15t^2 + 5t$$

$$15t^2 + 5t - 20 = 0$$

$$3t^2 + t - 4 = 0$$

$$(3t + 4)(t - 1) = 0$$

$$t = 1 // \quad (t > 0)$$

$$OA = \int_0^1 (v) dt = \int_0^1 \left[\frac{5t^2}{2} + \frac{5t}{6} \right] dt$$

$$= \left[\frac{5t^3}{6} + \frac{5t^2}{12} \right]_0^1 = \frac{5}{6} + \frac{5}{12} - 0 = \boxed{\frac{5}{4} \text{ m}}$$

Q3) Shape Mass (vol. ratio) Distance of c.o.m from O



$$\pi (4a)^2 (6a)$$

$$= \boxed{96\pi a^3}$$

$$3a$$

(-)



$$\frac{2}{3} \pi (3a)^3$$

$$= \boxed{18\pi a^3}$$

$$\frac{3(3a)}{8} = \frac{9a}{8}$$



$$\boxed{78\pi a^3}$$

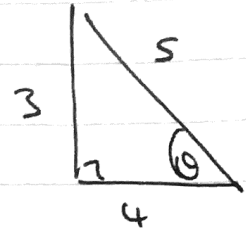
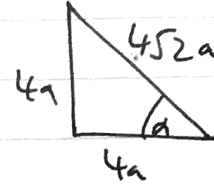
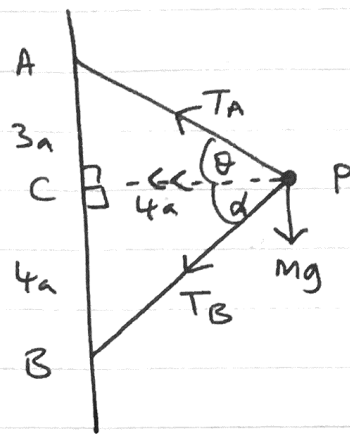
$$\bar{y}$$

moments about diameter through O

$$96\pi a^3 (3a) - 18\pi a^3 \left(\frac{9a}{8}\right) = 78\pi a^3 (\bar{y})$$

$$\frac{\left[96(3) - \frac{18(9)}{8}\right] a}{78} = \bar{y} = \frac{\frac{1071}{4} a}{78} = \boxed{\frac{357a}{104}}$$

Q4a)



$$\cos \theta = \frac{4}{5}$$

$$\sin \theta = \frac{3}{5}$$

$$\cos \alpha = \frac{\sqrt{2}}{2} = \sin \alpha$$

$$R(\uparrow\downarrow): T_A \sin \theta = mg + T_B \sin \alpha \quad \text{--- (1)}$$

$$\leftarrow \text{N2L(P)}: T_A \cos \theta + T_B \cos \alpha = m(4a)\omega^2$$

$$\sin \alpha = \cos \alpha$$

$$\text{so (1): } T_B \cos \alpha = T_A \sin \theta - mg$$

$$\text{and (2): } T_B \cos \alpha = 4m\omega^2 - T_A \cos \theta$$

$$\text{equating: } T_A \left(\frac{3}{5} \right) - mg = 4m\omega^2 - T_A \left(\frac{4}{5} \right)$$

$$T_A \left(\frac{3}{5} + \frac{4}{5} \right) = m(4a\omega^2 + g)$$

$$\frac{7}{5} T_A = m(4a\omega^2 + g)$$

$$\therefore T_A = \frac{5m}{7} (4a\omega^2 + g)$$

$$b) \text{ from } \textcircled{1} : T_B \left(\frac{\sqrt{2}}{2} \right) = T_A \left(\frac{3}{5} \right) - mg$$

$$T_B \cdot \frac{\sqrt{2}}{2} = \frac{3}{5} \left(\frac{5m}{7} \right) (4a\omega^2 + g) - mg$$

$$T_B \cdot \frac{\sqrt{2}}{2} = \frac{3m}{7} (4a\omega^2 + g) - mg$$

$$T_B \cdot \frac{\sqrt{2}}{2} = \frac{12ma\omega^2 + 3mg - mg}{7}$$

$$T_B \cdot \frac{\sqrt{2}}{2} = \frac{12ma\omega^2 - 4mg}{7}$$

$$T_B \cdot \frac{\sqrt{2}}{2} = \frac{4m}{7} (3a\omega^2 - g)$$

$$\therefore T_B = \frac{4m \times 2 (3a\omega^2 - g)}{7\sqrt{2}}$$

$$= \boxed{\frac{4m\sqrt{2} (3a\omega^2 - g)}{7}}$$

c) $T_B \geq 0$ for P to continue its circular motion:

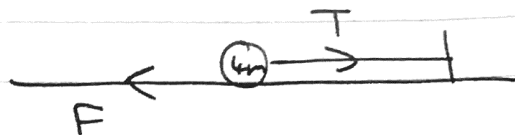
$$\Rightarrow \frac{4m\sqrt{2} (3a\omega^2 - g)}{7} \geq 0$$

$$\Rightarrow 3a\omega^2 - g \geq 0$$

$$\Rightarrow \omega^2 \geq \frac{g}{3a} \quad \therefore \omega \geq \sqrt{\frac{g}{3a}}$$

$$(k=3) //$$

Q5a)



$$F \leq \mu R \rightarrow \mu \geq \frac{F}{R}$$

since P is at rest, $F = T$.

$$T = \frac{\lambda x}{L} = \frac{3mg \left(\frac{L}{3}\right)}{L} = mg //$$

so $F = mg$ and $R = 4mg$.
(resolving perp to ground)

$$\text{and } \mu \geq \frac{F}{R} \rightarrow \mu \geq \frac{mg}{4mg}$$

$$\text{hence } \mu \geq \frac{1}{4}$$

b) Initially: $KE = 0$

$$EPE = \frac{3mg}{2L} (L)^2$$

When P comes to rest: $KE = 0$

$$EPE = \frac{3mg}{2L} (L-d)^2$$

$$W \cdot D \text{ by friction} = \frac{2}{5} (4mg)d$$

$$\text{C.O.E} : \frac{3mgL}{2} = \frac{3mg}{2L} (L^2 + d^2 - 2Ld) + \frac{8}{5} mgd$$

$$\frac{3mgL}{2} = \frac{3mgL}{2} + \frac{3mgd^2}{2L} - 3mgd + \frac{8}{5} mgd$$

$$\frac{3mgd^2}{2l} = \frac{7}{5} mgd$$

$$mgd \left(\frac{3d}{2l} - \frac{7}{5} \right) = 0$$

$$d \neq 0, \quad \frac{3d}{2l} - \frac{7}{5} = 0$$

$$d = \frac{14}{15} l < 0$$

hence P moves with S.H.M cen'

hence P comes to rest before the string becomes slack.

(Q6a)



$$T = 2mg \quad T = \frac{\lambda x}{l} = \frac{20mgx}{5l}$$

$$\frac{20mgx}{5l} = 2mgx$$

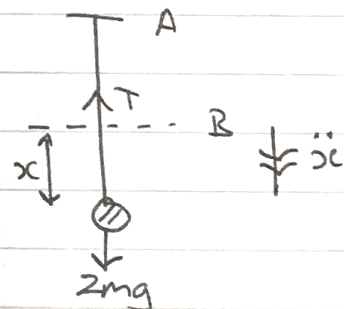
$$\therefore e = \frac{16l}{20} = \boxed{\frac{l}{2}}$$

$$b) \quad \underline{N2L(P)} \downarrow_+ : 2mg - T = 2m\ddot{x}$$

$$T = \frac{20mg}{5l} \left(x + \frac{l}{2} \right)$$

$$2mg - \frac{20mgx}{5l} - 2mg = 2m\ddot{x}$$

$$-\frac{4mgx}{l} = 2m\ddot{x}$$



$$\ddot{x} = -\frac{2g}{L}x = -\omega^2 x$$

hence P moves
with S.H.M
centre B.

$$c) \quad \omega = \sqrt{\frac{2g}{L}} \quad \therefore T = \boxed{2\pi \sqrt{\frac{L}{2g}}}$$

$$(T = \frac{2\pi}{\omega})$$

$$d) \quad v_{\max} = a\omega = \frac{\sqrt{g}L}{5} = a\sqrt{\frac{2g}{L}}$$

$$\therefore a = \frac{\sqrt{g}L}{5\sqrt{2}\sqrt{g}} = \boxed{\frac{L}{5\sqrt{2}}}$$

$$e) \quad x = \frac{L}{5\sqrt{2}} \cos\left(\sqrt{\frac{2g}{L}}t\right) = a \cos\left(t\sqrt{\frac{2g}{L}}\right)$$

$$\frac{a}{2} = a \cos\left(t\sqrt{\frac{2g}{L}}\right)$$

$$\therefore \cos^{-1}\left(\frac{1}{2}\right) = t\sqrt{\frac{2g}{L}} = \frac{\pi}{3}$$

$$\therefore t = \frac{\pi}{3} \sqrt{\frac{L}{2g}} = \text{time to D.}$$

P won't be slack at any point in the motion as $\frac{L}{5\sqrt{2}} < \frac{L}{2}$.

$$\text{time to E: } -a = a \cos\left(t\sqrt{\frac{2g}{L}}\right)$$

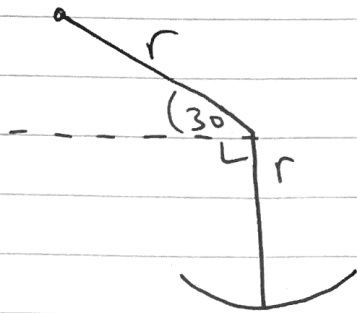
$$\cos^{-1}(-1) = t\sqrt{\frac{2g}{L}} = \pi$$

$$\therefore t = \pi \sqrt{\frac{L}{2g}}$$

hence time required = $(\pi - \frac{\pi}{3}) \sqrt{\frac{L}{2g}}$

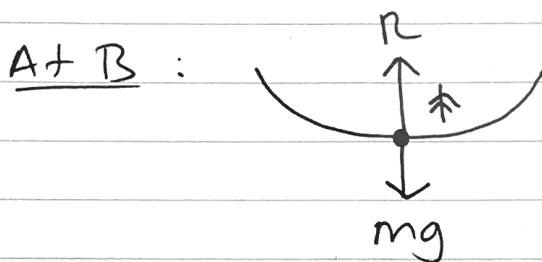
(= time to E - time to D) = $\frac{2\pi}{3} \sqrt{\frac{L}{2g}}$

Q7a) Energy to B : $\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = mgr(1 + \sin 30)$



$$\frac{v^2}{2} = \frac{u^2}{2} + \frac{3gr}{2}$$

$$v^2 = u^2 + 3gr //$$



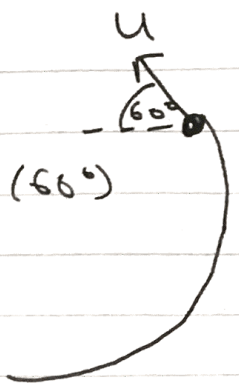
N2L(P) \uparrow : $R - mg = \frac{m}{r}(v^2)$

$$R - mg = \frac{mu^2}{r} + \frac{3mgr}{r}$$

$$R = mg + \frac{mu^2}{r} + 3mg$$

$$R = 4mg + \frac{mu^2}{r}$$

- b) P will leave the bowl at the same angle to the horizontal (60°) it started at.



$$\begin{array}{l}
 \uparrow \\
 \left. \begin{array}{l}
 s = d \\
 u = u \sin 60 \\
 v = 0 \\
 a = -g \\
 t = t
 \end{array} \right\} \begin{array}{l}
 v^2 = u^2 + 2as \\
 0 = u^2 \sin^2 60 - 2gd \\
 d = \frac{u^2 \sin^2 60}{2g} = \boxed{\frac{3u^2}{8g}}
 \end{array}
 \end{array}$$

\therefore greatest height above floor = $d + \frac{3r}{2}$

$$= \boxed{\frac{3u^2}{8g} + \frac{3r}{2}}$$

- c) Finding time when P returns

$$\left. \begin{array}{l}
 s = 0 \\
 u = u \sin 60 \\
 v = -u \sin 60 \\
 a = -g \\
 t = t
 \end{array} \right\} \begin{array}{l}
 v = u + at \\
 -u \sin 60 = u \sin 60 - gt \\
 t = \frac{2u \sin 60}{g} //
 \end{array}$$

horizontal distance travelled = $u \cos 60 \times \frac{2u \sin 60}{g}$
(in this time)

$$= \frac{2u^2 \sqrt{3}}{4g} = \frac{u^2 \sqrt{3}}{2g}$$

[bowl width = $2r \cos 30$]

but $u^2 > 2gr$: distance $> \frac{2gr \sqrt{3}}{2g}$

\Rightarrow distance $> r\sqrt{3} //$

hence distance \leftarrow $>$ bowl width

\therefore P doesn't fall back in.
