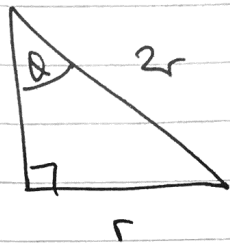
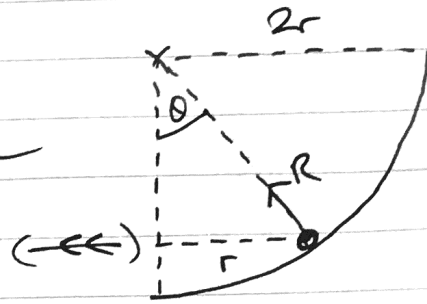


M3 Jan 2016 IAL (MA)

Q1)


 (\sim)


$$\therefore \sin \theta = \frac{r}{2r} = \frac{1}{2}$$

$$\therefore \theta = 30^\circ //$$

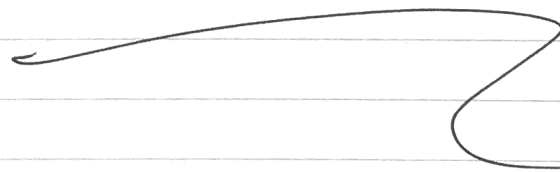
$$R (\uparrow): R \cos 30 = mg$$

$$\underline{N2L(P)}: R \sin 30 = m(r)\omega^2$$

$$\text{but } R = \frac{mg}{\cos 30} \therefore mg \tan 30 = m r \omega^2$$

$$\Rightarrow \omega^2 = \frac{g \tan 30}{r} = \frac{g \sqrt{3}}{3r}$$

$$\Rightarrow \omega = \sqrt{\frac{g \sqrt{3}}{3r}}$$



$$Q2a) \quad a = 6 - 2t = \frac{dv}{dt}$$

$$\int (6 - 2t) dt = \int (1) dv$$

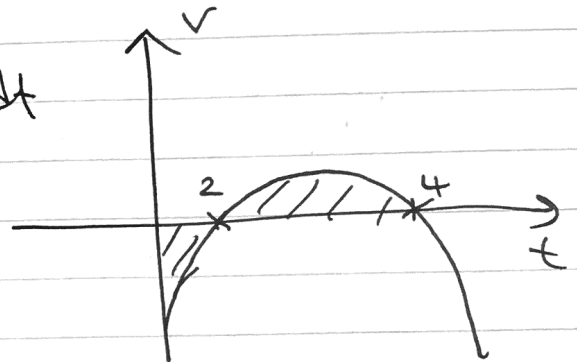
$$v = 6t - t^2 + c$$

$$t=0, v=-8 : -8 = c$$

$$\therefore \boxed{v = 6t - t^2 - 8}$$

$$b) \quad x_{\text{TOTAL}} = \left| \int_0^2 [v] dt \right| + \int_2^4 [v] dt$$

$$\Rightarrow \int_0^2 [6t - t^2 - 8] dt + \dots$$

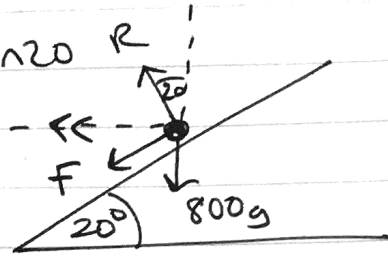


$$= \left[3t^2 - \frac{t^3}{3} - 8t \right]_0^2 = -\frac{20}{3} \therefore x = \frac{20}{3} //$$

$$\begin{aligned} \Rightarrow \int_2^4 (v) dt &= \left[3t^2 - \frac{t^3}{3} - 8t \right]_2^4 \\ &= \frac{4}{3} // \end{aligned}$$

$$\therefore \text{Distance} = \frac{4}{3} + \frac{20}{3} = \boxed{8} \text{ m}$$

$$Q3) R(\updownarrow): R \cos 20 = 800g + F \sin 20$$



$$\leftarrow \underline{N2L(car)}: R \sin 20 + F \cos 20 = \frac{800v^2}{20} = 40v^2$$

$$F = \mu R \rightarrow F = 0.5R$$

$$(1): R \cos 20 - F \sin 20 = 800g$$

$$(2): R \sin 20 + F \cos 20 = 40v^2$$

(v_{max} occurs when F is max.
ie when $F = 0.5R$)

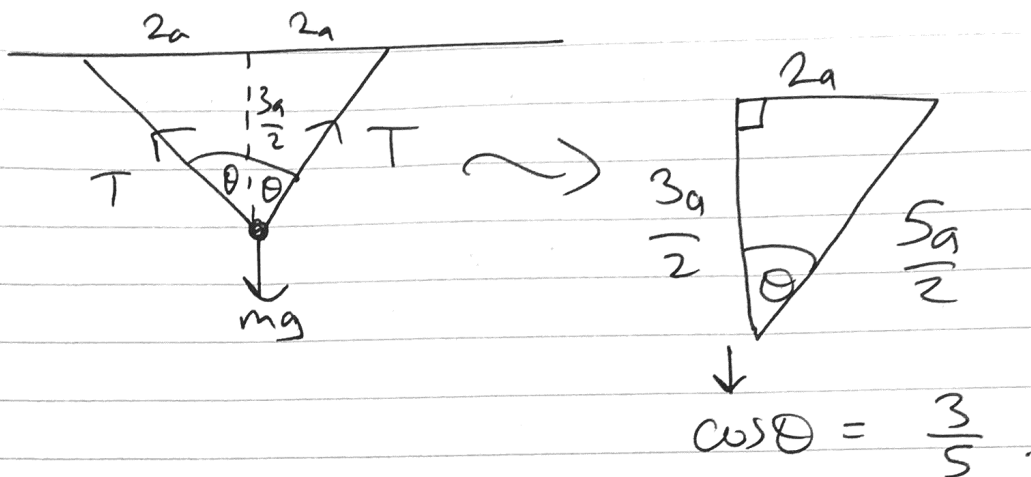
$$\frac{(2)}{(1)}: \frac{R \sin 20 + 0.5R \cos 20}{R \cos 20 - 0.5R \sin 20} = \frac{40v^2}{800g}$$

$$\therefore \frac{40v^2}{800g} = 1.05618\dots$$

$$\therefore v^2 = \frac{800g (1.05618\dots)}{40} = 207\dots$$

$$\therefore \boxed{v = 14.4 \text{ m/s}}$$

Q4a)



N2L(P) \uparrow : $2T \cos \theta = mg$

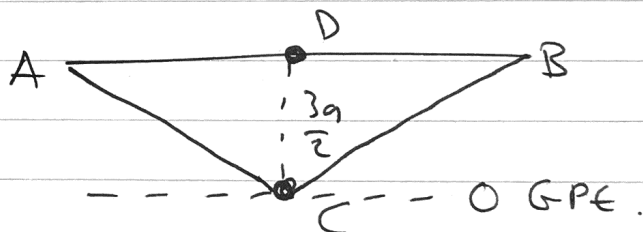
$$T = \frac{\lambda x}{l} = \frac{\lambda}{\frac{3a}{2}} \left(\frac{5a}{2} - \frac{3a}{2} \right) = \frac{2\lambda}{3}$$

for one half of the string.

$$\therefore 2 \times \frac{2\lambda}{3} \times \frac{3}{5} = mg$$

$$\Rightarrow \lambda = \frac{15mg}{2 \times 2 \times 3} = \boxed{\frac{5mg}{4}}$$

b) A+D : $KE = 0$
 $GPE = mg \left(\frac{3a}{2} \right)$
 $EPE = \frac{5mg}{4} (a)^2$



A+C : $KE = \frac{1}{2} mv^2$
 $GPE = 0$
 $EPE = \frac{5mg}{4} \left(\frac{4a^2}{6a} \right) = \frac{5mg}{4} \times \frac{2}{3} a = \frac{10amg}{12}$

By C.O.E : $\frac{3amg}{2} + \frac{5amg}{24} = \frac{mv^2}{2} + \frac{5amg}{6}$

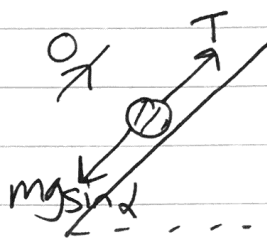
$$\left(\frac{3}{2} + \frac{5}{24} - \frac{5}{6}\right) mag = \frac{mv^2}{2}$$

$$\frac{v^2}{2} = \frac{7ag}{8}$$

$$v^2 = \frac{7ag}{4} \quad \therefore v = \frac{\sqrt{7ag}}{2}$$

Q5a) $mg \sin \alpha = T$

$$\frac{3}{5} mg = \frac{\lambda \left(\frac{l}{5}\right)}{l}$$

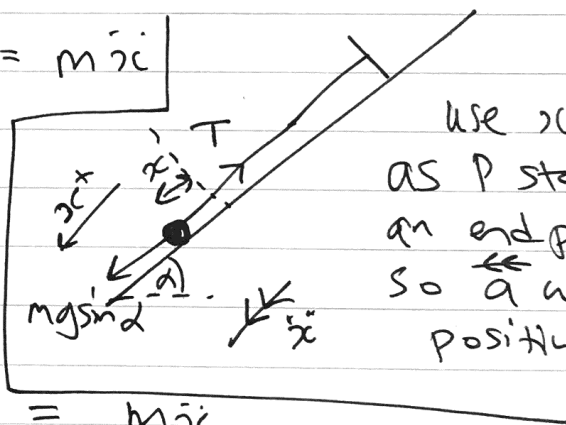


$$\therefore \frac{3}{5} mg = \frac{\lambda}{5} \xrightarrow{\times 5} \lambda = 3mg$$

b) N2L(P) : $mg \sin \alpha - T = m \ddot{x}$

$$T = \frac{\lambda x}{l} = \frac{3mg}{l} \left(x + \frac{l}{5}\right)$$

$$\therefore mg \left(\frac{3}{5}\right) - \frac{3mg}{l} \left(x + \frac{l}{5}\right) = m \ddot{x}$$



use $x = a \cos \omega t$
 as P starts at
 an endpoint.
 so a will be
 positive

$$\Rightarrow \frac{3mg}{5} - \frac{3mgx}{l} - \frac{3mg}{5} = m \ddot{x} = -\frac{3mgx}{l}$$

$$\therefore \ddot{x} = -\frac{3g}{c}x$$

hence P moves with
S.H.M, centre B.



$$c) |\dot{x}_{\max}| = \frac{3g}{c} \times \underset{\substack{\uparrow \\ \text{amplitude}}}{x(\max)} = \frac{3g}{c} \times \frac{L}{2} = \boxed{\frac{3g}{2}} \text{ ms}^{-2}$$

$$d) x = \frac{L}{2} \cos(\omega t) \quad \omega = \sqrt{\frac{3g}{c}}$$

time from C to slack point:

$$-\frac{L}{3} = \frac{L}{2} \cos\left(t \sqrt{\frac{3g}{c}}\right)$$

$$-\frac{2}{3} = \cos\left(t \sqrt{\frac{3g}{c}}\right)$$

$$t \sqrt{\frac{3g}{c}} = 1.982 \dots$$

$$t = \frac{1.982 \cdot \sqrt{c}}{\sqrt{3g}}$$

time from C to D:

$$\frac{L}{4} = \frac{L}{2} \cos\left(t \sqrt{\frac{3g}{c}}\right)$$

$$\cos\left(t \sqrt{\frac{3g}{c}}\right) = \frac{1}{2}$$

$$\cos^{-1} \frac{1}{2} = \frac{\pi}{3} = t \sqrt{\frac{3g}{c}}$$

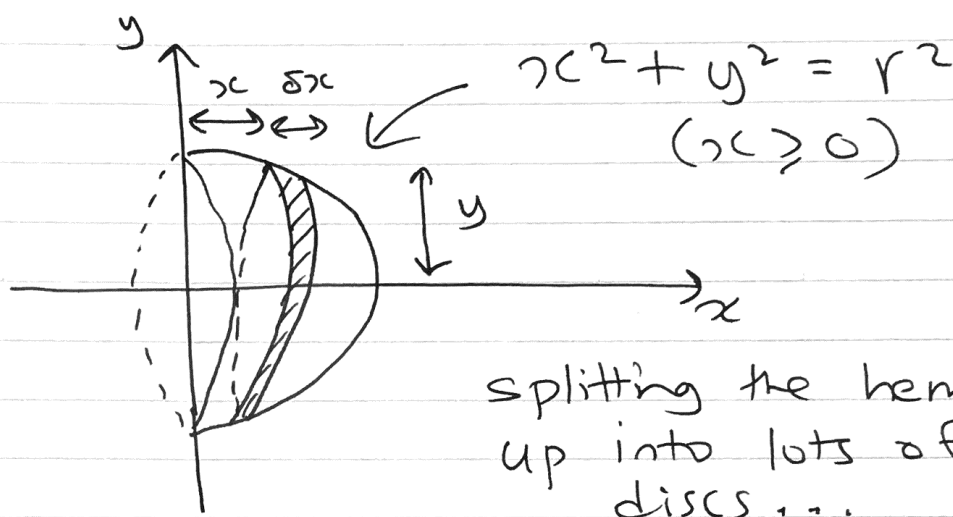
$$\therefore t = \left(\frac{\pi}{3\sqrt{3}}\right) \sqrt{\frac{c}{g}}$$

$$\text{hence required time} = \left(\frac{1.982}{\sqrt{3}} - \frac{\pi}{3\sqrt{3}} \right) \sqrt{\frac{L}{g}}$$

$$= 0.54 \sqrt{\frac{L}{g}}$$

hence $\boxed{u = 0.54}$

(Q6a)



for one disc

$$\left. \begin{aligned} \delta m &= \rho \pi y^2 \delta x \\ \delta \bar{x} &= x \end{aligned} \right\} \begin{aligned} \delta m \cdot \delta x &= (\rho \pi y^2 \delta x) x \\ &= m \cdot x \end{aligned}$$

entire "mass" = $\frac{2}{3} \rho \pi r^3$

$$\bar{x} \sum M_i = \sum M_i x_i$$

$$\bar{x} \left(\frac{2\rho\pi r^3}{3} \right) = \sum_{x=0}^r (\rho\pi x y^2) \delta x$$

$$\lim_{\delta x \rightarrow 0} \sum_{x=0}^r \rho\pi x y^2 \delta x = \rho\pi \int_0^r (x y^2) dx$$

$$= \rho \pi \int_0^r (r^2 - x^2) x \, dx = \rho \pi \int_0^r r^2 x - x^3 \, dx$$

$$= \rho \pi \left[\frac{r^2 x^2}{2} - \frac{x^4}{4} \right]_0^r = \rho \pi \left[\frac{r^4}{2} - \frac{r^4}{4} \right]$$

$$= \frac{\rho \pi r^4}{4}$$

$$\therefore \bar{x} \sum m_i = \bar{x} \left(\frac{2 \rho \pi r^3}{3} \right) = \frac{\rho \pi r^4}{4}$$

$$\Rightarrow \bar{x} \left(\frac{2}{3} \right) = \frac{r}{4}$$

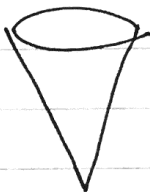
$$\Rightarrow \bar{x} = \frac{3r}{4 \times 2} = \boxed{\frac{3r}{8}}$$

b) Shape Mass (vol.) Distance of c.o.m from 0



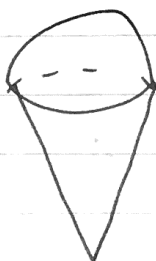
$$\boxed{m}$$

$$4r + \frac{3r}{8} = \boxed{\frac{35r}{8}}$$



$$\boxed{M}$$

$$\frac{3}{4}(4r) = \boxed{3r}$$



$$\boxed{m+M}$$

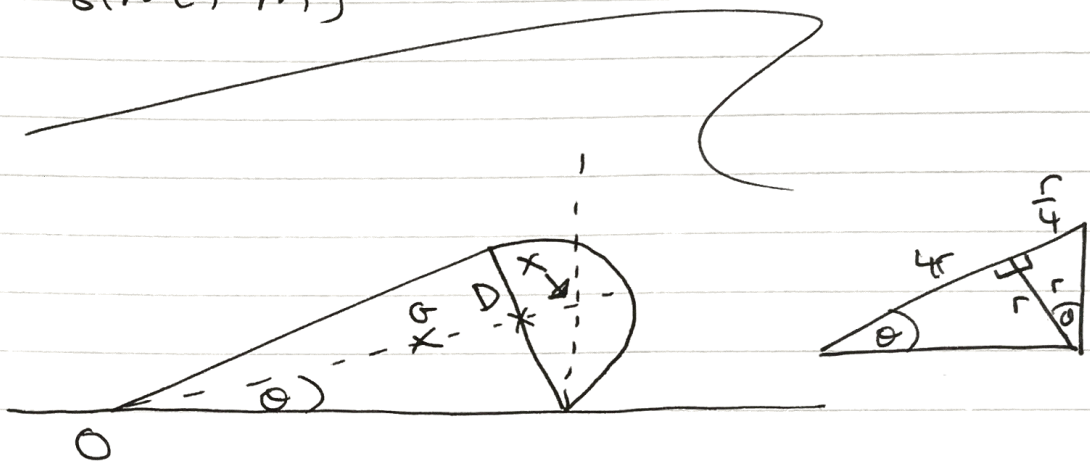
$$\boxed{\bar{y}}$$

Moments about vertex O

$$\frac{35mr}{8} + 3Mr = (m+M)\bar{y}$$

$$\frac{(35m + 24M)r}{8(m+M)} = \bar{y}$$

c)



\Rightarrow in equilibrium $\therefore OG \perp OX$

ie $\bar{y} \perp 4r + \frac{r}{4}$ ← similar triangles
(ODA and ADX)

$$\bar{y} \perp \frac{17r}{4}$$

$$\frac{(35m + 24M)r}{8(m+M)} \perp \frac{17r}{4}$$

$$35m + 24M \perp 34M + 34m$$

$$(35 - 34)m \perp (34 - 24)M$$

$$10M \geq m$$

hence $M \geq \frac{m}{10}$

(Q7a) $v > 0$ for complete circle (at top)

C.O.E: Total energy at A $>$ GPE at top.

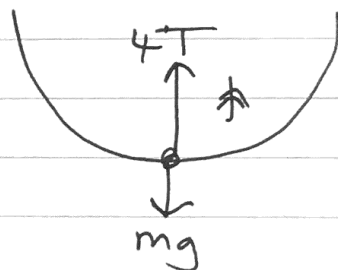
$$\frac{1}{2}mv^2 + mgL\left(\frac{4}{5}\right) > mgL$$

$$\frac{1}{2}mv^2 > \frac{mgL}{5}$$

$$v^2 > \frac{2gL}{5} \quad \therefore v > \sqrt{\frac{2gL}{5}}$$

b) Tension max at bottom.

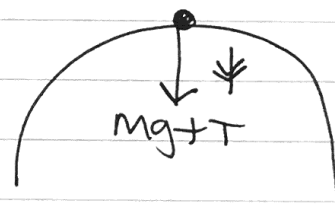
$$\underline{N2L(P)} \uparrow^+ : 4T - mg = \frac{mv^2}{L}$$



$$4T = mg + \frac{mv^2}{L} \quad \sim (1)$$

Tension minimum at top.

$$\underline{N2L(P)} \downarrow^+ : mg + T = \frac{mv^2}{L}$$



$$\underline{\times 4} : 4mg + 4T = \frac{4mv^2}{L}$$

$\Delta U \leftarrow$  ΔGPE 

$$\underline{\text{Energy to bottom}}: \frac{mv^2}{2} - \frac{mu^2}{2} = mgl(1 + \cos 2)$$

$$\therefore v^2 = 2gl(1 + \cos 2) + u^2$$

$$v^2 = \frac{18gl}{5} + u^2$$

$$\underline{\text{at top}}; \frac{mu^2}{2} - \frac{mv^2}{2} = mgl(1 - \cos 2)$$

$$u^2 - v^2 = 2gl\left(\frac{1}{5}\right)$$

$$v^2 = u^2 - \frac{2gl}{5}$$

now equate (1) and (2) and sub v^2 :

$$mg + \frac{m}{l}\left(u^2 + \frac{18gl}{5}\right) = \frac{4m}{l}\left(u^2 - \frac{2gl}{5}\right) - 4mg$$

$$mg + \frac{mu^2}{l} + \frac{18mg}{5} = \frac{4mu^2}{l} - \frac{8mg}{5} - 4mg$$

$$\frac{3mu^2}{l} = \left(\frac{18}{5} + \frac{8}{5} + 5\right)mg$$

$$\frac{3u^2}{l} = \frac{51}{5}g$$

$$\therefore u^2 = \frac{51gl}{5 \times 3} = \frac{17gl}{5} \text{ hence } u = \sqrt{\frac{17gl}{5}}$$