

Please check the examination details below before entering your candidate information

Candidate surname

Other names

**Pearson Edexcel
International
Advanced Level**

Centre Number

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Candidate Number

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Tuesday 13 November 2018

Morning (Time: 1 hour 30 minutes)

Paper Reference **WME02/01****Mechanics M2****Advanced/Advanced Subsidiary****You must have:**

Mathematical Formulae and Statistical Tables (Blue)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
– *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Whenever a numerical value of g is required, take $g = 9.8 \text{ m s}^{-2}$, and give your answer to either two significant figures or three significant figures.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
– *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ►

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1.

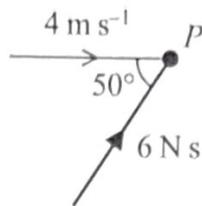


Figure 1

A particle, P , of mass 0.8 kg , moving with speed 4 m s^{-1} along a straight line on a smooth horizontal plane, receives a horizontal impulse of magnitude 6 N s . The angle between the initial direction of motion of P and the direction of the impulse is 50° , as shown in Figure 1.

Find the speed of P immediately after receiving the impulse.

(5)

$$\textcircled{1} \quad 0.8 \left(v - \begin{pmatrix} 4 \\ 0 \end{pmatrix} \right) = 6 \cos 50$$

$$v_H = \begin{pmatrix} 8.820907073 \\ 0 \end{pmatrix}$$

$$0.8 \left(v_v - \begin{pmatrix} 0 \\ 0 \end{pmatrix} \right) = 6 \cos 40$$

$$v_v = 5.74533323$$

$$\text{speed} = \sqrt{(8.8209)^2 + (5.7453)^2}$$

$$= \underline{\underline{10.5 \text{ m s}^{-1}}}$$



2.

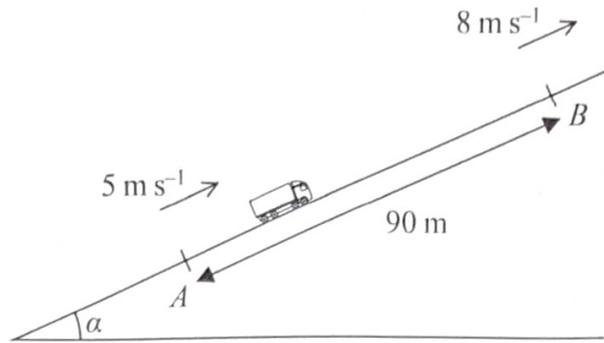


Figure 2

A truck of mass 1200 kg is being driven up a straight road that is inclined at an angle α to the horizontal, where $\sin \alpha = \frac{1}{15}$. The resistance to the motion of the truck from non-gravitational forces is modelled as a single constant force of magnitude 250 N. Two points, A and B , lie on the road, with $AB = 90$ m. The speed of the truck at A is 5 m s^{-1} and the speed of the truck at B is 8 m s^{-1} , as shown in Figure 2.

The truck is modelled as a particle and the road is modelled as a straight line.

- (a) Find the work done by the engine of the truck as the truck moves from A to B . (5)

On another occasion, the truck is being driven down the same road. The resistance to the motion of the truck is modelled as a single constant force of magnitude 250 N. The engine of the truck is working at a constant rate of 8 kW.

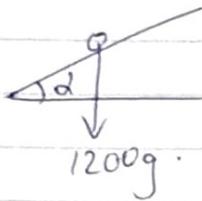
- (b) Find the acceleration of the truck at the instant when its speed is 6 m s^{-1} . (4)

(a) $D - \frac{1200g}{15} - 250 = 1200 \times a$	$D = \frac{1200g}{15} + 250 + 260.$
$s = 90$	$D = 1294 \text{ W}$
$u = 5$	
$v = 8$	Work = $F \times d$
$a = ?$	done
$t = x$	$= 1294 \times 90$
$v^2 = u^2 + 2as$	$= 116,460 \text{ J}$
$8^2 = 5^2 + 2(a)(90)$	
$a = \frac{13}{60}$	$= 116 \text{ kJ (3sf)}$



Question 2 continued

(b)



$$D + \frac{1200g}{15} - 250 = 1200 \times a$$

$$D \times 6 = 8000$$

$$D = \frac{4000}{3}$$

$$a = \frac{\frac{4000}{3} + \frac{1200g}{15} - 250}{1200}$$

$$a = \underline{\underline{1.56 \text{ ms}^{-2}}}$$



3. At time t seconds ($t \geq 0$) a particle P has position vector \mathbf{r} metres, with respect to a fixed origin O , where

$$\mathbf{r} = (16t - 3t^3)\mathbf{i} + (t^3 - t^2 + 2)\mathbf{j}$$

Find

- (a) the velocity of P at the instant when it is moving parallel to the vector \mathbf{j} , (5)
- (b) the magnitude of the acceleration of P when $t = 4$ (4)

$$(a) \frac{d\mathbf{r}}{dt} \rightarrow \mathbf{v}$$

$$\mathbf{v} = (16 - 9t^2)\mathbf{i} + (3t^2 - 2t)\mathbf{j}$$

when moving parallel to vector \mathbf{j} , \mathbf{i} component = 0.

$$\therefore 16 - 9t^2 = 0$$

$$t^2 = \frac{16}{9}$$

$$t = \pm \frac{4}{3}$$

Since t is scalar

$$t = \frac{4}{3}$$

$$\mathbf{v} = ? \text{ when } t = \frac{4}{3}$$

$$\mathbf{v} = \frac{8}{3} \mathbf{j} \text{ ms}^{-1}$$

$$(b) \mathbf{a} \rightarrow \frac{d\mathbf{v}}{dt}$$

$$-18t\mathbf{i} + (6t - 2)\mathbf{j}$$

$$\text{when } t = 4$$

$$-18(4)\mathbf{i} + (6(4) - 2)\mathbf{j}$$

$$\mathbf{a} = -72\mathbf{i} + 22\mathbf{j}$$

$$|\mathbf{a}| = \sqrt{72^2 + 22^2}$$

$$|\mathbf{a}| = \underline{75.3} \text{ ms}^{-2}$$



4. At time $t = 0$ a ball is projected from a fixed point A on horizontal ground to hit a target. The ball is projected from A with speed $u \text{ m s}^{-1}$ at an angle θ° to the horizontal. At time $t = 2 \text{ s}$ the ball hits the target. At the instant when it hits the target, the ball is travelling downwards at 30° below the horizontal with speed 12 m s^{-1} . The ball is modelled as a particle moving freely under gravity and the target is modelled as the point T .

(a) Find

- (i) the value of θ ,
- (ii) the value of u .

(7)

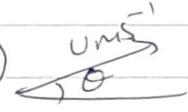
The height of T above the ground is h metres.

(b) Find the value of h .

(3)

(c) Find the length of time for which the ball is more than h metres above the ground during the flight from A to T .

(3)

<p>(a) i) </p> <p>$12 \cos 30 = u \cos \theta \quad \dots \textcircled{1}$</p> <p>Taking \uparrow as +ve</p> <p>$s = x$ $u = u \sin \theta$ $v = -12 \sin 30$ $a = -9.8$ $t = 2$</p> <p>$\frac{v-u}{t} = a$</p> <p>$-12 \sin 30 - u \sin \theta = -19.6$</p> <p>$u \sin \theta = 13.6 \quad \dots \textcircled{2}$</p>	<p>$\frac{\textcircled{2}}{\textcircled{1}} = \tan \theta$</p> <p>$= \frac{13.6}{12 \cos 30} = \tan \theta$</p> <p>$\tan^{-1}(\text{Ans}) = \underline{52.6^\circ} = \theta$</p> <p>(ii) $u = \frac{12 \cos 30}{\cos(52.6)}$</p> <p>$= \underline{17.1 \text{ m s}^{-1}} \text{ (3sf)}$</p> <p>$\uparrow$ +ve.</p> <p>(b) $s = ?$</p> <p>$u = 17.1 \sin 52.6$ $v = -12 \sin 30$ $a = -9.8$ $t = 2$</p>
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Question 4 continued

$$s = vt - \frac{1}{2}at^2$$

$$s = -6(2) + 4.9(2)^2$$

$$s = 7.6 \text{ m} \rightarrow h$$

c) $s = 7.6$

$$u = 17.1 \text{ s} \sin 2.6$$

$$v = x$$

$$a = -9.8$$

$$t = ?$$

$$s = ut + \frac{1}{2}at^2$$

$$7.6 = 13.6t - 4.9t^2$$

$$\frac{-13.6 \pm \sqrt{13.6^2 - 4(-4.9)(-7.6)}}{2 \times -4.9}$$

$$t = 2 \text{ or } t = \frac{38}{49}$$

$$\frac{2 - 38}{49} = \frac{60}{49}$$

$$= \underline{1.22 \text{ s}} \text{ (3sf)}$$



5.

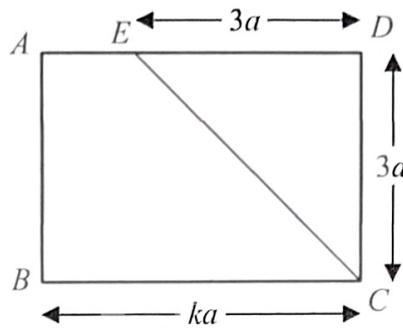


Figure 3

Figure 3 shows a uniform rectangular lamina $ABCD$ with sides of length $3a$ and ka , where $k > 3$. The point E on side AD is such that $DE = 3a$. Rectangle $ABCD$ is folded along the line CE to produce the folded lamina L shown in Figure 4.

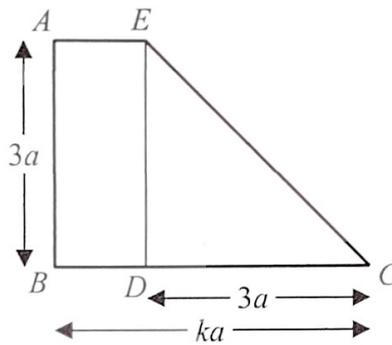


Figure 4

Find, in terms of a and k ,

(a) the distance of the centre of mass of L from AB , (5)

(b) the distance of the centre of mass of L from AE . (4)

The folded lamina L is freely suspended from A and hangs in equilibrium with AB at 45° to the downward vertical.

(c) Find, to 3 significant figures, the value of k . (4)

$A(0, 3a) \quad B(0, 0) \quad C(ka, 0) \quad D((k-3)a, 0) \quad E = ((k-3)a, 3a)$

mass ratio	$(3k-9)a^2$	$9a^2$	$3ka^2$
x	$\left(\frac{k-3}{2}\right)a$	$(k-2)a$	\bar{x}
y	$1.5a$	a	\bar{y}



Question 5 continued

a $[(3k-9) \left(\frac{k-3}{2}\right) + 9(k-2)] = 3k\bar{x}$ (b) From AE

a $[3k^2 - 18k + 27 + 18k - 36] = 6k\bar{x}$ $-4.5ka - 4.5a = 3k\bar{y}$

a $(3k^2 - 9) = 6k\bar{x}$

$\bar{y} = \left(\frac{-3}{2} - \frac{3}{2k}\right) a$

$\bar{x} = \left(\frac{k^2 - 3}{2k}\right) a$

Since it is a distance.

(b) From AE

$|\bar{y}| = \left(\frac{3}{2} + \frac{3}{2k}\right) a$

$3k-9(1.5a) + 9a = 3k\bar{y}$

$4.5ka - 4.5a = 3k\bar{y}$

(c) $\tan 45 = \frac{k/2 - 3/2k}{3/2 + 3/2k}$

$\bar{y} = \left(\frac{3}{2} - \frac{3}{2k}\right) a$

$\frac{k}{2} - \frac{3}{2k} = \frac{3}{2} + \frac{3}{2k}$

From AE

$= 3a - \frac{3}{2}a - \frac{3}{2k}a$

$\left(\frac{k}{2} - \frac{3}{2} - \frac{3}{2k} = 0\right) 2k$

$k^2 - 3k - 6 = 0$

$= \left(\frac{3}{2} - \frac{3}{2k}\right) a$

$\frac{3 \pm \sqrt{3^2 - 4(1)(-6)}}{2 \times 1}$

$k > 0$

$\therefore k = \underline{\underline{4.37}}$



6.

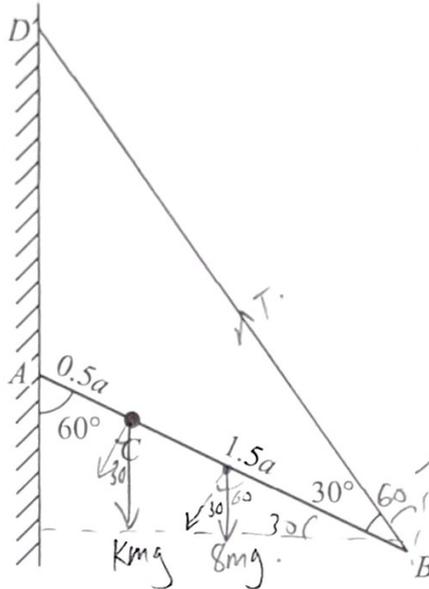


Figure 5

A uniform rod, AB , of mass $8m$ and length $2a$, has its end A resting against a rough vertical wall. One end of a light inextensible string is attached to the rod at B and the other end of the string is attached to the wall at the point D , which is vertically above A . The angle between the rod and the string is 30° . A particle of mass km is fixed to the rod at C , where $AC = 0.5a$. The rod is in equilibrium in a vertical plane perpendicular to the wall, and is at an angle of 60° to the wall, as shown in Figure 5. The tension in the string is T .

(a) Show that $T = \frac{\sqrt{3}}{4}(16 + k)mg$ (4)

The coefficient of friction between the wall and the rod is $\frac{2}{3}\sqrt{3}$.

Given that the rod is in limiting equilibrium,

(b) find the value of k . (6)

(a) $M(CA) =$ (Aligning forces to be perpendicular to rod).

$$0.5a \cdot kmg \cos 30 + 1.5a \cdot 8mg \cos 30 = T \cos 60 \cdot 2a$$

$$\frac{\sqrt{3}}{4} kmg + 4\sqrt{3} mg = T$$

$$\frac{\sqrt{3}}{4} mg (k + 16) = T \text{ as required.}$$



Question 6 continued

(b) Balancing Forces.

$$T \cos 60 = R_A \quad (\leftarrow)$$

$$\mu R_A + T \cos 30 = mg(k+8).$$

$$\mu = \frac{2\sqrt{3}}{3}$$

$$\frac{2\sqrt{3}}{3} \cdot \frac{\sqrt{3}}{4} mg(k+16) + \frac{1}{2} + \frac{\sqrt{3}}{4} mg(k+16) \cos 30 = mg(k+8).$$

$$\frac{1}{4}(k+16) + \frac{3}{8}(k+16) = k+8.$$

$$\frac{5}{8}k + 10 = k+8.$$

$$2 = \frac{3}{8}k.$$

$$k = \frac{16}{3}.$$



7. A particle, P , of mass km is moving in a straight line with speed $3u$ on a smooth horizontal surface. Particle P collides directly with another particle, Q , of mass $2m$ which is moving with speed u in the same direction along the same straight line. The coefficient of restitution between P and Q is e .

Given that immediately after the collision P and Q are moving in opposite directions and the speed of Q is $\frac{3}{2}u$,

- (a) find the range of possible values of e . (4)

It is now also given that $e = \frac{7}{8}$.

- (b) Show that the kinetic energy lost by P in the collision with Q is $\frac{11}{8}mu^2$. (6)

The collision between P and Q takes place at the point A . After the collision, Q hits a fixed vertical wall that is perpendicular to the direction of motion of Q . The distance from A to the wall is d . The coefficient of restitution between Q and the wall is $\frac{1}{3}$. Particle Q rebounds from the wall and moves so that P and Q collide directly at the point B .

- (c) Find, in terms of d and u , the time interval between the collision at A and the collision at B .

But $e \leq 1$
 $\therefore \frac{3}{4} \leq e \leq 1$ (6)

<p>(a)</p> <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="text-align: center; width: 50%;"> <p>(P)</p> <p style="text-align: center;">km</p> </td> <td style="width: 10%; border-left: 1px solid black;"></td> <td style="text-align: center; width: 50%;"> <p>(Q)</p> <p style="text-align: center;">$2m$</p> </td> </tr> <tr> <td style="text-align: center;"> <p>Initial \rightarrow</p> <p style="text-align: center;">$3u$</p> </td> <td style="border-left: 1px solid black;"></td> <td style="text-align: center;"> <p>\rightarrow</p> <p style="text-align: center;">u</p> </td> </tr> <tr> <td style="text-align: center;"> <p>Final \leftarrow</p> <p style="text-align: center;">v_2</p> </td> <td style="border-left: 1px solid black;"></td> <td style="text-align: center;"> <p>\rightarrow</p> <p style="text-align: center;">$\frac{3}{2}u$</p> </td> </tr> </table> $\frac{v_2 + \frac{3}{2}u}{2u} = e$ $\frac{3u + v_2}{2} = 2ue \quad \text{--- (1)}$ $v_2 = 2ue - \frac{3}{2}u \quad \text{--- (2)}$ <p>Since $v > 0$</p> $2ue > \frac{3}{2}u$ $e > \frac{3}{4}$	<p>(P)</p> <p style="text-align: center;">km</p>		<p>(Q)</p> <p style="text-align: center;">$2m$</p>	<p>Initial \rightarrow</p> <p style="text-align: center;">$3u$</p>		<p>\rightarrow</p> <p style="text-align: center;">u</p>	<p>Final \leftarrow</p> <p style="text-align: center;">v_2</p>		<p>\rightarrow</p> <p style="text-align: center;">$\frac{3}{2}u$</p>	<p>(b) Law of conservation of momentum.</p> <p>$R(\rightarrow)$</p> $km(3u) + 2m(u) = -kmv_1 + 3m \cdot u$ $3uk + 2u = -kv_2 + 3u$ <p>Knowing $v_2 = 2ue - \frac{3}{2}u$</p> $v_2 = 2u\left(\frac{7}{8}\right) - \frac{3}{2}u$ $v_2 = \frac{1}{4}u$ $3uk + 2u = -\frac{1}{4}uk + 3u$ $\frac{13}{4}k = 1$ $\therefore k = \frac{4}{13}$
<p>(P)</p> <p style="text-align: center;">km</p>		<p>(Q)</p> <p style="text-align: center;">$2m$</p>								
<p>Initial \rightarrow</p> <p style="text-align: center;">$3u$</p>		<p>\rightarrow</p> <p style="text-align: center;">u</p>								
<p>Final \leftarrow</p> <p style="text-align: center;">v_2</p>		<p>\rightarrow</p> <p style="text-align: center;">$\frac{3}{2}u$</p>								



Question 7 continued

KE Before

$$\frac{1}{2} \times \frac{4m}{13} (3u)^2$$

$$= \frac{18mu^2}{13}$$

KE after

$$\frac{1}{2} \times \frac{4m}{13} \times \left(\frac{1}{4}u\right)^2$$

$$= \frac{1}{104} mu^2$$

 ΔKE

$$= \frac{11}{8} mu^2 \text{ as req'}$$

$$(c) \frac{V_f}{V_i} = e$$

$$= \frac{V_f}{\frac{3}{2}u} = \frac{1}{3}$$

$$V_{\text{final}} = \frac{1}{2} u \text{ m s}^{-1}$$

$$\text{Time} = \frac{d}{s} = \frac{d}{V_i} = \frac{d}{\frac{3}{2}u}$$

$$= \frac{2d}{3u} \text{ -- (i)}$$

How far has P moved in this time?

$$d = s \times t$$

$$= \frac{u}{4} \rightarrow V_2$$

$$\frac{u}{4} \times \frac{2d}{3}$$

$$= \frac{1}{6} d \text{ meters}$$

\therefore Total distance between wall and P when Q just hits wall = $\frac{1}{6}d + d = \frac{7}{6}d$.

This means the velocities are closing in at

$$\frac{1}{2}u - \frac{1}{4}u = \frac{1}{4}u$$

$$t = \frac{d}{s} = \frac{\frac{7}{6}d}{\frac{1}{4}u}$$

$$= \frac{14d}{3u} \text{ -- (ii)}$$

\therefore Total time = (i) + (ii)

$$= \frac{14d}{3u} + \frac{2d}{3u} = \frac{16d}{3u}$$

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