M2 October 2017 (MA)

Q1) $I = m(v-u)$

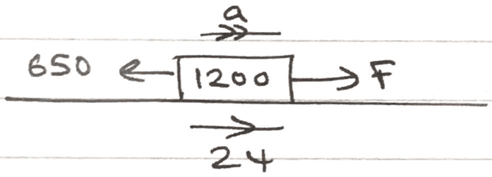
$I = 0.2(5\underline{i} + 8\underline{j} - 10\underline{i} + 17\underline{j})$

$I = 0.2(-5\underline{i} + 25\underline{j}) = -\underline{i} + 5\underline{j}$

$\therefore |I| = \sqrt{1^2 + 5^2} = \sqrt{26}\ \text{Ns}$

$= \boxed{5.1}\ \text{Ns}$

Q2a)



$P = 30000 = Fv$

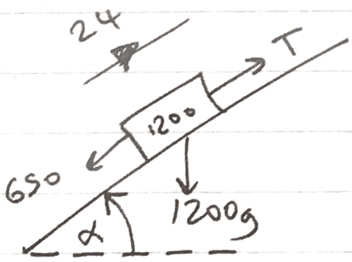
$30000 = 24F$

$\therefore F = 1250\text{N}$ //

$\xrightarrow{+}$ N2L (van) : $F - 650 = 1200a$

$\dfrac{1250 - 650}{1200} = a = \boxed{0.5\text{ms}^{-2}}$

b)



$\sin\alpha = \dfrac{1}{12}$

$P = Tv \longrightarrow T = \dfrac{P}{24}$ //

$\nearrow^{+}$ N2L (van) : $T - 650 - \dfrac{1200g}{12} = 1200(0)$

$\therefore \dfrac{P}{24} = 650 + \dfrac{1200g}{12} = 1630$ //

$\therefore P = 24 \times 1630 = 39120\,W$

$= \boxed{39.1kW}$

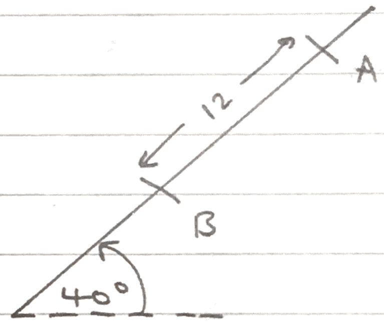
Q3a) W.D due to friction $= F \times d$

$= \mu R \times d$

$= 0.5(4g\cos 40) \times 12 = \boxed{180J}$

b) at A : KE $= 2v^2$

GPE $= 4g(12\sin 40)$



at B : KE $= 2(24)^2$

GPE $= 0$

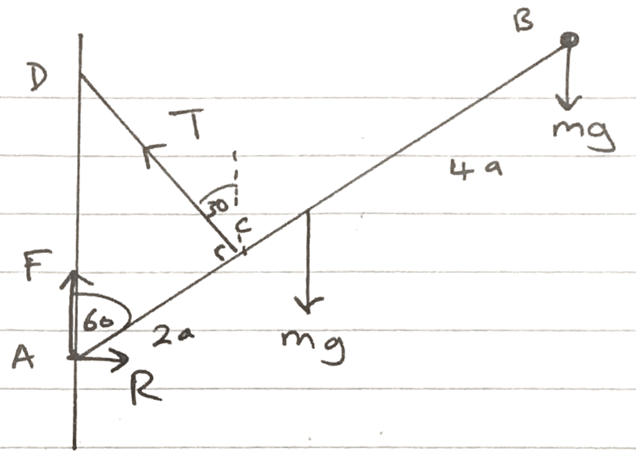
W.D due to friction $\simeq 180J$.

so... $2v^2 + 4g(12\sin 40) = 2(24)^2 + 180$

$v^2 = \dfrac{2(24)^2 + 180 - 4g(12\sin 40)}{2} \simeq 515$

so $\boxed{v \simeq 22.7\ m/s}$

Q4a)



$$M(A): \quad T(2a) = mg(3a\sin 60) + mg(6a\sin 60)$$

$$T = \frac{9\cancel{a}mg\sin 60}{2\cancel{a}} = 9mg \times \frac{\sqrt{3}}{4}$$

$$= \boxed{\frac{9\sqrt{3}}{4}mg}$$

b) $R(\leftrightarrow)$: $R = T\cos 60$ //

$$\text{so } R = \frac{9\sqrt{3}}{4} \times \frac{1}{2} \times mg = \boxed{\frac{9\sqrt{3}}{8}mg}$$

c) $R(\updownarrow)$: $F + T\cos 30 = 2mg$

$$F = 2mg - \frac{9\sqrt{3}}{4}\cos 30(mg)$$

$$F = \frac{-11}{8}mg \; //$$

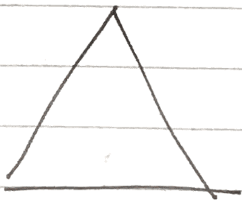
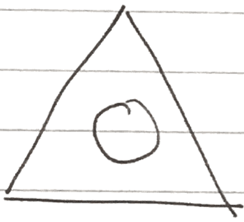
$F < 0$ so F acts <u>downwards</u>.

$$|F| = \frac{11}{8}mg.$$

$F = \mu R$ ← limiting equilibrium

$$\therefore \mu = \frac{F}{R} = \frac{\frac{11}{8}\cancel{mg}}{\frac{9\sqrt{3}}{8}\cancel{mg}} = \boxed{0.71}$$

(Q5a)

	Shape	Mass (area)	Distance of c.o.m from D
(+)		$\boxed{4a^2\sqrt{3}}$	$\frac{4a\sin 60}{3} = \boxed{\frac{2a}{3}\sqrt{3}}$
(−)		$\boxed{\pi a^2}$	$\boxed{\frac{3a}{2}}$
(=)		$\boxed{a^2(4\sqrt{3}-\pi)}$	$\boxed{\bar{y}}$

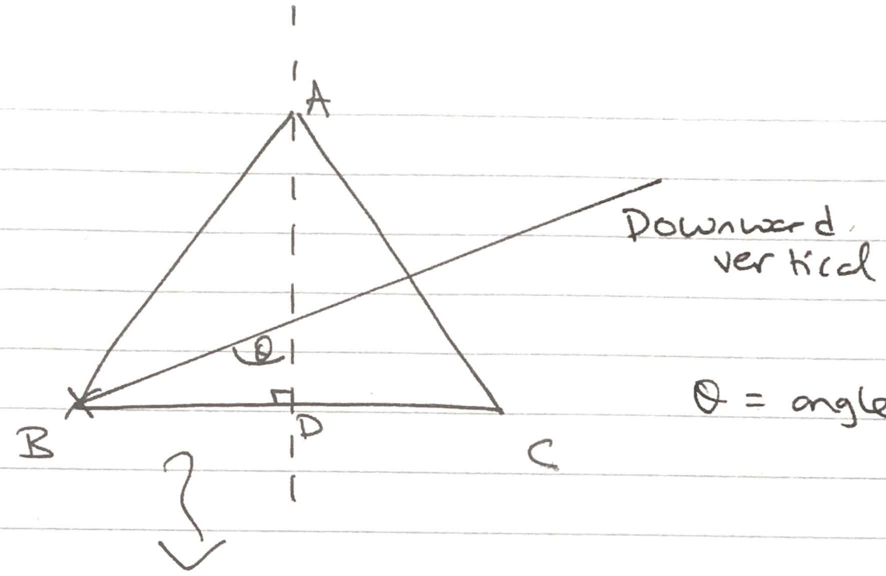
$$\underline{\bar{x}\sum m_i = \sum m_i x_i}$$

$$4a^2\sqrt{3}\left(\frac{2a}{3}\sqrt{3}\right) - \pi a^2\left(\frac{3a}{2}\right) = a^2(4\sqrt{3}-\pi)\bar{y}$$

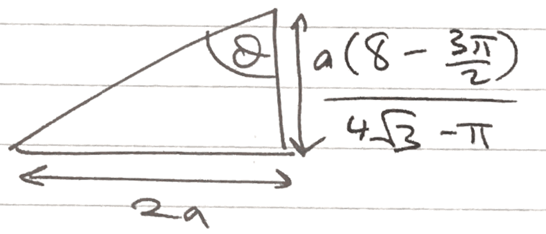
$$8a - \frac{3\pi a}{2} = (4\sqrt{3}-\pi)\bar{y}$$

$$\therefore \bar{y} = \frac{a\left(8 - \frac{3\pi}{2}\right)}{4\sqrt{3}-\pi}$$ = distance from D of c.o.m

b)



θ = angle required.



$$\tan\theta = \frac{2a}{\frac{a(8-\frac{3\pi}{2})}{4\sqrt{3}-\pi}} = \frac{8\sqrt{3}-2\pi}{8-\frac{3\pi}{2}}$$

$$\approx 2.304\ldots$$

so $\theta = \tan^{-1}(2.304\ldots) \approx \boxed{67°}$

Q6a) $a = 2t - 3$

$v = \int (a)\,dt = \int (2t-3)\,dt = t^2 - 3t + c = v$

$t = 3, v = 2$: $2 = 9 - 3(3) + c$

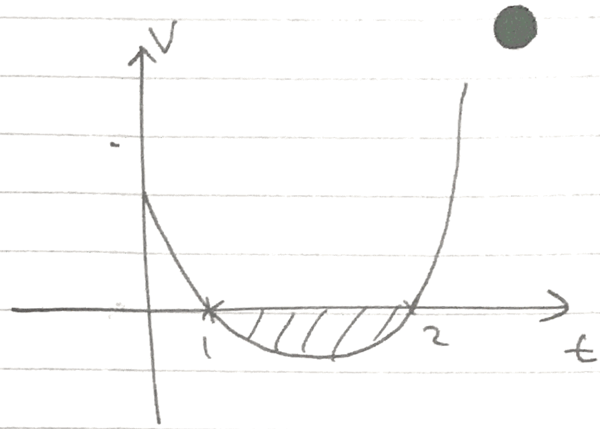
$\therefore c = 2$

so $\boxed{v = t^2 - 3t + 2}$

b) $V = t^2 - 3t + 2$

$V = (t-2)(t-1) = 0$

so P is at A at $t=1$
and P is at B at $t=2$



$\therefore AB = \left| \int_1^2 (t^2 - 3t + 2)\,dt \right|$

$\Rightarrow \int_1^2 (t^2 - 3t + 2)\,dt = \left[\frac{t^3}{3} - \frac{3t^2}{2} + 2t\right]_1^2$

$= \left[\frac{2}{3}\right] - \left[\frac{5}{6}\right] = -\frac{1}{6}$

$\therefore AB = \left|-\frac{1}{6}\right| = \boxed{\frac{1}{6}}\,m$

Q7a) <u>At A</u>: $KE = \frac{1}{2}(m)(15)^2$

$GPE = mg(47.5)$

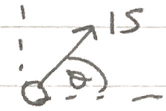
<u>At B</u>: $KE = \frac{1}{2}(m)(v)^2$

$GPE = 0$

<u>C.O.E</u>: $\frac{225m}{2} + 47.5mg = \frac{mv^2}{2}$

$\therefore v^2 = 225 + 95g$

$v = \sqrt{225 + 95g} = \boxed{34 \text{ m/s}}$

b) 

$$\uparrow \left.\begin{array}{l} s = s \\ u = 15\sin\theta = 9 \\ v = 0 \\ a = -g \\ t = \end{array}\right\} \quad \begin{array}{l} v^2 = u^2 + 2as \\ 0^2 = 9^2 - 2gs \end{array}$$

$s = \frac{9^2}{2g} = \frac{405}{98}$ m

So greatest height $= \frac{405}{98} + 47.5$

$= \boxed{51.6 \text{ m}}$

c) least speed occurs at highest point (when vert component of speed $= 0$)

least speed $= \vec{u} = 15\cos\theta = 15 \times \frac{4}{5} = \boxed{12}$ m/s

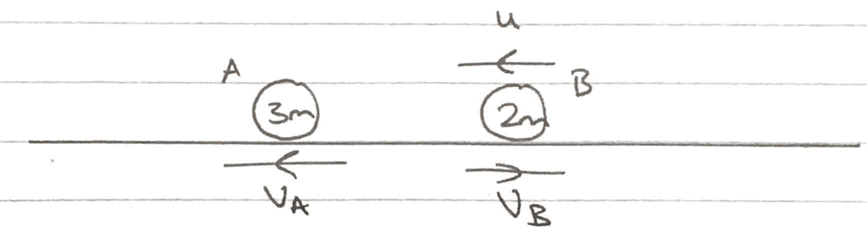
d)

$$\uparrow \left.\begin{array}{l} s = -47.5 \\ u = 9 \\ v = \\ a = -g \\ t = t \end{array}\right\} \quad \begin{array}{l} s = ut + \frac{1}{2}at^2 \\ -47.5 = 9t - 4.9t^2 \\ 4.9t^2 - 9t - 47.5 = 0 \end{array}$$

By Quadratic formula, $t = 4.16\ldots$
$t > 0$ so reject other solution.

$\vec{s} = ut = 12 \times t = 12 \times 4.16\ldots \simeq \boxed{50 \text{ m}}$

Q8ai)



$\overleftarrow{\text{C.L.M}}$: $2mu = 3mV_A - 2mV_B$

$$2u = 3V_A - 2V_B \sim ①$$

$\overleftarrow{\text{N.I.L}}$: $e = \dfrac{V_A + V_B}{u}$

$$\therefore ue = V_A + V_B$$

$\times 2$: $2ue = 2V_A + 2V_B$

$+ ①$: $+[2u = 3V_A - 2V_B]$

$$2u(1+e) = 5V_A + 0$$

$$\therefore V_A = \boxed{\frac{2u}{5}(1+e)} = \text{speed}$$

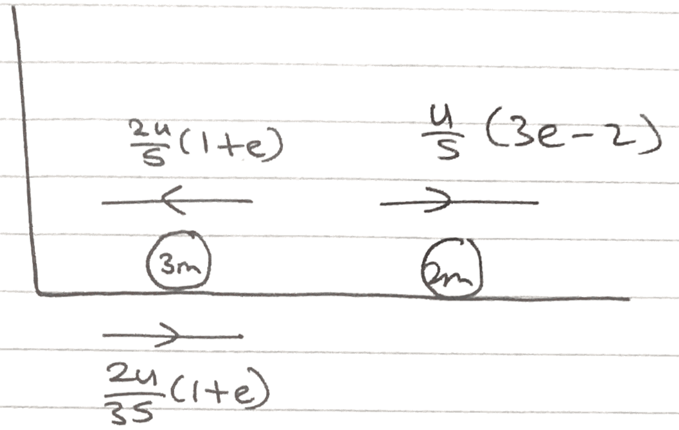
ii) $V_B = ue - V_A = ue - \dfrac{2u}{5} - \dfrac{2ue}{5}$

$$V_B = \frac{3ue}{5} - \frac{2u}{5} = \frac{u}{5}(3e-2)$$

$$\therefore \text{speed}_B = \boxed{\left|\frac{u}{5}(3e-2)\right|}$$

Modulus not required since we know B changes direction so $\frac{u}{5}(3e-2) > 0$

b)



Second collision occurs

so $\frac{2u}{35}(1+e) > \frac{u}{5}(3e-2)$

$\Rightarrow \frac{2}{35} + \frac{2e}{35} > \frac{3e}{5} - \frac{2}{5}$

$\Rightarrow \frac{19e}{35} < \frac{16}{35} \longrightarrow e < \frac{\frac{16}{35}}{\frac{19}{35}}$

$\Rightarrow e < 16/19$ //

and $V_B > 0$ since we are told B's direction is reversed . . .

$\Rightarrow \frac{u(3e-2)}{5} > 0 \quad \therefore \quad 3e - 2 > 0$ //

$\Rightarrow 3e > 2 \quad \therefore e > 2/3$ //

hence $\boxed{\frac{2}{3} < e < \frac{16}{19}}$