

M2 October 2016 (MA)

Q1a)  $\underline{\underline{\sum m_i x_i = \bar{x} \sum m_i}}$

$$m \begin{pmatrix} -3 \\ 2 \end{pmatrix} + 4m \begin{pmatrix} 4 \\ 3 \end{pmatrix} + 4m \begin{pmatrix} 6 \\ -4 \end{pmatrix} = (5+4u)m \begin{pmatrix} c \\ 0 \end{pmatrix}$$

consider j component:  $2m + 12m - 44m = 0$

$$14 = 4u$$

$$\boxed{u = \frac{7}{2}}$$

b) consider i:  $-3m + 16m + 6um = (5 + \frac{7}{2})m \times c$

$$\frac{16 - 3 + 6(\frac{7}{2})}{5 + \frac{7}{2}} = \boxed{c = 4}$$

Q2)  $I = m(v-u)$

$$\lambda \underline{i} - 2\lambda \underline{j} = 2(v - 3\underline{i})$$

$$\lambda \underline{i} - 2\lambda \underline{j} = 2v - 6\underline{i}$$

$$(6 + \lambda) \underline{i} - 2\lambda \underline{j} = 2v$$

$$\therefore v = \left(3 + \frac{\lambda}{2}\right) \underline{i} + (-\lambda) \underline{j}$$

but  $|v| = 6$ .  $\therefore \sqrt{\left(3 + \frac{\lambda}{2}\right)^2 + (-\lambda)^2} = 6$

$$\left(3 + \frac{\lambda}{2}\right)^2 + \lambda^2 = 36$$

$$\lambda^2 + \frac{\lambda^2}{4} + 3\lambda + 9 = 36$$

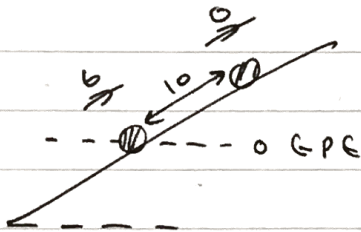
$$\frac{5}{4}\lambda^2 + 3\lambda - 27 = 0$$

By Quadratic formula:

$$\lambda = \frac{18}{5}$$

$$\lambda = -6$$

Q3a)



$$\sin \alpha = \frac{1}{7}$$

$$\therefore \cos \alpha = \sqrt{1 - \left(\frac{1}{7}\right)^2} = \frac{4\sqrt{3}}{7}$$

Initial energy:  $KE = \frac{1}{2}(4)(6)^2 = 72 \text{ J}$   
 $GPE = 0$

Final energy:  $KE = 0$   
 $GPE = 4g(10 \sin \alpha) = \frac{40g}{7}$

Initial energy - Final energy = W.D due to friction

$$72 - \frac{40g}{7} = \text{W.D against friction} = \boxed{16 \text{ J}}$$

b)

At B:  $KE = 0$   
 $GPE = \frac{40g}{7}$

At A:  $KE = 2v^2$   
 $GPE = 0$

$$\text{\$ W.D due to friction} = 16 //$$

$$\text{so } \frac{40g}{7} = 2v^2 + 16$$

$$v^2 = \frac{\frac{40g}{7} - 16}{2} = 20$$

$$\text{so } v = \sqrt{20} = \boxed{4.47 \text{ ms}^{-1}}$$

$$\text{(Q4a)} \quad \frac{dr}{dt} = v = (3t^2 - 9t - 24)\underline{i} + (-3t^2 + 6t + 12)\underline{j}$$

When P moves parallel to  $-\underline{i} - \underline{j}$ , components of velocity ( $i$  and  $j$ ) are equal.

$$\text{so } 3t^2 - 9t - 24 = -3t^2 + 6t + 12$$

$$6t^2 - 15t - 36 = 0$$

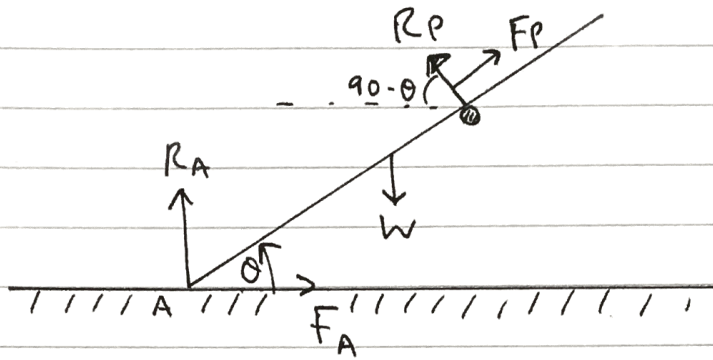
By Quadratic formula:  $t = 4$ ,  $t = -\frac{3}{2}$

$$t > 0 \text{ so } t = \boxed{4} = T$$

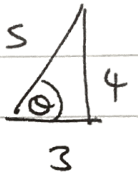
$$\text{b) } a = \frac{dv}{dt} = (6t - 9)\underline{i} + (-6t + 6)\underline{j}$$

$$|a| = \sqrt{(6 \times 4 - 9)^2 + (-6(4) + 6)^2} = 3\sqrt{61} = \boxed{23.4 \text{ m/s}}$$

Q5a)



$$\tan \theta = \frac{4}{3}$$



$$\cos \theta = \frac{3}{5}$$

$$\sin \theta = \frac{4}{5}$$

$$M(A): R_p(s) = W \cos \theta \quad (4)$$

$$R_p = \frac{4W \cos \theta}{5} = \frac{4}{5} \times \frac{3}{5} \times W = \boxed{0.48W}$$

$$b) R(\updownarrow): R_p \cos \theta + R_A + F_p \sin \theta = W \quad (1)$$

$$R(\leftrightarrow): F_A + F_p \cos \theta = R_p \sin \theta \quad (2)$$

$$\underline{F_p = \frac{1}{4} R_p}$$

$$(2): F_A + \frac{1}{4} R_p \left( \frac{3}{5} \right) = R_p \left( \frac{4}{5} \right)$$

$$F_A = \frac{13}{20} R_p //$$



$$(1): R_p \cos \theta + \frac{F_A}{\mu} + F_p \sin \theta = W$$

$$\underline{(F_A = \mu R_A)}$$

$$\Rightarrow R_p \left( \frac{3}{5} \right) + \frac{13 R_p}{20\mu} + \frac{1}{4} R_p \left( \frac{4}{5} \right) = W$$

$$\Rightarrow R_p \left( \frac{3}{5} + \frac{13}{20\mu} + \frac{1}{5} \right) = W$$

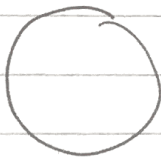



$$0.48W \left( \frac{4}{5} + \frac{13}{20\mu} \right) = W$$

$$\frac{4}{5} + \frac{13}{20\mu} = \frac{25}{12}$$

$$\frac{13}{20\mu} = \frac{77}{60}$$

$$\mu \left( \frac{20}{13} \right) = \frac{60}{77}$$

$$\mu = \frac{\frac{60}{77}}{\frac{20}{13}} = \boxed{0.506} = \frac{39}{77}$$

Q6a)	<u>Shape</u>	<u>Mass (area)</u>	<u>Distance of c.o.m from R</u>
(+)		$\pi(8a)^2$ $= \boxed{64\pi a^2}$	$\boxed{8a}$
(-)		$\boxed{\pi a^2}$	$\boxed{12a}$
(-)		$\boxed{4\pi a^2}$	$\boxed{4a}$
(=)		$\boxed{59\pi a^2}$	$\boxed{\bar{x}}$

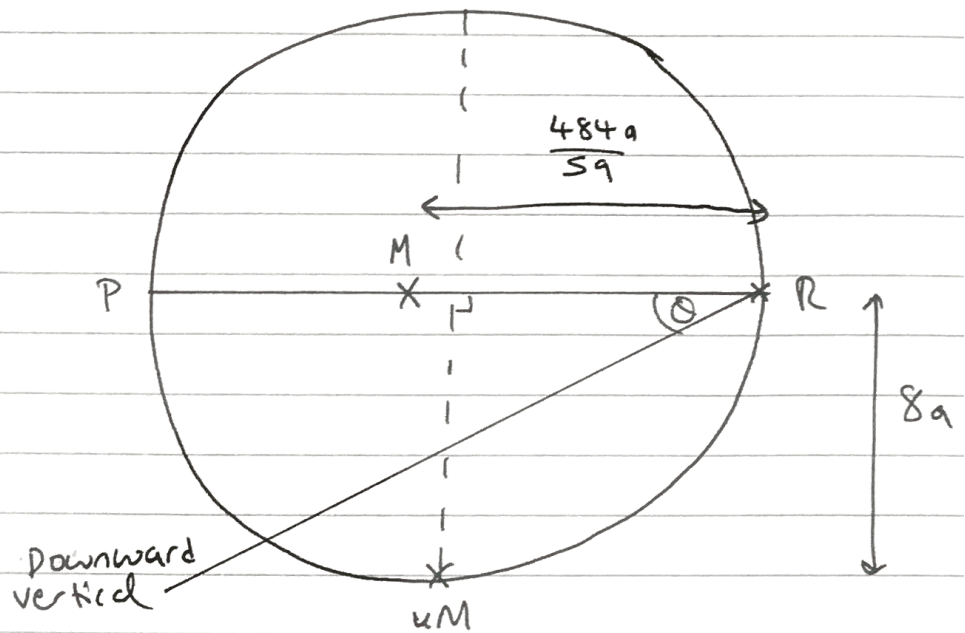
$$\sum m_i x_i = \bar{x} \sum m_i$$

$$64(8a) - 1(12a) - 4(4a) = 59(\bar{x})$$

$$484 = 59\bar{x}$$

hence  $\bar{x} = \boxed{\frac{484a}{59}}$

b)  $\tan \theta = \frac{1}{4}$

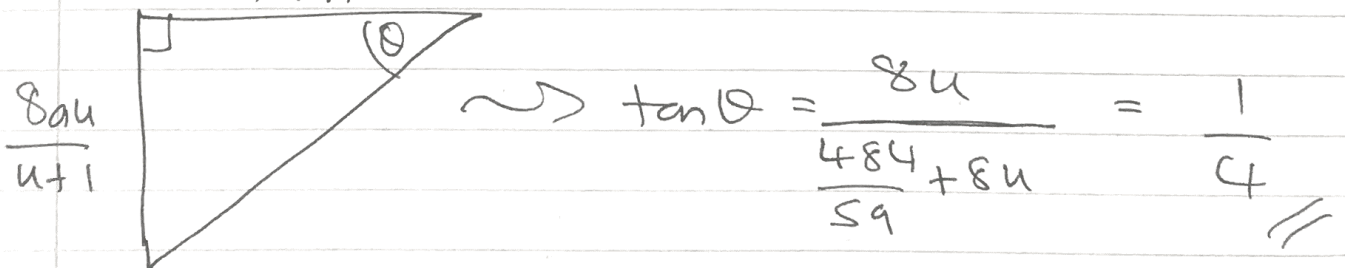


$$\bar{x} \sum m_i = \sum m_i x_i$$

$$M \begin{pmatrix} \frac{484a}{59} \\ 0 \end{pmatrix} + uM \begin{pmatrix} 8a \\ -8a \end{pmatrix} = (u+1)M \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix}$$

$$\begin{pmatrix} \frac{a \left( \frac{484}{59} + 8u \right)}{u+1} \\ \frac{-8au}{u+1} \end{pmatrix} = \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} //$$

$a \left( \frac{484}{59} + 8u \right) / (u+1)$

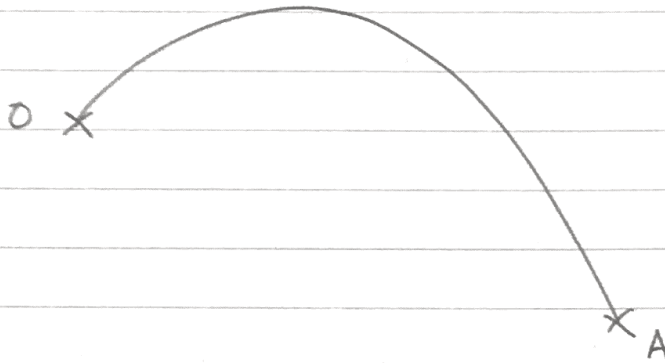




$$\Rightarrow \frac{484}{59} + 8u = 324$$

$$\Rightarrow u = \frac{484}{59} = \boxed{\frac{121}{354}}$$

Q7)



expressing position as a vector relative to O,

$$r = \begin{pmatrix} 3t \\ \lambda t - \frac{9.8t^2}{2} \end{pmatrix} \quad \left( \begin{array}{l} \uparrow \\ \uparrow \\ \uparrow \\ \uparrow \end{array} \right) \left. \begin{array}{l} s = 5 \\ u = \lambda \\ v = \\ a = -9 \\ t = t \end{array} \right\} \begin{array}{l} s = \lambda t - \frac{9}{2}t^2 \\ = \end{array}$$

$$\therefore v = \dot{r} = \begin{pmatrix} 3 \\ \lambda - 9.8t \end{pmatrix} //$$

$$KE \text{ at } O = \frac{1}{2} m (9 + \lambda^2)$$

$$KE \text{ at } A = \frac{1}{2} m (25)$$

$$KE \text{ at } A = \frac{1}{2} \times KE \text{ at } O$$

$$\therefore \frac{25m}{2} = \frac{m}{2} (9 + \lambda^2) \times \frac{1}{2}$$

$$\frac{2S}{2} = \frac{1}{4} (9 + \lambda^2)$$

$$\lambda^2 + 9 = 50$$

$$\lambda^2 = 41$$

$$\lambda = \pm \sqrt{41}$$

$$\lambda > 0 \quad \therefore \lambda = \sqrt{41}$$

at A,  $\underline{r} = 3\underline{i} - 4\underline{j}$  and  $\underline{r} = \begin{pmatrix} 3 \\ \lambda - 9.8t \end{pmatrix}$

$$\therefore -4 = \lambda - 9.8t$$

$$t = \frac{\lambda + 4}{9.8} = \frac{4 + \sqrt{41}}{9.8} = 1.062\dots$$

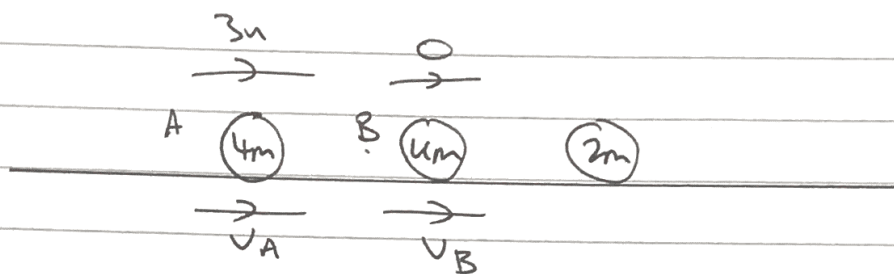
at  $t = 1.062$ ,  $\underline{r} = \begin{pmatrix} 3 \times 1.062 \\ \sqrt{41}(1.062) - 4.9(1.062)^2 \end{pmatrix}$

$$\underline{r} = \begin{pmatrix} 3.18 \\ 1.27 \end{pmatrix}$$

$$\underline{r} = \boxed{\begin{pmatrix} 3.2 \\ 1.3 \end{pmatrix}}$$



(Q8a)



$$\underline{\text{C.L.M}} : 4m(3u) = 4mv_A + 4mv_B$$

$$12u = 4v_A + 4v_B \quad \sim (1)$$

$$\underline{\text{N.T.L}} : e = \frac{2}{3} = \frac{v_B - v_A}{3u}$$

$$\therefore 2u = v_B - v_A \quad \sim (2)$$

$$\times 4 : 8u = 4v_B - 4v_A$$

$$+ (1) : + [12u = 4v_A + 4v_B]$$

$$20u = (4+4)v_B$$

$$\therefore v_B = \frac{20u}{4+4}$$

$$(2) : \text{so } v_A = v_B - 2u = \frac{20u}{4+4} - 2u$$

$$v_A = \frac{20u - 2u(4+4)}{4+4}$$

$$v_A = \frac{12u - 24u}{4+4}$$

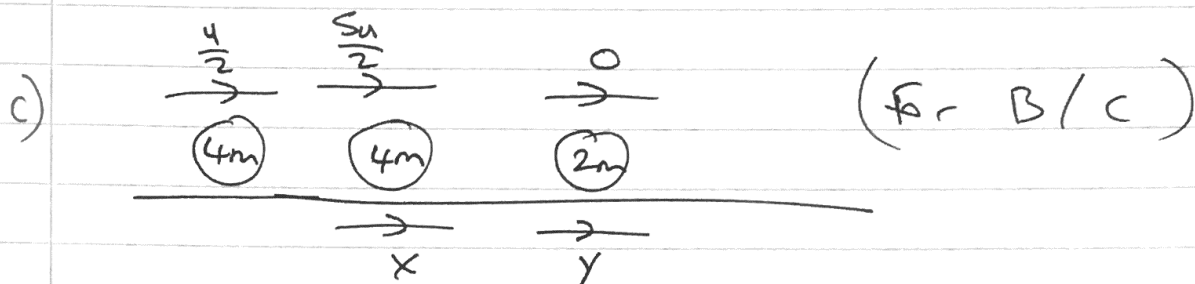
$$\text{hence } V_A = \frac{2u(6-u)}{u+4}$$

$$\text{so speed} =$$

$$\left| \frac{2u(6-u)}{u+4} \right|$$

$$\text{b) } V_A > 0 : 2u(6-u) > 0$$

$$0 < u < 6$$



$$\underline{\text{C.L.M}} : 4m \left( \frac{5u}{2} \right) = 4mx + 2my$$

$$10u = 4x + 2y$$

$$5u = 2x + y \quad \parallel \quad \textcircled{1}$$

$$\underline{\text{N.P.L}} : \frac{2}{3} = \frac{y-x}{\frac{5u}{2}}$$

$$\therefore \frac{5}{3}u = y - x \quad \parallel \quad \textcircled{2}$$

solve  $\textcircled{1}$  and  $\textcircled{2}$  for  $x$ .

$$\underline{\textcircled{1}} - \textcircled{2} : \left( 5u - \frac{5u}{3} \right) = 3x$$

$$x = \frac{10u}{9} //$$

$\frac{10u}{9} > \frac{u}{2}$  here A won't collide with B.