Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel International Advanced Level

Centre Number	Candidate Number

Thursday 20 June 2019

Morning (Time: 1 hour 30 minutes)	Paper Reference **WME02/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Mechanics M2

You must have: Mathematical Formulae and Statistical Tables (Blue), calculator	Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Whenever a numerical value of g is required, take $g = 9.8\,\text{m s}^{-2}$, and give your answer to either 2 significant figures or 3 significant figures.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for each question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over

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Answer ALL questions. Write your answers in the spaces provided.

1. A truck of mass 800 kg is moving on a straight road that is inclined at an angle α to the horizontal, where $\sin\alpha = \frac{1}{10}$. When the truck is moving up the road at a constant speed of $12\,\mathrm{m\,s^{-1}}$, the engine of the truck is working at a constant rate of 15 kW. The resistance to the motion of the truck from non-gravitational forces is modelled as a constant force of magnitude R newtons.

(a) Find the value of R.

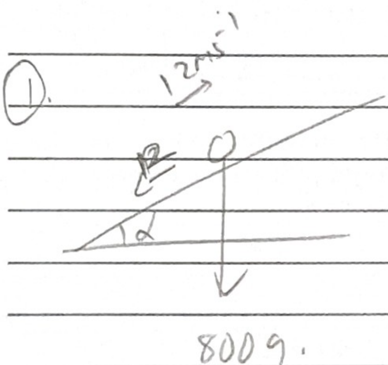
(4)

The truck now moves down the same road. The resistance to the motion of the truck is now modelled as a constant force of magnitude 500 N. The engine of the truck is again working at a constant rate of 15 kW.

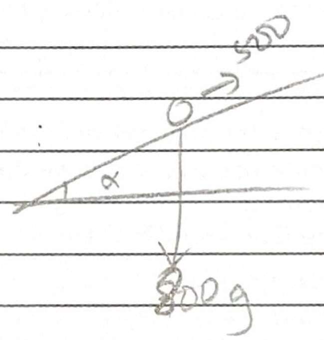
(b) Find the acceleration of the truck at the instant when it is moving at $12\,\mathrm{m\,s^{-1}}$.

(3)

1.



$\sin\alpha = \frac{1}{10}.$



$P = Fv$

$F = R + 800g\sin\theta$

$15{,}000 = D \times 12$

$D = 1250.$

$1250 - R - \frac{800g}{10} = 0$

$R = 466\,N$

$D \times 12 = 15{,}000$

$D = 1250.$

$1250 + \frac{800g}{10} - 500 = 800 \times a$

$a = 1.9175$

$= 1.92\,ms^{-2}$

P 5 5 8 7 6 A 0 2 2 8

Leave blank

2. A particle P moves along the x-axis. At time t seconds, the acceleration of P is a m s^{-2} in the positive x direction, where

$$a = 8 - 6t \quad t \geqslant 0$$

When t = 0, P is at the origin O and is moving with speed 3 m s^{-1} in the positive x direction.

Find

(i) the distance of P from O at the instant when P is instantaneously at rest,

(ii) the total distance travelled by P in the interval $0 \leqslant t \leqslant 4$

(10)

2. $a = 8 - 6t$

$\int a\,dt \rightarrow v$

$8t - 3t^2 + c$

When $t = 0$
$v = 3$

$c = 3$

$8t - 3t^2 + 3 \Rightarrow v$

$\frac{-8 \pm \sqrt{8^2 - 4(-3)(3)}}{2 \times -3}$

$t = 3$ $\quad t = -\frac{1}{3}$

↑
time cannot be -ve ∴ N/A

$\int v\,dt \rightarrow s$

$\int 8t - 3t^2 + 3\,dt$

$= 4t^2 - t^3 + 3t$

$\left[4t^2 - t^3 + 3t\right]_0^3$

$= 18m$

ii) $\left[4t^2 - t^3 + 3t\right]_3^4$

$12 - 18 = -6$

$18 + 6 = 24m$

P 5 5 8 7 6 A 0 6 2 8

Leave blank

3. A particle P of mass 0.4 kg is moving with velocity $u\mathbf{i}$ m s^{-1}, where u is a positive constant. The particle receives an impulse $(3\mathbf{i} + 6\mathbf{j})$ N s.

Immediately after receiving the impulse, the speed of P is $2u$ m s^{-1}.

Find the value of u.

(5)

3

$v = \begin{pmatrix} x \\ y \end{pmatrix}$

$0.4\left(v - \begin{pmatrix} u \\ 0 \end{pmatrix}\right) = \begin{pmatrix} 3 \\ 6 \end{pmatrix}$

$v - \begin{pmatrix} u \\ 0 \end{pmatrix} = \begin{pmatrix} 7.5 \\ 15 \end{pmatrix}$

$v = \begin{pmatrix} 7.5 + u \\ 15 \end{pmatrix}$

$\sqrt{(7.5+u)^2 + 15^2} = 2u$

$56.25 + 15u + u^2 + 225 = 4u^2$

$-3u^2 + 15u + 281.25 = 0$

$\dfrac{-15 \pm \sqrt{15^2 - 4(-3)(281.25)}}{2 \times -3}$

$u = 12.5$ or -7.5

since $u > 0$

$\underline{u = 12.5}$

P 5 5 8 7 6 A 0 8 2 8

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4.

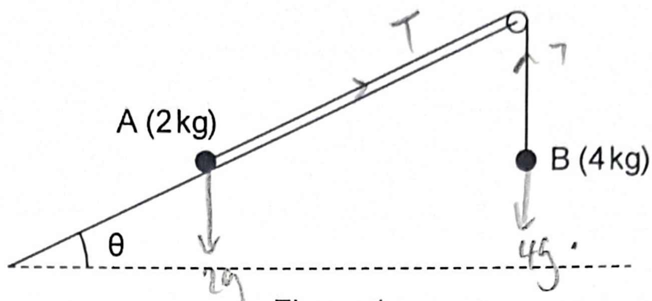

Figure 1

A particle A of mass 2kg is attached to one end of a light inextensible string. A particle B of mass 4kg is attached to the other end of the string. The string passes over a small smooth pulley. The pulley is fixed at the top of a fixed rough plane, which is inclined to the horizontal at an angle θ, where $\tan\theta = \frac{3}{4}$. Initially the particles are held at rest with A on the plane, B hanging freely below the pulley and the string taut, as shown in Figure 1. The part of the string from A to the pulley lies along a line of greatest slope of the plane. The coefficient of friction between A and the plane is $\frac{1}{5}$.

At time t = 0, the particles are released from rest, with A more than 1.5m from the pulley and B more than 1.5m above the ground.

At time t = T seconds, the speed of B is v m s^{-1} and B is 1.5m below its initial position.

(a) Find the total potential energy lost by the system in the interval $0 \leqslant t \leqslant T$.
(3)

(b) Find the work done against friction in the interval $0 \leqslant t \leqslant T$.
(3)

(c) Use the work-energy principle to find the value of v.
(3)

4.

$\tan\theta = \frac{3}{4}$	(a) PE A (gain)
$\sin\theta = \frac{3}{5}$	$2g \times 1.5 \sin\theta$
	$2g \times 1.5 \times 3/5$
$\cos\theta = \frac{4}{5}$	$= 17.64$
$\mu = \frac{1}{5}$	

Leave blank

Question 4 continued

PE B (loss)

$4g \times 1.5 = 58.8.$

$\therefore$ loss in PE

$= 58.8 - 17.64$

$= \underline{41.16}$

(b) Friction

$\mu R = \frac{1}{5} \times 2g \times 0.8$

$= 3.136.$

$3.136 \times 1.5 = 4.7043 = 4.7$ (2sf)

(c) $41.16 - \frac{1}{2} \times 6 \times (v^2 - 0^2) = 4.704$

$36.456 = 3v^2$

$v = \underline{3.49 ms^{-1}}$

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5.

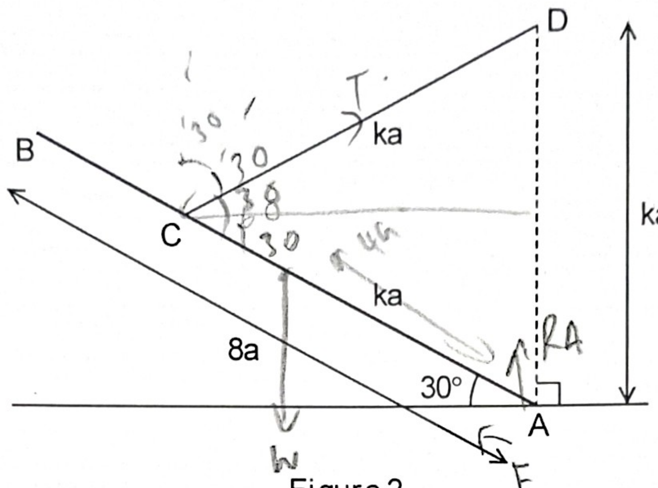

Figure 2

A uniform rod AB, of weight W and length 8a, rests with one end A on rough horizontal ground. The rod is held in limiting equilibrium at 30° to the horizontal by a light inextensible string of length ka, where k is a constant. One end of the string is attached to the rod at C, where AC = ka. The other end of the string is attached to the fixed point D which is vertically above A such that AD = ka, as shown in Figure 2. The string lies in the vertical plane which contains the rod.

The coefficient of friction between the rod and the ground is $\frac{\sqrt{3}}{2}$.

(a) Show that the tension in the string is $\frac{4W}{k}$.

(2)

(b) Find the value of k.

(6)

The magnitude of the force exerted on the rod by the ground at A is λW.

(c) Find the value of λ.

(3)

(5) M(A).

$W(4\cancel{a}) \times \cancel{\cos 30} = T\cancel{\cos 30} \times k\cancel{a}$

$T = \frac{4W}{K}$

as required

(b) Balancing Forces.

(↕)

$T\cos 60 + R_A = W$

(↔)

$T\cos 30 = \mu R_A$.

P 5 5 8 7 6 A 0 1 4 2 8

Leave blank

Question 5 continued

$R_A = W - \frac{2W}{K}$

$\frac{2\sqrt{3}W}{K} = \frac{\sqrt{3}}{2} \times \left(\frac{WK - 2W}{K}\right)$

$\frac{2W}{K} = \frac{1}{2}\left(\frac{WK-2W}{K}\right)$

$\frac{4W}{K} = \frac{WK - 2W}{K}$

$4W = WK - 2W$

$6W = WK$

$K = 6$

(c) $R_A = \frac{2}{3}W$

$F = \frac{\sqrt{3}W}{3}$

$\sqrt{\left(\frac{2}{3}W\right)^2 + \left(\frac{\sqrt{3}}{3}W\right)^2}$

$\frac{4}{9}W^2 + \frac{1}{3}W^2$

$\sqrt{\frac{7}{9}W^2}$

$\frac{\sqrt{7}}{3}W \rightarrow$ Force

$T = \frac{\sqrt{7}}{3}$

P 5 5 8 7 6 A 0 1 5 2 8

Leave blank

6. A particle P of mass m is moving in a straight line with speed v on a smooth horizontal surface. The particle P collides directly with a particle Q of mass km which is moving with speed w, $(w < v)$, along the same straight line and in the same direction as P. The direction of motion of P is unchanged by the collision and, immediately after the collision, the speed of P is w and the speed of Q is 2w.

The coefficient of restitution between P and Q is $\frac{2}{3}$.

(a) Find the value of k.

(6)

When P and Q collide they are at the point A, which is a distance d from a smooth fixed vertical wall. The wall is perpendicular to the direction of motion of the particles. After the collision with P, particle Q hits the wall and rebounds towards P.

The coefficient of restitution between Q and the wall is $\frac{1}{3}$.

There is a second direct collision between P and Q at the point B.

(b) Find, in terms of d and w, the time taken for P to travel from A to B.

(5)

P Q

m km

$\rightarrow$ V $\rightarrow$ w.

$\rightarrow$ w $\rightarrow$ 2w.

$e = 2/3$.

$\frac{2w - w}{v - w} = 2/3$.

$w = \frac{2}{3}(v - w)$.

$\frac{5}{3}w = \frac{2}{3}v$.

$5w = 2v$ — (1)

$v = \frac{5}{2}w$

LCM.

$m(v) + km(w) = m(w) + km(2w)$

$v + kw = w + 2kw$.

$kw = v - w$.

$kw = \frac{5}{2}w - w$

$k = \frac{3}{2}$

P 5 5 8 7 6 A 0 1 8 2 8

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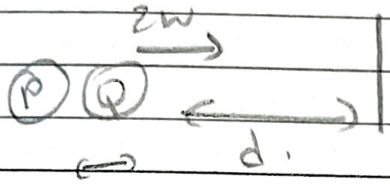
Question 6 continued



Time taken for Q to reach the wall.

$\frac{d}{2w}$

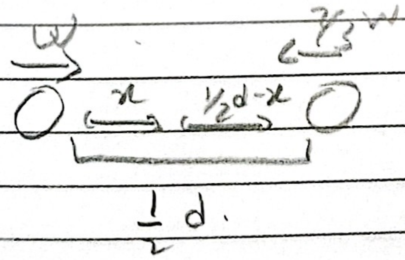
Speed of rebound.

$\frac{r}{2w} = \frac{1}{3}$

$r = \frac{2}{3}w$

Distance moved by P ~~as~~ in $\frac{d}{2w}$ time.

$w \times \frac{d}{2w} = \frac{d}{2}$



time

$\frac{x}{w} = \frac{\frac{1}{2}d - x}{\frac{2}{3}w}$

$\frac{2}{3}wx = \frac{1}{2}dw - xw$

$\frac{5}{3}\cancel{w}x = \frac{1}{2}d\cancel{w}$

$x = \frac{3}{10}d$

$\frac{\frac{3}{10}d}{w} = \frac{3d}{10w}$

$\frac{d}{2w} + \frac{3d}{10w} =$

$= \frac{4d}{5w}$

Leave blank

7.

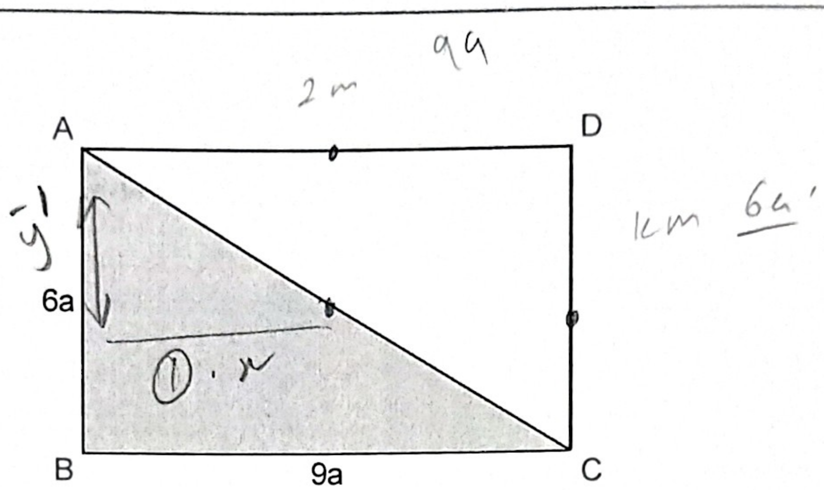


Figure 3

The model of the design for a pendant consists of a uniform triangular lamina, ABC, and two uniform rods AD and DC, as shown in Figure 3. The rods are in the same plane as the lamina. The lamina ABC has mass 8M, sides AB = 6a and BC = 9a, and angle ABC = 90°. The rod AD has mass 2M and length 9a. The rod DC has mass kM and length 6a.

(a) Show that the centre of mass of the model is $\left(\dfrac{33 + 9k}{10 + k}\right)a$ from AB. **(4)**

The model is suspended from A and hangs freely in equilibrium with AC vertical.

(b) Find the value of k. **(7)**

7 ①

Area of Δ

$\frac{1}{2} \times 9a \times 6a$

$27a^2$

com

(0, 0) (9a, 0) (0, 6a)

$\frac{9a}{3} = 3a$

$\frac{6a}{3} = 2a$

(3a, 2a)

ROD AD

(4.5a, 6a)

length 9a

ROD DC

(9a, 3a) →

length (6a)

P 5 5 8 7 6 A 0 2 2 2 8

Question 7 continued

$8\cancel{M}\begin{pmatrix}3a\\2a\end{pmatrix} + 2\cancel{M}\begin{pmatrix}4.5a\\6a\end{pmatrix}$

$+ k\cancel{M}\begin{pmatrix}9a\\3a\end{pmatrix} = (10+k)\cancel{M}\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$

$\begin{pmatrix}24a + 9a + 9ka\\16a + 12a + 3ka\end{pmatrix} = (10+k)\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$

$\begin{pmatrix}(33+9k)a\\(28+3k)a\end{pmatrix} = (10+k)\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$

$\bar{x} = \left(\frac{33+9k}{10+k}\right)a$

$\bar{y} = \left(\frac{28+3k}{10+k}\right)a$

$\bar{x} \rightarrow$ distance from AB $\therefore$ as required

(b) $\tan\theta = \frac{9a}{6a}$

$\tan\theta = 3/2$

$\tan\theta = \dfrac{\left(\frac{33+9k}{10+k}\right)\cancel{a}}{\left(6 - \frac{(28+3k)}{10+k}\right)\cancel{a}}$

$60 + 6k - 28 - 3k$

$\frac{32+3k}{10+k}$

$\frac{33+9k}{\cancel{10+k}} \times \frac{\cancel{10+k}}{32+3k}$

$\frac{33+9k}{32+3k} = \frac{3}{2}$

$66 + 18k = 96 + 9k$

$9k = 30$

$k = \frac{10}{3}$

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8.

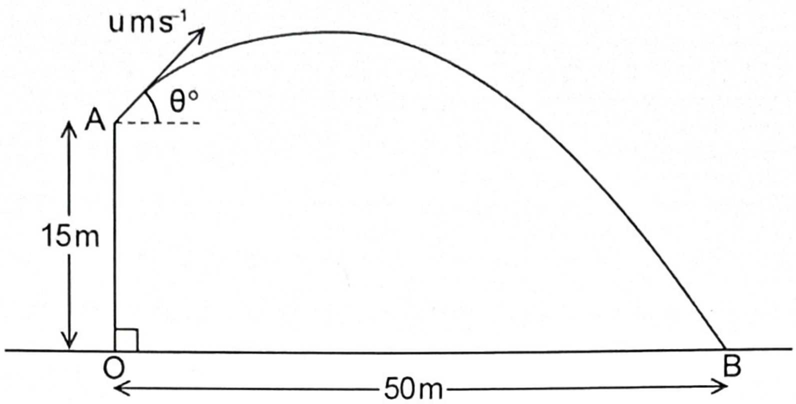


Figure 4

A small ball is thrown from the point A with speed $u\,\text{m s}^{-1}$ at an angle $\theta°$ above the horizontal. The point A is vertically above the point O, which is on horizontal ground, such that AO is 15 m.

The ball takes 3 seconds to travel from A to B, where B is on the ground and OB = 50 m, as shown in Figure 4. By modelling the motion of the ball as that of a particle moving freely under gravity,

find

(a) (i) the value of θ,

(ii) the value of u,

(6)

(b) the speed of the ball as it hits the ground at B,

(3)

(c) the direction of motion of the ball as it hits the ground at B.

(2)

8 $u\cos\theta \times t = 50$ - (1) | $t = \frac{50}{u\cos\theta}$

$s = -15$ ↑+ve

$u = u\sin\theta$ | $t = 3$

v

$a = -9.8$ | $3 = \frac{50}{u\cos\theta}$

$t = t$

$-15 = u\sin\theta t - 4.9t^2$ | $u\cos\theta = \frac{50}{3}$

P 5 5 8 7 6 A 0 2 6 2 8

Leave blank

Question 8 continued

$-15 = u\sin\theta(3) - 4.9(3)^2$

$u\sin\theta = 9.7$

$u\cos\theta = 50/3$.

~~$\tan\theta = \frac{291}{500}$~~

~~$\theta = \underline{30.2^\circ}$~~

$u = 19.28387352$

$u = 19.3$ (3sf)

(b) $s = -15$

$u = u\sin\theta$

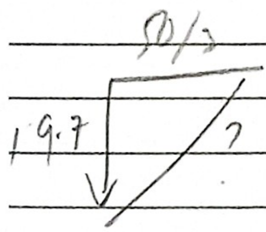
$v = ?$

$a = -9.8$

$t = 3$.

$\frac{v - 9.7}{3} = -9.8$

$v = -19.7$.



$\sqrt{19.7^2 + (50/3)^2}$

$= \underline{25.8 \text{ ms}^{-1}}$

c) $\tan^{-1}\left(\frac{19.7}{50/3}\right)$

$\theta = 49.8^\circ$ below Horizontal