Write your name here

Surname	Other names

Pearson Edexcel
International
Advanced Level

Centre Number | Candidate Number

Mechanics M2

Advanced/Advanced Subsidiary

Friday 22 June 2018 – Morning
Time: 1 hour 30 minutes

Paper Reference
WME02/01

You must have:
Mathematical Formulae and Statistical Tables (Blue)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}, and give your answer to either two significant figures or three significant figures.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

P 5 1 4 1 6 A 0 1 2 8

Pearson

June 2018
M2

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1. A particle P of mass 0.7 kg is moving with velocity $(\mathbf{i} - 2\mathbf{j})\,\mathrm{m\,s^{-1}}$ when it receives an impulse. Immediately after receiving the impulse, P is moving with velocity $(3\mathbf{i} + 4\mathbf{j})\,\mathrm{m\,s^{-1}}$.

(a) Find the impulse.

(3)

(b) Find, in degrees, the size of the angle between the direction of the impulse and the direction of motion of P immediately before receiving the impulse.

(3)

(a) $0.7\left(\begin{pmatrix}3\\4\end{pmatrix} - \begin{pmatrix}1\\-2\end{pmatrix}\right)$

$= 0.7\begin{pmatrix}2\\6\end{pmatrix}$

$= \begin{pmatrix}1.4\\4.2\end{pmatrix} \rightarrow$ Impulse

$I = 1.4\mathbf{i} + 4.2\mathbf{j}$ Ns

(b) $\tan^{-1}\left(\frac{2}{1}\right) = 63.43$

$\tan^{-1}\left(\frac{4.2}{1.4}\right)$

$= 71.6$.

$63.43 + 71.6 = 135^\circ$.

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2.

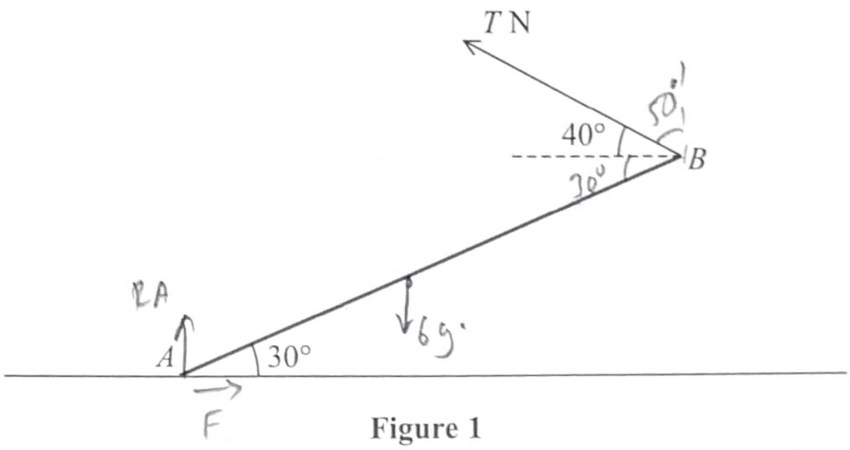

Figure 1

A uniform rod AB, of mass 6 kg and length 1.6 m, rests with its end A on rough horizontal ground. The rod is held in equilibrium at 30° to the horizontal by a light string attached to the rod at B. The string is at 40° to the horizontal and lies in the same vertical plane as the rod, as shown in Figure 1. The tension in the string is T newtons. The coefficient of friction between the ground and the rod is μ.

(a) Show that, to 3 significant figures, $T = 27.1$

(4)

(b) Find the set of values of μ for which equilibrium is possible.

(5)

(a) M(A)

$6g\cos 30 \times 0.8 = T\cos 20 \times 1.6$

$T = 27.1$ N (3 s.f.)

(b) $F \le \mu R$

Balancing Forces

$F = T\cos 40$

$R_A + T\cos 50 = 6g$

$R_A = 41.3835$

$F = T\cos 40$

$27.1 \cdot \cos 40 \le \mu \times 41.3835$

$\mu \geq 0.50$

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3. A cyclist and his bicycle, with a combined mass of 75 kg, move along a straight horizontal road. The cyclist is working at a constant rate of 180 W. There is a constant resistance to the motion of the cyclist and his bicycle of magnitude R newtons. At the instant when the speed of the cyclist is 4 m s^{-1}, his acceleration is 0.2 m s^{-2}.

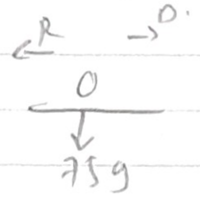
(a) Find the value of R.

(4)

Later, the cyclist moves up a straight road with a constant speed v m s^{-1}. The road is inclined at an angle θ to the horizontal, where $\sin\theta = \dfrac{1}{21}$. The cyclist is working at a rate of 180 W and the resistance to the motion of the cyclist and his bicycle from non-gravitational forces is again the same constant force of magnitude R newtons.

(b) Find the value of v.

(4)



$P = 180W$

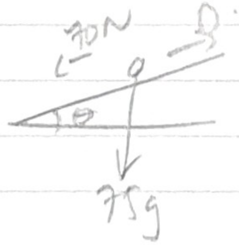
$180 = D \times 4$

$D = 45N$

$45 - R = 75 \times 0.2$

$R = 30N$

(b)



$\sin\theta = \frac{1}{21}$

$180 = D \times v$

$D - 30 - \frac{75g}{21} = 0$

$D = 65N$

$\frac{180}{65} = 2.77 ms^{-1}$

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4.

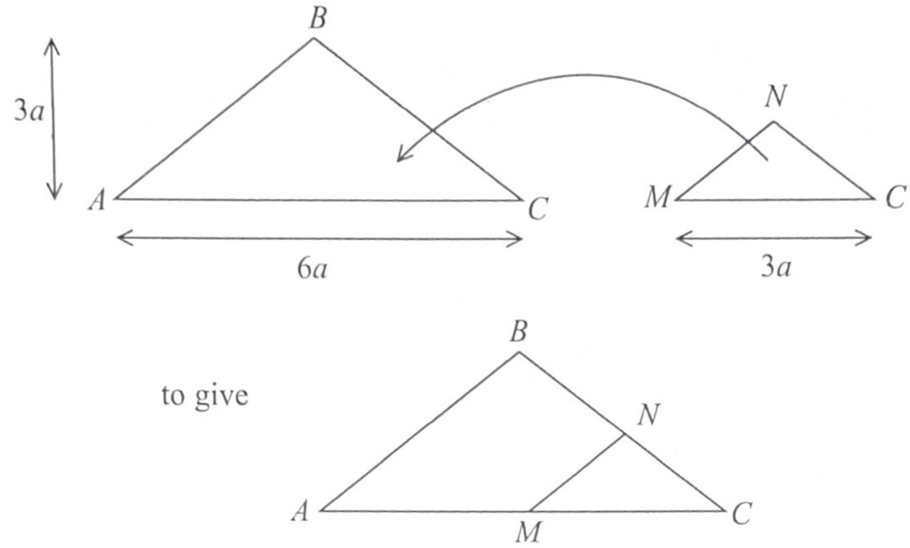


Figure 2

The uniform lamina ABC is an isosceles triangle with $AB = BC$, $AC = 6a$ and the distance from B to AC is $3a$.

The uniform lamina MNC is an isosceles triangle with $MN = NC$ and $MC = 3a$. Triangles ABC and MNC are similar and are made of the same material.

The lamina L is formed by fixing triangle MNC on top of triangle ABC, as shown in Figure 2.

(a) Show that the distance of the centre of mass of L from AC is $\frac{9}{10}a$ **(5)**

The lamina L is freely suspended from B and hangs in equilibrium.

(b) Find, to the nearest degree, the size of the angle between AB and the downward vertical.

(7)

(a) ABC Area.

$\frac{1}{2} \times 6a \times 3a = 9a^2$

com

$\frac{1}{3} \times 3a = a$.

$\begin{pmatrix} 3a \\ a \end{pmatrix}$

Area of MNC

$\frac{1}{2} \times 3a \times 1.5a = \frac{9}{4}a^2$.

com

$\frac{1}{3} \times 1.5a = \frac{1}{2}a$.

$= \begin{pmatrix} 1.5a \\ 0.5a \end{pmatrix}$

Leave blank

Question 4 continued

$$9a^2\begin{pmatrix}3a\\a\end{pmatrix} + \frac{9a^2}{4}\begin{pmatrix}1.5a\\0.5a\end{pmatrix}$$

$$= \frac{45}{4}a^2\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

$$\begin{pmatrix}27a + \frac{27}{8}a\\ 9a + \frac{9}{8}a\end{pmatrix} = \frac{45}{4}\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

$\bar{x} = \frac{27}{10}a$

$\bar{y} = \frac{9}{10}a$

$\therefore$ distance from $AC = \frac{9}{10}a$

as req.

(b) $\tan^{-1}\left(\frac{0.3}{2.1}\right)$

$= 8.13$

$45 + 8.13 = 53.13$

$= 53^\circ$ nearest degree.

Leave blank

5. A particle P of mass 0.3 kg moves under the action of a single force **F** newtons. At time t seconds ($t \geqslant 0$), P has velocity $\mathbf{v}$ m s^{-1}, where

$$\mathbf{v} = (3t^2 - 4t)\mathbf{i} + (3t^2 - 8t + 4)\mathbf{j}$$

(a) Find **F** when $t = 4$

(3)

At the instants when P is at the points A and B, particle P is moving parallel to the vector **i**.

(b) Find the distance AB.

(9)

$v = (3t^2 - 4t)i$

$+ (3t^2 - 8t + 4)j$

$\frac{dv}{dt} = a$

$(6t - 4)i + (6t - 8)j$

when $t = 4$

$= 20i + 16j$

$0.3 \begin{pmatrix} 20 \\ 16 \end{pmatrix} = \begin{pmatrix} 6 \\ 4.8 \end{pmatrix} \Rightarrow F$

$F = 6i + 4.8j$

(b) $3t^2 - 8t + 4 = 0$

$t = 2/3 \quad t = 2$

$\int v\,dt \to r$

$\left[(t^3 - 2t^2)i + (t^3 - 4t^2 + 4t)j\right]_{2/3}^{2}$

$= (0i + 0j) - \left(\frac{16}{27}i + \frac{32}{27}j\right)$

$|AB| \sqrt{\left(\frac{16}{27}\right)^2 + \left(\frac{32}{27}\right)^2}$

$= \frac{16\sqrt{5}}{27}$

$\simeq 1.3\text{ m}$

6. A particle P is projected from a fixed point A with speed 12 m s^{-1} at an angle α above the horizontal and moves freely under gravity. As P passes through the point B on its path, P is moving with speed 8 m s^{-1} at an angle β below the horizontal.

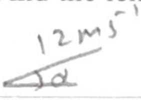
(a) By considering energy, find the vertical distance between A and B.
(4)

Particle P takes 1.5 seconds to travel from A to B.

(b) Find the size of angle α.
(3)

(c) Find the size of angle β.
(3)

(d) Find the length of time for which the speed of P is less than 8 m s^{-1}.
(4)



$\frac{1}{2}\cancel{m}(12^2-8^2)=\cancel{m}gh.$

$40=gh$

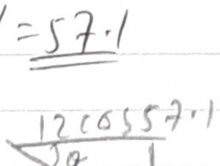
$h=\underline{\underline{4.1m.}}$

(b) $s=200/49$
$u=12\sin\alpha.$
v
$a=-9.8$
$t=1.5.$

$\frac{200}{49}=18\sin\alpha-4.9(1.5)^2$

$\alpha=\underline{\underline{57.1}}$

(c)



$\cos\theta=\frac{12\cos 57.1}{8.}$

$\beta=\underline{\underline{35.4^\circ}}$

(d) $s=200/49$
$u=12\sin 57.1$
v
$a=-9.8$
$t=?$

$\frac{200}{49}=12\sin 57.1.t-4.9t^2.$

$t=1.5$ or $0.55.$

$1.5-0.55=0.945s.$

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7. Three particles A, B and C have masses $2m$, $3m$ and $4m$ respectively. The particles lie at rest in a straight line on a smooth horizontal surface, with B between A and C. Particle A is projected towards B with speed u and collides directly with B. The coefficient of restitution between A and B is e. The kinetic energy of A immediately after the collision is one ninth of the kinetic energy of A immediately before the collision.

Given that the direction of motion of A is unchanged by the collision,

(a) find the value of e.

(7)

After the collision between A and B there is a direct collision between B and C. The coefficient of restitution between B and C is f, where $f < \frac{3}{4}$. The speed of B immediately after the collision with C is V.

(b) (i) Express V in terms of f and u.

(ii) Hence show that there will be a second collision between A and B.

(7)

(a) $\xrightarrow{u}$ A B C

2m 3m. 4m.

$\rightarrow$ v $\rightarrow$ w

v $\rightarrow$ w $\rightarrow$

Law of conservation of momentum.

$2mu = 2mv + 3mw$. - ①.

Newton's Law of Restitution.

$eu = w - v$. - - ②

Eliminating w gives

$5v = u(2-3e)$. - - ③

Kinetic Energy.

~~$v = \frac{u}{3}$~~ $\frac{1}{2}mu^2 = E$.

$\frac{1}{2} \times 2m \times v^2 = \frac{1}{9}E$.

$mv^2 = \frac{mu^2}{9}$.

$9v^2 = u^2$.

$v = \sqrt{\frac{u^2}{9}} = \frac{u}{3}$.

∴ substituting this in ③

$e = \frac{1}{9}$.

Leave blank

Question 7 continued

bi) substituting $e = \frac{1}{9}$

we know $v = \frac{u}{3}$

and $w = \frac{4}{9}u$

LCM

$3m \times w = 3mV + 4mW$.

NLR

$wf = W - V$.

$\therefore V = \frac{4u}{21} - \frac{16uf}{63}$

(ii) Since $f \geq 0$

$V \leq \frac{4u}{21} < \frac{u}{3}$

$\therefore$ A collides with B.