# M2 June 2017 IAL (MA)

Q1) $I = m(v-u) = 4(v-u) = 7\underline{i} - 5\underline{j}$

$$4(v - 2\underline{i} - 3\underline{j}) = 7\underline{i} - 5\underline{j}$$

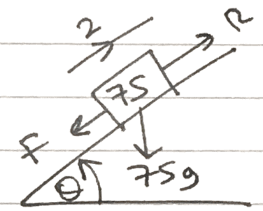
$$v - 2\underline{i} - 3\underline{j} = \left(\tfrac{7}{4}\right)\underline{i} - \left(\tfrac{5}{4}\right)\underline{j}$$

$$v = \left(\tfrac{7}{4} + 2\right)\underline{i} + \left(3 - \tfrac{5}{4}\right)\underline{j}$$

$$v = \frac{15}{4}\underline{i} + \frac{7}{4}\underline{j} \quad \therefore \text{speed} = \sqrt{\left(\tfrac{15}{4}\right)^2 + \left(\tfrac{7}{4}\right)^2}$$

$$= \boxed{4.14\,ms^{-1}}$$

Q2a)




$P = Fv$

$280 = F(2)$

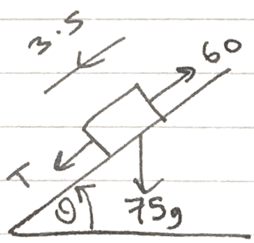
$F = \frac{280}{2} = 140N$ //

N2L (Parallel to slope): $F - R - 75g\sin\theta = 75(0)$

$$R = F - 75g\sin\theta = 140 - \frac{75g}{21}$$

$$= \boxed{105N}$$

b)




$P = Tv$

$280 = T(3.5)$

$T = \frac{280}{3.5} = 80N$ //

N2L (Parallel to slope): $T + 75g\sin\theta - 60 = 75a$

$$\therefore a = \frac{80 + \frac{75g}{21} - 60}{75} = \boxed{-0.73\,ms^{-2}} \quad (|a| = 0.73)$$

Q3ai) $a = 4t - 8$

(constant)

$v = \int (a)\,dt = \int (4t-8)\,dt = 2t^2 - 8t + c = v$

$\underline{t=0,\ v=6}$ : $6 = 2(0) - 8(0) + c \quad \therefore 6 = c$

hence $v = 2t^2 - 8t + 6$

$\underline{t=1}$ : $v = 2 - 8 + 6 = 0$ hence P is at instantaneous rest at $t=1$.

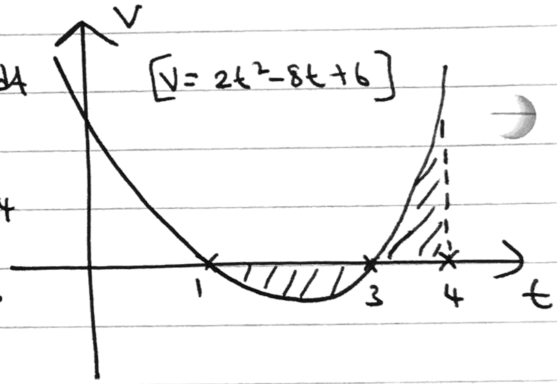
ii) $\underline{v=0}$ : $2t^2 - 8t + 6 = 0$

$(2t-6)(t-1) = 0$

$2t - 6 = 0 \quad \therefore \boxed{t = 3}$

b) Total distance $= \left| \int_1^3 (v)\,dt \right| + \int_3^4 (v)\,dt$

$= \int_1^3 [2t^2 - 8t + 6]\,dt + \int_3^4 [2t^2 - 8t + 6]\,dt$

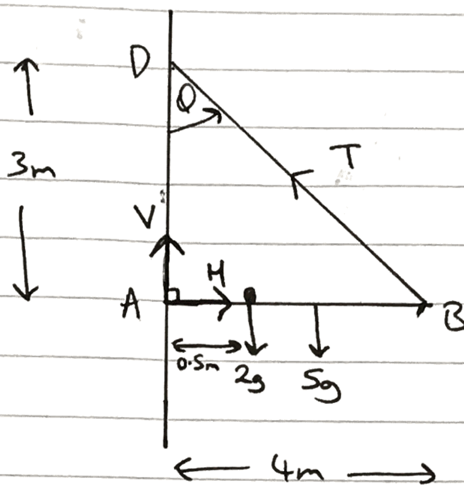



$= \left[\frac{2t^3}{3} - 4t^2 + 6t\right]_1^3 + \left[\frac{2t^3}{3} - 4t^2 + 6t\right]_3^4$

$= \left| [18 - 4(9) + 18] - \left[\frac{8}{3}\right] \right| + \left[\frac{128}{3} - 64 + 24\right] - [18 - 4(9) + 18]$

$= \left| -\frac{8}{3} \right| + \frac{8}{3} = \boxed{\frac{16}{3}}$ m.

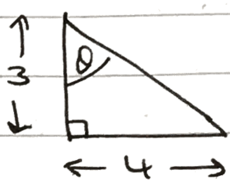
Q4a)




$F_{max} = \mu R$

$\Downarrow$

$V_{max} = \mu H$




$M(A): \quad T\sin\theta(3) = 2g(0.5) + 5g(2)$

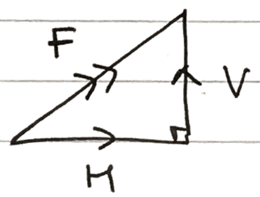
$$T = \frac{11g}{3\sin\theta} = \frac{11g}{3 \times \frac{4}{5}} = \boxed{44.9N}$$

b) $R_f \uparrow: \quad V + T\cos\theta = 7g \sim (1)$

$\overrightarrow{R_f}: \quad H = T\sin\theta \sim (2)$

(1): $V = 7g - T\cos\theta = 7(9.8) - 44.9\left(\frac{3}{5}\right) = \underline{\underline{41.66N}}$

(2): $H = T\sin\theta = 44.9 \times \frac{4}{5} = \underline{\underline{35.9\dot{3}N}}$




$|\text{Force exerted}| = F$

$= \sqrt{H^2 + V^2}$

$= \sqrt{(41.66)^2 + (35.9\dot{3})^2}$

$= \boxed{55.0 \ N}$

c) $V_{max} = \mu H \longrightarrow V \leq \mu H \quad \therefore \mu \geq \frac{V}{H}$

$$\therefore \mu \geq \frac{41.66}{35.9\dot{3}}$$

$$\boxed{\mu \geq 1.16}$$

Q5a)

**At A**

$KE = 0.35u^2$

$GPE = 0$

**At B**

$KE = 0.35(4.2^2)$

$GPE = 0.7g(4)$

**By C.O.E**

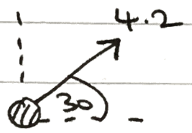
$\Rightarrow 0.35u^2 = 0.35(4.2^2) + 0.7g(4)$

$\Rightarrow 0.35u^2 = 33.614$

$\Rightarrow u = \sqrt{\dfrac{33.614}{0.35}} = \boxed{9.8}$

b) i) We can deduce $w = 9.8$ as P does not lose any energy during the entire motion $\therefore$ speed at A $= w = \boxed{9.8}$ //

ii)

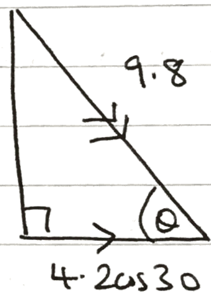



$\vec{u} = \vec{v} = 4.2\cos 30$

at the ground:

$\cos\theta = \dfrac{4.2\cos 30}{9.8}$

$\cos\theta = \dfrac{3\sqrt{3}}{14}$




$\therefore \theta = \cos^{-1}\left(\dfrac{3\sqrt{3}}{14}\right)$

$= \boxed{68.2^\circ}$

c) (FROM B TO GROUND) $\downarrow +$

$s = 4.9$

$u = -4.2\sin 30$

$v = \sqrt{9.8^2 - (4.2\cos 30)^2} = 9.1$

$a = g$

$t = ?$

$v = u + at$

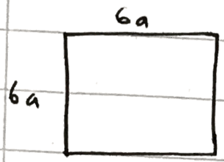
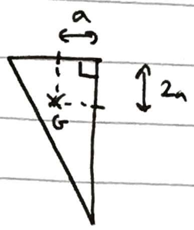
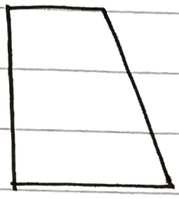
$9.1 = -2.1 + gt$

$t = \dfrac{11.2}{9.8} = \dfrac{8}{7}$ //

horizontal distance $= \vec{u} \times t = 4.2\cos 30 \times \dfrac{8}{7}$

$= \boxed{4.16\text{m}}$

Q6a)

| | Shape | Area | Distance of c.o.m from: AB | AE |
|---|---|---|---|---|
| (+) |   | $\boxed{36a^2}$ | $3a$ | $3a$ |
| (−) |   | $\frac{1}{2}\times 3a \times 6a$ $= \boxed{9a^2}$ | $5a$ | $2a$ |
| = |  | $\boxed{27a^2}$ | $\bar{x}$ | $\bar{y}$ |

$$\underset{\sim\sim\sim}{\sum m_i x_i = \bar{x} \sum m_i}$$

$$\Rightarrow 36\cancel{a^2}\begin{pmatrix}3a\\3a\end{pmatrix} - 9\cancel{a^2}\begin{pmatrix}5a\\2a\end{pmatrix} = 27\cancel{a^2}\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

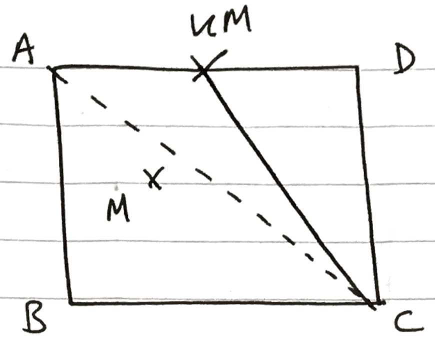
$$\begin{pmatrix}63a\\90a\end{pmatrix} = 27\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

$$\therefore \begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix} = \begin{pmatrix}7a/3\\10a/3\end{pmatrix} //$$

$\therefore$ Distance from AB $= \dfrac{7a}{3}$

b) Distance from AE $= \dfrac{10a}{3}$

c)




$\bar{x} = \overrightarrow{\text{distance}}$ of c.o.m from A
$\bar{y} = \downarrow$ distance of c.o.m from A

The diagonal AC is vertical which means that $\bar{x} = \bar{y}$ as ABCD is a square.

$\underline{\sum m_i x_i = \bar{x} \sum m_i}$

$M\left(\frac{7a}{3}\right) + kM(3a) = M(k+1)\bar{x}$ //

$\therefore \bar{x} = \dfrac{3ak + \frac{7a}{3}}{(k+1)}$ //

and $M\left(\frac{10a}{3}\right) + (kM)(0) = M(k+1)\bar{y}$

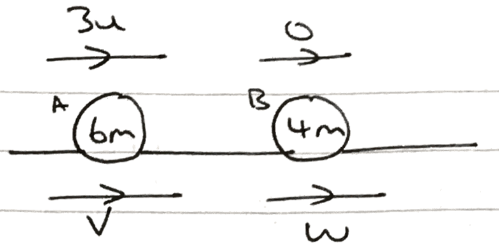
$\therefore \bar{y} = \dfrac{\frac{10a}{3}}{k+1}$ //

$\underline{\bar{x} = \bar{y}}$ : $\dfrac{3ak + \frac{7a}{3}}{k+1} = \dfrac{\frac{10a}{3}}{k+1}$

$\Rightarrow \quad 3ak + \frac{7\cancel{a}}{3} = \frac{10\cancel{a}}{3}$

$\Rightarrow \quad 3k = 1 \quad \therefore \boxed{k = \frac{1}{3}}$

Q7a)




$e = \frac{1}{6}$

$\overrightarrow{C \cdot L \cdot M}$ : $6m(3u) = 4mw + 6mv$

$18u - 6v = 4w$ ~ (1)

$\overrightarrow{N \cdot I \cdot L}$ : $\frac{1}{6} = \frac{w - v}{3u}$ $\therefore \frac{u}{2} = w - v$

$\therefore v = w - \frac{u}{2}$ //

↳ (1) : $18u - 6(w - \frac{u}{2}) = 4w$

$18u - 6w + 3u = 4w$

$10w = 21u$

$\therefore \boxed{w = \frac{21u}{10}}$ □

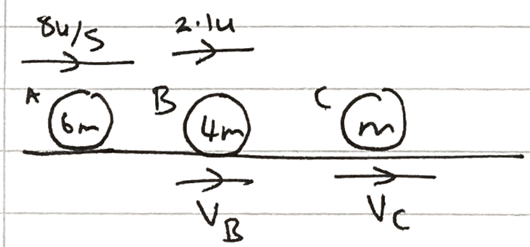
b) $v = w - \frac{u}{2} = \frac{21u}{10} - \frac{u}{2} = \frac{8u}{5}$ //

KE before: $\frac{1}{2}(6m)(3u)^2 = 27mu^2$

KE after: $\frac{1}{2}(6m)(\frac{8u}{5})^2 + \frac{1}{2}(4m)(\frac{21u}{10})^2 = \frac{33}{2}mu^2$

Fraction required $= \frac{27 - \frac{33}{2}}{27} = \boxed{\frac{7}{18}}$

c)




$\overrightarrow{C \cdot L \cdot M}$ : $4m(2.1u) = mV_C + 4mV_B$

$\frac{42}{5}u = V_C + 4V_B$

(cont.) $\overrightarrow{N \cdot T \cdot L}$ : $e = \dfrac{V_C - V_B}{\frac{21u}{10}}$ $\therefore$ $\dfrac{21ue}{10} = V_C - V_B$

$$\left[\frac{21ue}{10} = V_C - V_B\right]$$

$$- \left[\frac{84u}{10} = V_C + 4V_B\right]$$

$$\frac{21ue}{10} - \frac{84u}{10} = -5V_B$$

$$\Rightarrow \quad 5V_B = \frac{u}{10}\left[84 - 21e\right]$$

$$\Rightarrow \quad V_B = \frac{21u}{50}\left[4 - e\right]$$

but $V_B \geqslant \dfrac{8u}{5}$ for no more collisions.

$$\therefore \frac{21\cancel{u}}{50}\left[4 - e\right] \geqslant \frac{8\cancel{u}}{5}$$

$$4 - e \geqslant \frac{80}{21}$$

$$e \leqslant 4 - \frac{80}{21}$$

$$e \leqslant \frac{4}{21} \quad //$$

$$\boxed{0 \leqslant e \leqslant \frac{4}{21}}$$