M2 June 2016 (IAL) (MA)

Q1a) $I = m(v-u)$

$-4\underline{i} + 3\underline{j} = 3(v - 3\underline{i} - 5\underline{j})$

$-4\underline{i} + 3\underline{j} = 3v - 9\underline{i} - 15\underline{j}$

$3v = 5\underline{i} + 18\underline{j}$

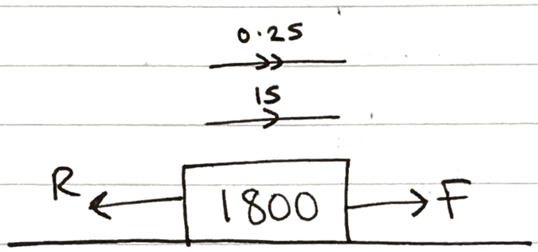
$v = \left(\frac{5}{3}\right)\underline{i} + (6)\underline{j}$

$|v| = \sqrt{\left(\frac{5}{3}\right)^2 + (6)^2} = \boxed{6.23\,ms^{-1}}$

b) KE gained $= \frac{1}{2}(3)\left[6.23\right]^2 - \frac{1}{2}(3)\left[\sqrt{5^2+3^2}\right]^2$

$= \frac{349}{6} - 51 = \boxed{\frac{43}{6}\,J} = 7.17J.$

Q2a)



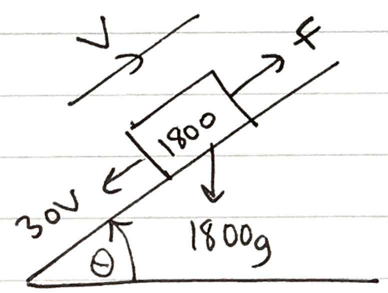
$P = 10000 = Fv$

$F = \frac{10000}{15} = \frac{2000}{3}$

$\overrightarrow{\text{N2L}}^{+}$ (Truck): $\frac{2000}{3} - R = 1800(0.25)$

$R = \frac{2000}{3} - \frac{1800}{4} = \boxed{\frac{650}{3}\,N}$

b)



$P = 12000 = FV.$

$\therefore\ F = \frac{12000}{V}.$

$\underline{\text{N2L (Truck)}}$: $F - 30V - 1800g \sin\theta = 1800(0)$

$$\Rightarrow \frac{12000}{V} - 30V = \frac{1800g}{14}$$

$\times V$

$$\Rightarrow 12000 - 30V^2 = \left(\frac{1800g}{14}\right)V$$

$$\Rightarrow 30V^2 + \left(\frac{1800g}{14}\right)V - 12000 = 0$$

$\underline{\text{By Quadratic formula}}$: $V = 8$ // , $V = -50$

reject. $(V>0)$.

$\therefore \boxed{V = 8}$

(Q3a) GPE lost = GPE lost by Q − GPE gained by P

$$= 3mgd - 2mgd\sin\theta$$

$$= 3mgd - \frac{4mgd}{5}$$

$$= \boxed{\frac{11}{5}mgd}$$

b) GPE lost = W.D by friction + K.E gained

$$\frac{11}{5}mgd = \frac{3}{5}mg \cdot d + \frac{1}{2}(2m)v^2 + \frac{1}{2}(3m)v^2$$

$$\left(\frac{11}{5} - \frac{3}{5}\right)\cancel{m}gd = \frac{5}{2}\cancel{m}v^2$$

$$\left(\frac{8}{5}gd\right) \times \left(\frac{2}{5}\right) = v^2 = \boxed{\frac{16gd}{25}}$$

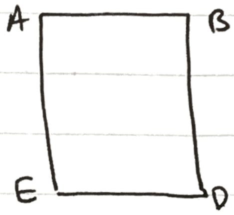
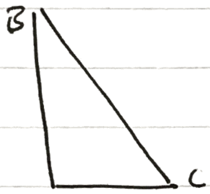
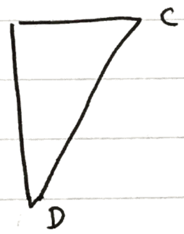
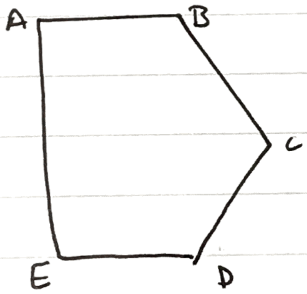
c) $V^2 = \frac{16g}{25} \times \frac{3}{2}$ $\quad \therefore V = \left(\frac{2\sqrt{6}}{5}\right)\sqrt{g} = \frac{14\sqrt{30}}{25}$

$$\left.\begin{aligned} s &= 1.5 \\ u &= 0 \\ V &= \frac{14\sqrt{30}}{25} \\ a &= \\ t &= ? \end{aligned}\right\} \quad s = \frac{(u+v)t}{2}$$

$$3 = \left(\frac{14\sqrt{30}}{25}\right)t$$

$$\therefore t = \frac{3}{\frac{14\sqrt{30}}{25}} = \boxed{0.98}$$

(Q4a)

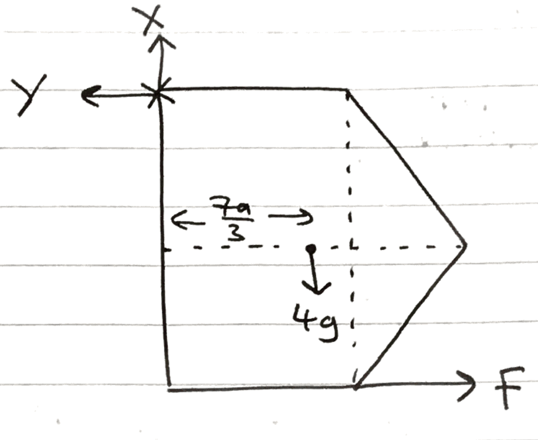
<u>Shape</u>	<u>Area</u>	<u>Distance of c.o.m from AE</u>
	$18a^2$	$\frac{3a}{2}$
	$\frac{1}{2} \cdot 3a \cdot 3a = \frac{9a^2}{2}$	$4a$
	$\frac{9a^2}{2}$	$4a$
	$27a^2$	$\bar{x}$

$$\underline{\sum m_i x_i = \bar{x} \sum m_i}$$

$$18\cancel{a^2}\left(\frac{3a}{2}\right) + \left[\frac{9\cancel{a^2}}{2}(4a)\right] \times 2 = 27\cancel{a^2}\,(\bar{x})$$

$$\Rightarrow \frac{27a + 36a}{27} = \bar{x} = \frac{63a}{27} = \boxed{\frac{7a}{3}}$$

b)



F acts to the right to counteract the turning effect of the weight.

$$R(\updownarrow): X = 4g \qquad |\text{Force exerted}| = \sqrt{X^2 + Y^2}$$

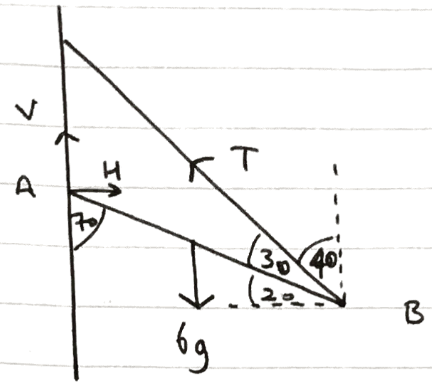
$$R(\leftrightarrow): Y = F \qquad = \sqrt{F^2 + (4g)^2}$$

$$M(A): F(6a) = 4g\left(\frac{7a}{3}\right)$$

$$F = \frac{4g \times \frac{7}{3}}{6} = \frac{686}{45}$$

$$\therefore |\text{Force}| = \sqrt{\left(\frac{686}{45}\right)^2 + 16g^2} = \boxed{42.1\text{N}}$$

Q5a)



$$M(A):\ 6g\sin 70(1) = T\sin 30(2)$$

$$T = \frac{6g\sin 70}{2\sin 30} = \boxed{55.3N}$$

b) Limiting equilibrium $\longrightarrow$ $V = \mu H$.

$$R(\updownarrow):\ T\cos 40 + V = 6g$$

$$V = 6g - 55.3\cos 40 = \boxed{16.5N}$$

$$R(\leftrightarrow):\ H = T\cos 50 = 55.3\cos 50 = \underline{\underline{35.52N}}$$

$$\therefore\ 16.5 = \mu \cdot 35.52$$

$$\Rightarrow\ \mu = \frac{16.5}{35.52} = \boxed{0.46}$$

c)

$\rightsquigarrow$ $\tan\theta = 0.46$ $(=\mu)$

$$\therefore\ \theta = \tan^{-1}(0.46)$$

$$= \boxed{24.9° \text{ to the horizontal}}$$

Q6a) $\uparrow^{+}$ $\left.\begin{array}{l} s = y \\ u = 7\sqrt{3} \\ v = \\ a = -g \\ t = \end{array}\right\}$ $s = ut + \frac{1}{2}at^2$

$y = 7t\sqrt{3} - \frac{g}{2}t^2$

$\xrightarrow{+}$ $s = ut \}$ $x = 7t$ $\therefore$ $t = \frac{x}{7}$ //

<u>substituting into y</u>: $y = 7\sqrt{3}\left(\frac{x}{7}\right) - \frac{g}{2}\left(\frac{x^2}{49}\right)$

$y = \sqrt{3}x - \frac{g}{98}x^2$

$\square$

b) <u>$x = 20$</u>: $y = 20\sqrt{3} - \frac{g}{98}(20)^2 = 20\sqrt{3} - 40$ //

from (a), $y = 7t\sqrt{3} - \frac{g}{2}t^2$

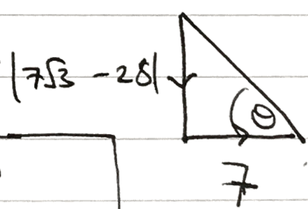
$\Rightarrow (20\sqrt{3} - 40) + \frac{g}{2}t^2 - (7\sqrt{3})t = 0$

<u>By Quadratic formula</u>: $t = \frac{20}{7}$ // , $t = -0.38..$ (reject, $t > 0$)

$+\uparrow$ $\left.\begin{array}{l} s = \\ u = 7\sqrt{3} \\ v = ? \\ a = -g \\ t = \frac{20}{7} \end{array}\right\}$ $v = u + at$

$v = 7\sqrt{3} - \frac{20g}{7} = 7\sqrt{3} - 28$ //

$\tan\theta = \frac{|7\sqrt{3} - 28|}{7}$



$\boxed{\therefore \theta = 66.2° \text{ below horizontal}}$

c) from (a), $y = \sqrt{3}(x) - \frac{9}{98}(x^2)$

$$\lambda = \sqrt{3}(2\lambda) - \frac{9}{98}(4\lambda^2)$$

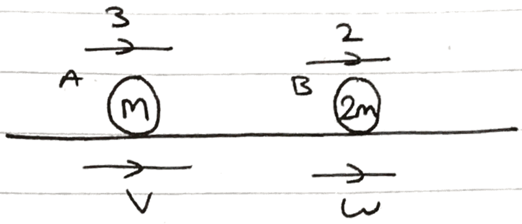
$$\lambda(1-2\sqrt{3}) = -0.4\lambda^2$$

$$\lambda(2\sqrt{3}-1) = 0.4\lambda^2$$

$$\lambda = \frac{2\sqrt{3}-1}{0.4} = 6.16..$$

$(x=7t)$ $\quad \therefore T = \frac{2\lambda}{7} = \boxed{1.76}$

Q7a i)



$e = \frac{2}{3}$

$\underline{\overrightarrow{C.L.M}}$: $3m + 2m(2) = mv + 2mw$

$\Rightarrow 7 = v + 2w \quad \sim (1)$

$\underline{\overrightarrow{N.I.L}}$: $\frac{2}{3} = \frac{w - v}{1}$

$\therefore \frac{2}{3} = w - v \quad \sim (2)$

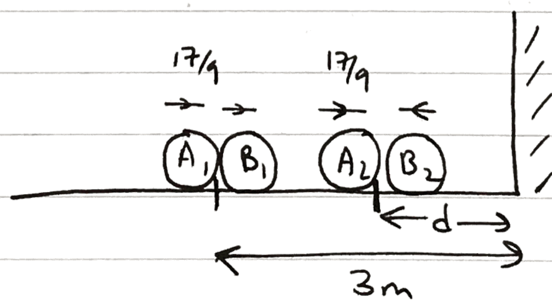
$\underline{(1) + (2)}$: $\frac{2}{3} + 7 = 3w + \cancel{v} - \cancel{v}$

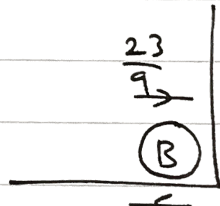
$3w = \frac{23}{3} \quad \therefore w = \frac{23}{9}$

$\square$

ii) $v = 7 - 2w = 7 - 2\left(\frac{23}{9}\right) = \boxed{\frac{17}{9}}$

b)





$\frac{23}{9} \times \frac{1}{2} = \frac{23}{18}$

B rebounds with speed $\frac{23}{18}$ ms^{-1}.

[t_T = total time.
t_1 = time from collision to wall
t_2 = time from wall to collision]

For A : $s = ut$: $(3-d) = \frac{17}{9} t_T$

$$\frac{9(3-d)}{17} = t_T$$

For B : $s = ut_1$: $3 = \frac{23}{9} t_1$

$$\therefore t_1 = \frac{9 \times 3}{23} = \frac{27}{23}$$

$s = ut_2$: $d = \frac{23}{18} t_2$

$$\therefore t_2 = \frac{18d}{23}$$

but $t_1 + t_2 = t_T$

$$\frac{27}{23} + \frac{18d}{23} = \frac{9(3-d)}{17}$$

$$\frac{3+2d}{23} = \frac{3-d}{17}$$

$$3 + 2d = \frac{23}{17}(3) - \frac{23}{17}(d)$$

$$d\left(2 + \frac{23}{17}\right) = \frac{23(3)}{17} - 3$$

$$\boxed{d = \frac{6}{19}}$$