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M2 IAC

1. A particle of mass 0.3 kg is moving with velocity $(5\mathbf{i} + 3\mathbf{j})\ \mathrm{m\,s^{-1}}$ when it receives an impulse $(-3\mathbf{i} + 3\mathbf{j})$ N s. Find the change in the kinetic energy of the particle due to the impulse.

(6)

Initial Mom $= 0.3\begin{pmatrix}5\\3\end{pmatrix} = \begin{pmatrix}1.5\\0.9\end{pmatrix}$ Initial speed $= \sqrt{34}$

$\therefore$ final Mom $= \begin{pmatrix}1.5\\0.9\end{pmatrix} + \begin{pmatrix}-3\\3\end{pmatrix} = \begin{pmatrix}-1.5\\3.9\end{pmatrix} = 0.3v$

$\therefore 3v = \begin{pmatrix}-15\\39\end{pmatrix} \quad \therefore v = \begin{pmatrix}-5\\13\end{pmatrix} \Rightarrow$ speed $= \sqrt{194}$

final KE $= \frac{1}{2}(0.3)(\sqrt{194})^2 = 29.1$

Initial KE $= \frac{1}{2}(0.3)(\sqrt{34})^2 = 5.1$

$\therefore$ Change $= 24$ J

2

2. At time t seconds, $t \geqslant 0$, a particle P has velocity $\mathbf{v}$ m s^{-1}, where

$$\mathbf{v} = (27 - 3t^2)\mathbf{i} + (8 - t^3)\mathbf{j}$$

When $t = 1$, the particle P is at the point with position vector $\mathbf{r}$ m relative to a fixed origin O, where $\mathbf{r} = -5\mathbf{i} + 2\mathbf{j}$

Find

(a) the magnitude of the acceleration of P at the instant when it is moving in the direction of the vector $\mathbf{i}$,

(5)

(b) the position vector of P at the instant when $t = 3$

(5)

a) $v = (27-3t^2)i + (8-t^3)j = \begin{pmatrix} 27-3t^2 \\ 8-t^3 \end{pmatrix}$

$a = \frac{dv}{dt} = \begin{pmatrix} -6t \\ -3t^2 \end{pmatrix}$

moving parallel to i when j component = 0

$8 - t^3 = 0 \Rightarrow t = 2 \therefore a = \begin{pmatrix} -12 \\ -12 \end{pmatrix}$ $|a| = 17.0 \text{ ms}^{-2}$

b) $s = \int v\,dt = \begin{pmatrix} 27t - t^3 + c_1 \\ 8t - \frac{1}{4}t^4 + c_2 \end{pmatrix}$ $t = 1$ $s = \begin{pmatrix} -5 \\ 2 \end{pmatrix}$

$\Rightarrow \begin{pmatrix} 26 + c_1 \\ 7.75 + c_2 \end{pmatrix} = \begin{pmatrix} -5 \\ 2 \end{pmatrix}$ $\therefore c_1 = -31$, $c_2 = -5.75$

$s = \begin{pmatrix} 27t - t^3 - 31 \\ 8t - \frac{1}{4}t^4 - 5.75 \end{pmatrix}$ $t = 3$ $s = \begin{pmatrix} 23 \\ -2 \end{pmatrix}$

2

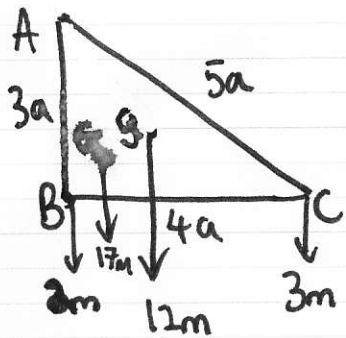
3. A thin uniform wire of mass $12m$ is bent to form a right-angled triangle ABC. The lengths of the sides AB, BC and AC are $3a$, $4a$ and $5a$ respectively. A particle of mass $2m$ is attached to the triangle at B and a particle of mass $3m$ is attached to the triangle at C. The bent wire and the two particles form the system S.

The system S is freely suspended from A and hangs in equilibrium.

Find the size of the angle between AB and the downward vertical.

(10)

$a = \frac{3a}{3}$



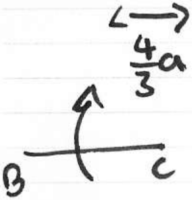
$\cancel{12mg} \times \frac{4}{3}a + 3\cancel{mg} \times 4a = 17\cancel{mg} \times \bar{x}$

$16a + 12a = 17\bar{x}$

$28a = 17\bar{x}$

$\bar{x} = 1.65a = \frac{28}{17}a$

2

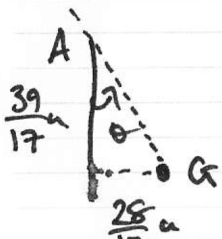


$12\cancel{m} \times a = 17\cancel{m} \times \bar{y}$

$\bar{y} = \frac{12}{17}a$

$= 3a - \frac{12}{17}a$

$= \frac{39}{17}a$ from A



$\theta = \tan^{-1}\left(\frac{\frac{28}{\cancel{17}}}{\frac{39}{\cancel{17}}}\right) = 35.7°$

2

4.

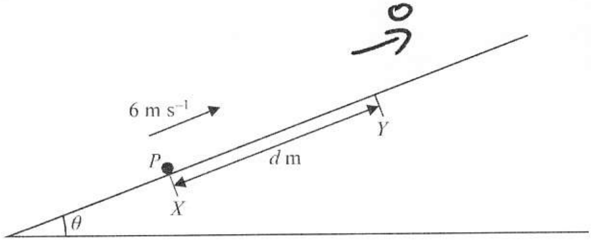
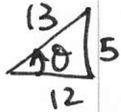


Figure 1



$\sin\theta = \frac{5}{13}$

$\cos\theta = \frac{12}{13}$

A particle P of mass 6.5 kg is projected up a fixed rough plane with initial speed 6 m s^{-1} from a point X on the plane, as shown in Figure 1. The particle moves up the plane along the line of greatest slope through X and comes to instantaneous rest at the point Y, where $XY = d$ metres. The plane is inclined at an angle θ to the horizontal, where $\tan\theta = \frac{5}{12}$.

The coefficient of friction between P and the plane is $\frac{1}{3}$.

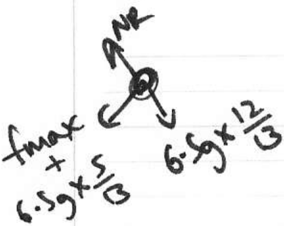
(a) Use the work-energy principle to show that, to 2 significant figures, $d = 2.7$

(7)

After coming to rest at Y, the particle P slides back down the plane.

(b) Find the speed of P as it passes through X.

(4)



$f_{max} = \mu \times 6.5g \times \frac{12}{13} = 2g$

$\therefore$ wd against friction $= 2g \times d$.

KE_X − WD against friction $= PE_Y$

$\frac{1}{2}(6.5)6^2 - 2gd = 6.5g \times d\,\frac{5}{13}$

$117 = \frac{9}{2}gd \quad \therefore d = \frac{234}{9g} = 2.653 \simeq 2.7$

b) PE_Y − wd against friction $= KE_X$

$6.5g \times 2.653 \times \frac{5}{13} - 2g \times 2.653 = \frac{1}{2}(6.5)v^2$

$\therefore v \simeq 2\ \text{ms}^{-1}$

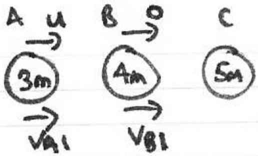
5. Three particles A, B and C lie at rest in a straight line on a smooth horizontal table with B between A and C. The masses of A, B and C are $3m$, $4m$, and $5m$ respectively. Particle A is projected with speed u towards particle B and collides directly with B. The coefficient of restitution between A and B is $\frac{1}{3}$.

(a) Show that the impulse exerted by A on B in this collision has magnitude $\frac{16}{7}mu$ **(7)**

After the collision between A and B there is a direct collision between B and C.

After this collision between B and C, the kinetic energy of C is $\frac{72}{245}mu^2$

(b) Find the coefficient of restitution between B and C. **(6)**



$$e = \frac{\text{sep}}{\text{app}} = \frac{V_{B1} - V_{A1}}{u} = \frac{1}{3}$$

$$u = 3V_{B1} - 3V_{A1} \quad (1)$$

$$CLM \Rightarrow 3mu = 3mV_{A1} + 4mV_{B1} \quad (11)$$

Mom B before $= 0$

Mom B after $= 4mV_{B1}$

$$\Rightarrow 4mV_{B1} = 3m(u - V_{A1})$$

$$\therefore \text{Impulse} = \text{Change in Mom} = 4mV_{B1} = 3m(u - V_{A1}) \quad (111)$$

$$(1) \times 4m \Rightarrow 4mu = 12mV_{B1} - 12mV_{A1}$$

$$(11) \times 3 \Rightarrow 9mu = 12mV_{B1} + 9mV_{A1}$$

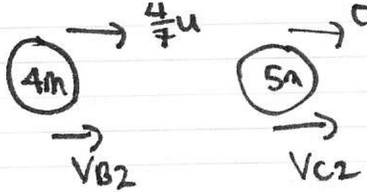
$$5mu = 21\cancel{m}V_{A1} \qquad = V_{A1} = \frac{5u}{21}$$

$$(111) \quad 4\cancel{m}V_{B1} = 3m\left(\frac{21u}{21} - \frac{5u}{21}\right) = \frac{48\cancel{m}u}{21}$$

$$V_{B1} = \frac{12}{21}u \qquad \therefore \text{Impulse} = 4m\left(\frac{12}{21}u\right)$$

$$V_{B1} = \frac{4}{7}u \qquad = \frac{48}{21}mu = \frac{16}{7}mu$$

$$V_{A1} = \frac{5u}{21}$$



$\frac{1}{2}(5\cancel{m})(V_{C2})^2 = \frac{72}{245}\cancel{m}u^2 \Rightarrow (V_{C2})^2 = \frac{144}{1225}u^2 \therefore V_{C2} = \frac{12}{35}u$

CLM $\Rightarrow 4\cancel{m}\left(\frac{4}{7}u\right) = 4\cancel{m}V_{B2} + 5\cancel{m} \times \frac{12}{35}u$

$\Rightarrow 4V_{B2} = \frac{16}{7}u - \frac{12}{7}u = \frac{4}{7}u \therefore V_{B2} = \frac{1}{7}u$

$e = \frac{\text{Sep}}{\text{app}} = \frac{\frac{12}{35}u - \frac{5}{35}u}{\frac{4}{7}u} = \frac{\frac{1}{5}\cancel{u}}{\frac{4}{7}\cancel{u}} = \frac{7}{20}$

6.

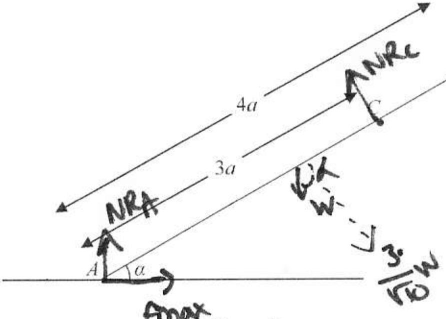
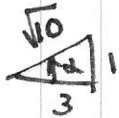


Figure 2



$\cos\alpha = \frac{3}{\sqrt{10}}$

$\sin\alpha = \frac{1}{\sqrt{10}}$

A uniform rod AB has length $4a$ and weight W. A particle of weight kW, $k < 1$, is attached to the rod at B. The rod rests in equilibrium against a fixed smooth horizontal peg. The end A of the rod is on rough horizontal ground, as shown in Figure 2. The rod rests on the peg at C, where $AC = 3a$, and makes an angle α with the ground, where $\tan \alpha = \frac{1}{3}$. The peg is perpendicular to the vertical plane containing AB.

(a) Give a reason why the force acting on the rod at C is perpendicular to the rod.

(1)

(b) Show that the magnitude of the force acting on the rod at C is

$$\frac{\sqrt{10}}{5}W(1 + 2k)$$

(4)

The coefficient of friction between the rod and the ground is $\frac{3}{4}$.

(c) Show that for the rod to remain in equilibrium $k \leqslant \frac{2}{11}$.

(7)

a) The force at C is the 'Normal' reaction, therefore considered perpendicular to the surface (Rod).

b) A↺ $\frac{3}{\sqrt{10}}W \times 2\cancel{a} + \frac{3}{\sqrt{10}}kW \times 4\cancel{a} = NR_C \times 3\cancel{a}$

$\therefore NR = \frac{1}{\sqrt{10}}W \times 2 + \frac{1}{\sqrt{10}}kW \times 4$

$\frac{2}{\sqrt{10}}W(1+2k) = \frac{2\sqrt{10}}{10}W(1+2k) = \frac{\sqrt{10}}{5}W(1+2k)$

c) $\overrightarrow{RF} = 0$ if in equilibrium

friction $= NR_C \times \sin\alpha$

friction $= \frac{1}{\cancel{\sqrt{10}}} \times \frac{\cancel{\sqrt{10}}}{5}W(1+2k) = \frac{W}{5}(1+2k)$

$\therefore$ to remain in equilibrium $f_{max} \geq \frac{1}{5}(1+2k)$

$\frac{3}{4}NR_A \geq \frac{1}{5}(1+2k)$ (1)

$RF\uparrow = 0$ $\quad NR_A + NR_C \times \cos\alpha = W + kW$

$\Rightarrow NR_A + \frac{3}{\cancel{\sqrt{10}}} \times \frac{\cancel{\sqrt{10}}}{5}\cancel{W}(1+2k) = \cancel{W}(1+k)$

$NR_A = (1+k) - \left(\frac{3}{5} + \frac{6}{5}k\right) = \frac{2}{5} - \frac{1}{5}k = \frac{1}{5}(2-k)$

(1) $\Rightarrow \frac{3}{4} \times \frac{1}{\cancel{5}}(2-k) \geq \frac{1}{\cancel{5}}(1+2k)$

$6 - 3k \geq 4 + 8k$

$2 \geq 11k \quad \therefore k \leq \frac{2}{11}$

Q5

7. [*In this question, the unit vectors* **i** *and* **j** *are in a vertical plane,* **i** *being horizontal and* **j** *being vertically upwards.*]

At time $t = 0$, a particle P is projected with velocity $(4\mathbf{i} + 9\mathbf{j})$ m s^{-1} from a fixed point O on horizontal ground. The particle moves freely under gravity. When P is at the point H on its path, P is at its greatest height above the ground.

(a) Find the time taken by P to reach H.

(2)

At the point A on its path, the position vector of P relative to O is $(k\mathbf{i} + k\mathbf{j})$ m, where k is a positive constant.

(b) Find the value of k.

(4)

(c) Find, in terms of k, the position vector of the other point on the path of P which is at the same vertical height above the ground as the point A.

(3)

At time T seconds the particle is at the point B and is moving perpendicular to $(4\mathbf{i} + 9\mathbf{j})$

(d) Find the value of T.

(4)

a) s $\quad v = u + at$
$u = 9$ $\quad 0 = 9 - 9.8t \quad \therefore t = 0.918$ sec
$v = 0$
$a = -9.8$
t

2

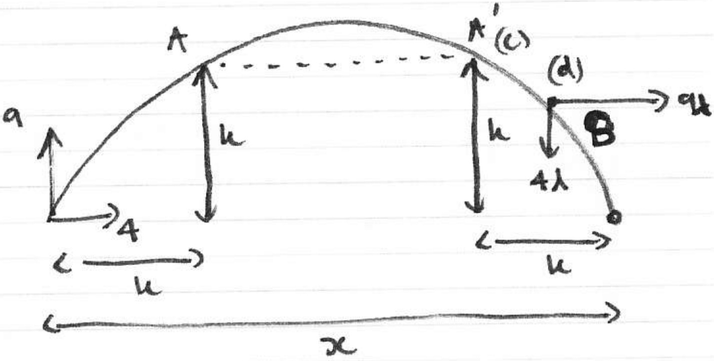
b) $s = k$ $\quad s = ut + \frac{1}{2}at^2$
$\uparrow$ $u = 9$ $\quad k = 9t - 4.9t^2$
v
$a = -9.8$
t

$\overrightarrow{H}$ speed $= 4 \Rightarrow$ dist $= 4t = k$

$\therefore 4t = 9t - 4.9t^2 \Rightarrow 4.9t^{\cancel{2}} = 5\cancel{t}$

$t = \frac{5}{4.9}$

$\therefore k = \frac{20}{4.9} = 4.08\ldots$



c) when P hits ground

V $s = 0$ $\quad s = ut + \frac{1}{2}at^2 \quad \therefore t = \frac{18}{9}$

$u = 9 \quad 0 = 9t - 4.9t^2$

$v = -9$

$a = -9.8 \quad 9\cancel{t} = \frac{9}{2}t^{\cancel{2}}$

t

H $x = \text{speed} \times \text{time} = 4 \times \frac{18}{9} = \frac{72}{9}$

$\therefore A'\begin{pmatrix} \frac{72}{9} - h \\ h \end{pmatrix} = \begin{pmatrix} \frac{4}{5}h \\ h \end{pmatrix} \qquad \frac{40}{9} = h \therefore \frac{72}{9} = \frac{9}{5}h$

d) perp at B $\Rightarrow V_c \begin{pmatrix} 9 \\ -4 \end{pmatrix}\lambda \Rightarrow 9\lambda = 4$ (constant)

$\therefore V_c \begin{pmatrix} 4 \\ -\frac{16}{9} \end{pmatrix}$

V s $\quad v = u + at$

$u = 9 \quad -\frac{16}{9} = 9 - 9.8T \Rightarrow 9.8T = \frac{97}{9}$

$v = -\frac{16}{9}$

$a = -9.8 \quad \therefore T \simeq 1.1$

$t = T$

(Total 16 marks)

Q7

TOTAL FOR PAPER: 75 MARKS