M2 June 2014 IAL (MA)

Q1a) $a = 2t - 3$

$v = \int (a)\,dt = \int (2t-3)\,dt = [t^2 - 3t] + c$ //

$\underline{t=0,\ v=2}$: $2 = 0 + c$

$\therefore c = 2$ //

so: $\boxed{v = t^2 - 3t + 2}$

b) $\underline{v=0}$: $(t-2)(t-1) = 0$

$\therefore \boxed{\begin{matrix} t_1 = 1 \\ t_2 = 2 \end{matrix}}$

c) distance $= \left| \int_1^2 [v]\,dt \right|$

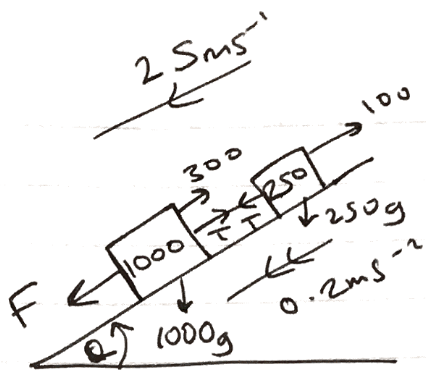
$\int_1^2 (t^2 - 3t + 2)\,dt = \left[\frac{t^3}{3} - \frac{3t^2}{2} + 2t\right]_1^2$

$= \left[\frac{8}{3} - \frac{3}{2}(4) + 4\right] - \left[\frac{1}{3} - \frac{3}{2} + 2\right]$

$= \frac{2}{3} - \frac{5}{6} = -\frac{1}{6}$

$\therefore$ Distance $= \boxed{\frac{1}{6}}$ m.

Q2a)



$\sin\theta = \frac{1}{20}$.

$P = Fv$:

$P = 25F$.

$+\swarrow$ N2L (system) : $F - 300 - 100 + 1250g\sin\theta = 1250(0.2)$

$\Rightarrow F = 1250(0.2) + 400 - \frac{1250g}{20}$

$\Rightarrow F = 37.5N$ //

$\therefore P = 25 \times 37.5 = \boxed{938W}$ (3sf)

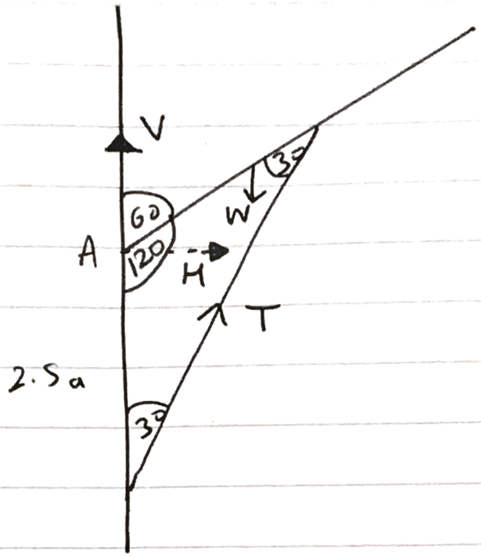
b) $+\swarrow$ N2L (car) : $F + 1000g\sin\theta - 300 - T = 1000(0.2)$

$T = 37.5 + \frac{1000g}{20} - 300 - 1000(0.2)$

$T = \boxed{27.5N}$

(you could also consider the trailer)

Q3a)



$$M(A):\quad W\sin60(2\cancel{a}) = T\sin30(2.5\cancel{a})$$

$$T = \frac{2W\sin60}{\frac{5}{2}\sin30} = \frac{\sqrt{3}\,W}{\frac{5}{4}} = \boxed{\frac{4\sqrt{3}}{5}W}$$

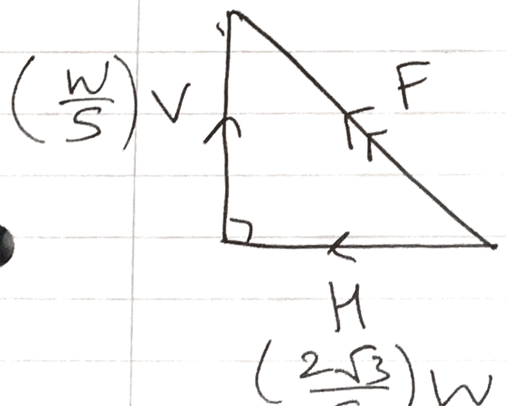
b) $R(\updownarrow):\quad V + T\cos30 = W$

$$V = W - T\cos30 = W - \left(\frac{4\sqrt{3}}{5}\right)\left(\frac{\sqrt{3}}{2}\right)W$$

$$V = W - \frac{6}{5}W = -\frac{1}{5}W \;//$$ so V acts downwards.

$R(\leftrightarrow):\quad H + T\sin30 = 0$

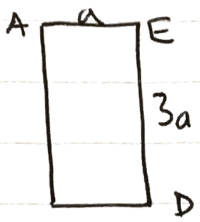
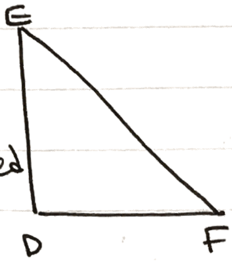
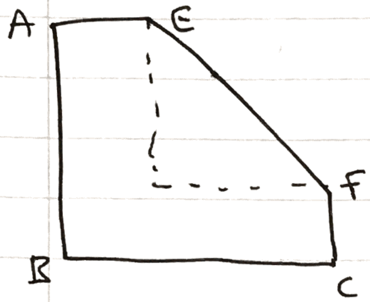
$$H = -T\sin30 = -\frac{2\sqrt{3}}{5}W \;//$$ so H acts to the left.



$$|F| = \sqrt{\left(\frac{W}{5}\right)^2 + \left(\frac{2\sqrt{3}W}{5}\right)^2}$$

$$= \boxed{\frac{W\sqrt{13}}{5}}$$

Q4a)

Shape	Area	Displacement of c.o.m from AB	BC
	$3a^2$	$\frac{a}{2}$	$2a$
	$4a^2$	$2a$	$\frac{a}{2}$
×2 ← Because this bit is folded 	$2\left(\frac{1}{2}\cdot 3a\cdot 3a\right) = 9a^2$	$a+a = 2a$	$a+a = 2a$
	$16a^2$	$\bar{x}$	$\bar{y}$

(Due to symmetry, $\bar{x} = \bar{y}$).

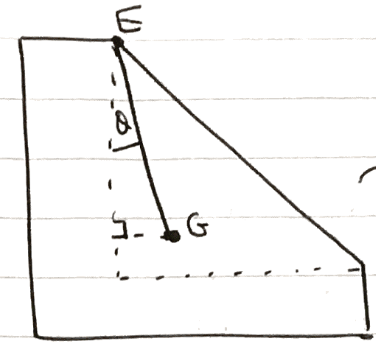
$$\bar{x}\sum m_i = \sum m_i x_i$$

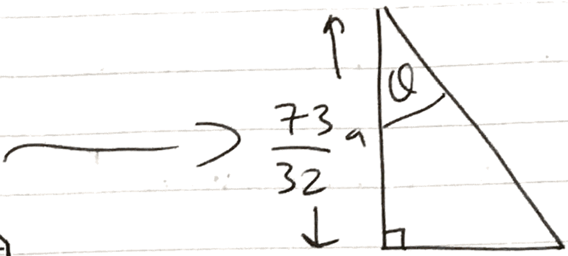
$$3\cancel{a^2}\left(\frac{a}{2}\right) + 4\cancel{a^2}(2a) + 9\cancel{a^2}(2a) = \frac{23\cancel{a^2}}{2}(\bar{x})$$

$$\frac{\left(\frac{3a}{2} + 8a + 2(9a)\right)}{\frac{23}{2}} = \bar{x} = \boxed{\frac{55}{23}a}$$

$$4a - \frac{55}{32}a = \frac{73}{32}a$$

b)





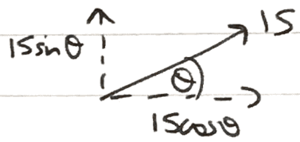
$$\left(\frac{55a}{32} - a\right) = \frac{23}{32}a$$

$$\therefore \tan\theta = \frac{\frac{23}{32}\cancel{a}}{\frac{73}{32}\cancel{a}} = \frac{23}{73}$$

$$\therefore \theta = \tan^{-1}\left(\frac{23}{73}\right) = \boxed{17.5^\circ}$$

Q5ai) $I = 0.5(v-u)$

after



$$k(\underline{i}+\underline{j}) = 0.5(v - 12\underline{i})$$

$$2k\underline{i} + 2k\underline{j} = v - 12\underline{i}$$

$$v = (12+2k)\underline{i} + (2k)\underline{j}$$

$\underline{|v| = 15}$: $\sqrt{(2k+12)^2 + (2k)^2} = 15$

$$\Rightarrow 4k^2 + 48k + 144 + 4k^2 = (15)^2$$

$$\Rightarrow 8k^2 + 48k + 144 - 225 = 0$$

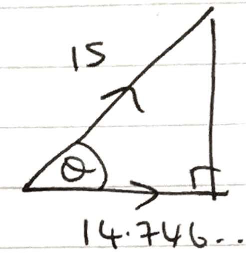
$$\Rightarrow 8k^2 + 48k - 81 = 0$$

By Quadratic formula, $k = \frac{-12+3\sqrt{34}}{4} = \boxed{1.37}$

reject other solution as $u > 0$.

$\therefore V = (12 + 2(1.37))\underline{i} + (2(1.37))\underline{j}$

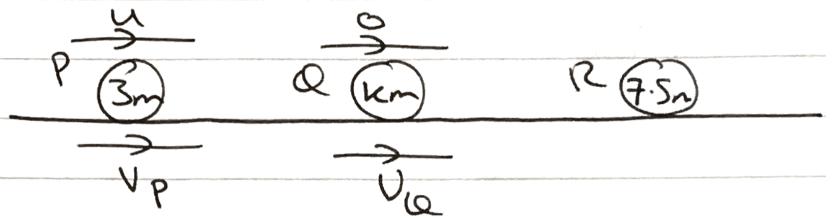
$\underline{i}$ component $= 14.746\ldots$



$\cos\theta = \frac{\text{adjacent}}{\text{hypotense}} = \frac{14.746\ldots}{15}$

$\therefore \theta = \boxed{10.6°}$

Q6a)



$\overrightarrow{C.L.M}$: $3\cancel{m}u = 3\cancel{m}V_P + k\cancel{m}V_Q$.

$$3u = 3V_P + kV_Q. \sim (1)$$

$\overrightarrow{N.I.L}$: $\frac{1}{9} = \frac{V_Q - V_P}{u}$

$$\therefore \frac{u}{9} = V_Q - V_P \sim (2)$$

$3 \times (2)$: $\frac{u}{3} = 3V_Q - 3V_P$

$+ (1)$: $\left(\frac{1}{3} + 3\right)u = (3+k)V_Q = \frac{10u}{3}$

hence $V_Q = \frac{10u}{3(3+k)}$.

b) $V_P = V_Q - \frac{u}{9} = \frac{10u}{3(u+3)} - \frac{u}{9}$

$\underline{V_P < 0}$: $\frac{10\cancel{u}}{3(u+3)} - \frac{\cancel{u}}{9} < 0$

$\frac{10}{3(u+3)} < \frac{1}{9}$

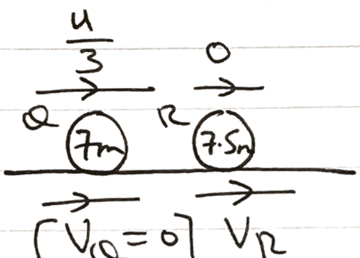
take reciprocal : so signs flip. $\quad \frac{3(u+3)}{10} > 9$

$u + 3 > \frac{90}{3}$

$u + 3 > 30$

$\boxed{u > 27}$

c)



$V_Q = \frac{10u}{3(10)} = \frac{u}{3}$

$\underline{\overrightarrow{C.L.M}}$: $7\cancel{m}\left(\frac{u}{3}\right) = 7m(0) + 7.5\cancel{m}(V_R)$

$\frac{7}{3}u = \frac{15}{2}V_R \quad \therefore V_R = \frac{14}{45}u.$

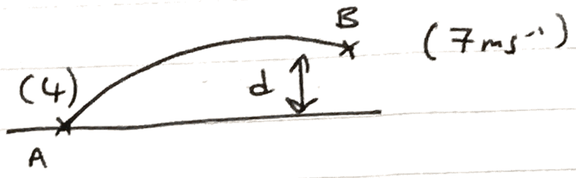
$\underline{KE \text{ before}}$: $\frac{1}{2}(7m)\left(\frac{u}{3}\right)^2 = \frac{7mu^2}{18}$

$\underline{KE \text{ after}}$: $\frac{1}{2}(7.5m)\left(\frac{14}{45}u\right)^2 = \frac{49}{135}mu^2$

$KE \text{ lost} = \left(\frac{7}{18} - \frac{49}{135}\right)mu^2 = \boxed{\frac{7}{270}mu^2}$

Q7a) **Initially**: $KE = \frac{1}{2}m(4^2)$

$GPE = 0$



Finally: $KE = \frac{1}{2}m(7^2)$

$GPE = mgd$

C.O.E: $8\cancel{m} = \frac{49\cancel{m}}{2} + \cancel{m}gd$

$$\left| \frac{8 - \frac{49}{2}}{9} \right| = d = \boxed{1.68m}.$$

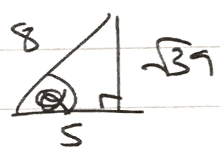
[d comes out as negative because B is <u>below</u> A]

b) minimum speed is at the highest point. [only the horizontal component of velocity is > 0]

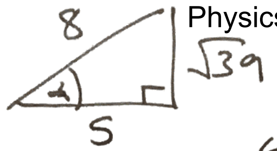
∴ horizontal component = 2.5m/s.

$2.5 = 4\cos\alpha$

$\cos\alpha = \frac{2.5}{4} = \frac{5}{8}$



$\alpha = \cos^{-1}\left(\frac{5}{8}\right) = \boxed{51°}$ (2 s.f.)



$\leftarrow x = -1.68$ from (a).

c)

$$\left.\begin{array}{l} s = x \\ u = 4\sin\alpha \\ v = \\ a = -g \\ t = t \end{array}\right\} \quad s = ut + \frac{1}{2}at^2$$

$$x = \frac{4\sqrt{39}}{8}t - \frac{g}{2}t^2 = -1.68$$

$$4.9t^2 - \frac{\sqrt{39}}{2}t - 1.68 = 0$$

By Quadratic formula; $t = 0.9865$. $(t > 0$

Horizontal distance $= ut$

$= 4\cos\alpha \times 0.986$

$= 4 \times \frac{5}{8} \times 0.986$

$= \boxed{2.46m}$