

M2 January 2018 (IAL) (MA)

Q1)  $I = m(v - u)$

$$4\mathbf{i} + 5\mathbf{j} = 0.5(v - (2\mathbf{i} - 3\mathbf{j}))$$

$$\times 2 : 8\mathbf{i} + 10\mathbf{j} = v - 2\mathbf{i} + 3\mathbf{j}$$

$$v = 10\mathbf{i} + 7\mathbf{j}$$

$$\therefore \text{KE after} = \frac{1}{2} (0.5) (10^2 + 7^2) = \boxed{37.25}$$

$$\text{and KE before} = \frac{1}{2} (0.5) (2^2 + 3^2) = \underline{\underline{3.25}}$$

$$\therefore \text{Increase in KE} = 37.25 - 3.25 \\ = \boxed{34 \text{ J}}$$

Q2a)  $v = 2t - 2t^2 - 1 + t$

$$v = -2t^2 + 3t - 1$$

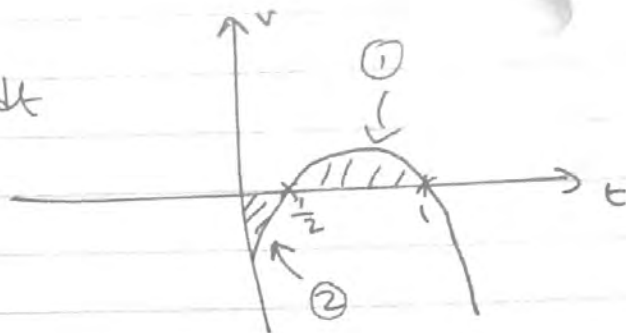
$$a = \frac{dv}{dt} = -4t + 3$$

$$t = \frac{1}{2} : a = -4\left(\frac{1}{2}\right) + 3 = \boxed{1 \text{ ms}^{-2}}$$

b) P is at rest at  $t = \frac{1}{2}$  and  $t = 1$

$$\left[ (2t - 1)(1 - t) = 0 \right]$$

$$\therefore \text{distance} = \int_{\frac{1}{2}}^1 [-2t^2 + 3t - 1] dt$$



$$= \left[ -\frac{2t^3}{3} + \frac{3t^2}{2} - t \right]_{\frac{1}{2}}^1$$

$$= \left[ -\frac{2}{3} + \frac{3}{2} - 1 \right] - \left[ -\frac{1}{12} + \frac{3}{8} - \frac{1}{2} \right]$$

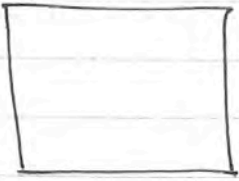
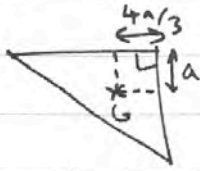
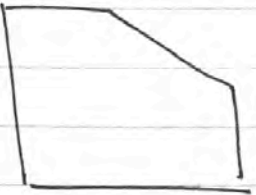
$$= \boxed{\frac{1}{24}} = \textcircled{1}$$

$$\text{distance} = \left| \int_0^{\frac{1}{2}} [-2t^2 + 3t - 1] dt \right| = \left| \left[ -\frac{2t^3}{3} + \frac{3t^2}{2} - t \right]_0^{\frac{1}{2}} \right|$$

$$= \left| -\frac{5}{24} \right| = \frac{5}{24} //$$

$$\therefore \text{total required distance} = \frac{5}{24} + \frac{1}{24}$$

$$= \boxed{\frac{1}{4}}$$

	Shape	Mass (area)	Distance of c.o.m from ...	
			<u>OD</u>	<u>OA</u>
(+)		$36a^2$	$3a$	$3a$
(-)		$\frac{1}{2} \cdot 4a \cdot 3a$ $= 6a^2$	$\frac{14a}{3}$	$5a$
(=)		$30a^2$	$\bar{x}$	$\bar{y}$

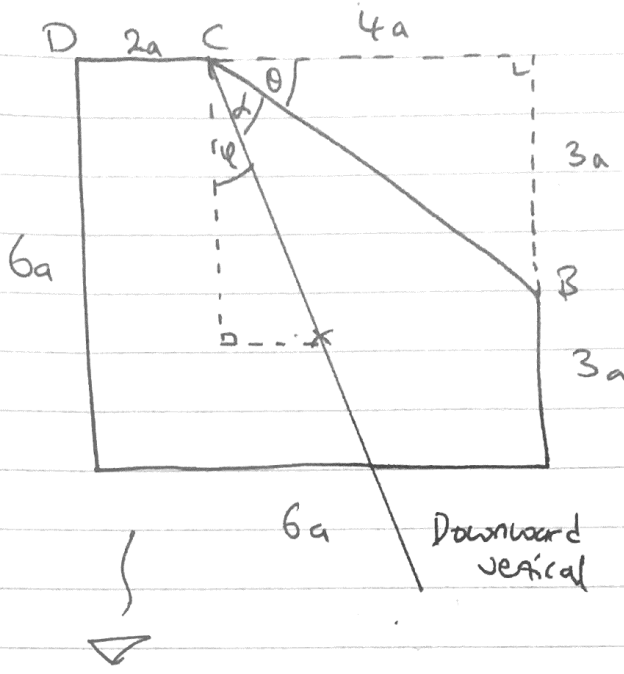
$$\bar{x} \sum M_i = \sum m_i x_i$$

$$36a^2 \begin{pmatrix} 3a \\ 3a \end{pmatrix} - 6a^2 \begin{pmatrix} 14a/3 \\ 5a \end{pmatrix} = 30a^2 \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix}$$

$$\begin{pmatrix} 108a & -28a \\ 108a & -30a \end{pmatrix} = 30 \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix}$$

$$\therefore \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = \begin{pmatrix} 8a/3 \\ 13a/5 \end{pmatrix}$$

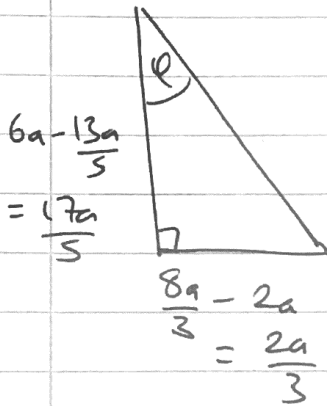
b)



$$\alpha = 90 - \theta - \phi$$

$$\tan \theta = \frac{3}{4}$$

$$\therefore \theta = \arctan \frac{3}{4}$$



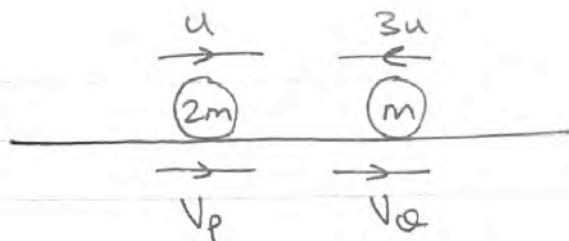
$$\therefore \tan \phi = \frac{\frac{2}{3}}{\frac{17}{5}} = \frac{10}{51}$$

$$\therefore \phi = \arctan \frac{10}{51}$$

$$\therefore \alpha = 90 - \arctan \frac{10}{51} - \arctan \frac{3}{4}$$

$$\alpha \approx 42^\circ$$

(Q4ai)



$$\begin{aligned} \underline{\text{C.L.M}} : 2mu - 3mu &= 2mv_p + mv_q \\ -u &= 2v_p + v_q \quad \text{--- (1)} \end{aligned}$$

$$\underline{\text{N.I.L}} : e = \frac{v_q - v_p}{4u}$$

$$\therefore 4ue = v_q - v_p$$

$$\underline{\text{--- (1)}} : -[-u = 2v_p + v_q]$$

$$4ue + u = -2v_p - v_p + v_q - v_p$$

$$u(4e + 1) = -3v_p$$

$$\therefore v_p = -\frac{u}{3}(4e + 1)$$

$$\begin{aligned} \therefore \text{speed} &= \left| -\frac{u}{3}(4e + 1) \right| \\ &= \boxed{\frac{u}{3}(4e + 1)} \end{aligned}$$

$$\text{ii) and } v_q = 4ue + v_p$$

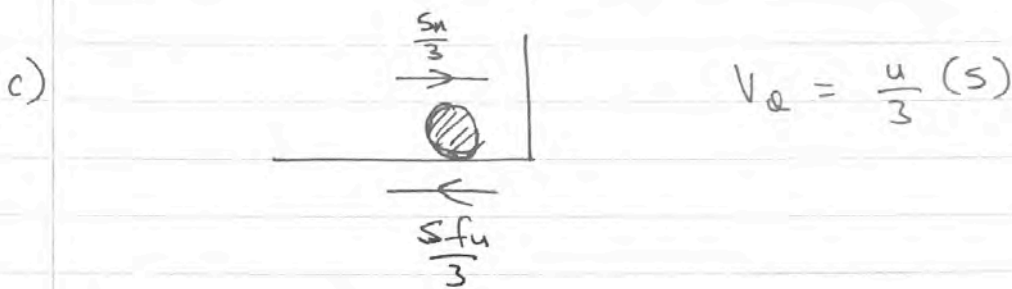
$$= 4ue + -\left(\frac{4ue}{3} + \frac{u}{3}\right)$$

$$= \frac{8ue}{3} - \frac{u}{3} = \boxed{\frac{u}{3}(8e - 1)}$$

$8e - 1 > 0$  since  $e > \frac{1}{8}$   
 so no need for modulus sign

b)  $V_p = -\frac{4}{3}(1+4e)$

$V_p < 0$  for all values of  $e$  ( $e > \frac{1}{8}$ )  
 so  $P$  has its direction reversed.



final speed =  $\frac{5u}{3} \times f$

there's another collision...

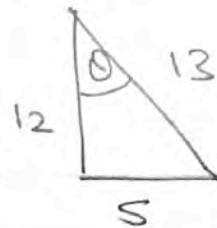
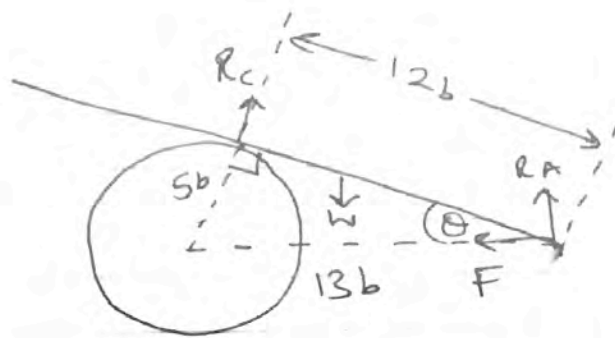
so  $\frac{5fu}{3} \geq \frac{u}{3}(4)$

$\Rightarrow 5f \geq 4$

$\Rightarrow f \geq \frac{4}{5}$

so  $\boxed{1 \geq f \geq \frac{4}{5}}$

(Q5a)



$$M(A): R_c(12b) = W \cos \theta (8b)$$

$$\cos \theta = \frac{12}{13}$$

$$R_c = \frac{8 \times W \times \frac{12}{13}}{12}$$

$$\sin \theta = \frac{5}{13}$$

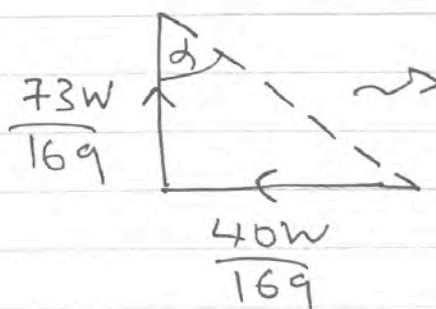
$$R_c = \boxed{\frac{8W}{13}}$$

$$b) R(\updownarrow): R_A + R_c \cos \theta = W$$

$$R_A = W - \frac{8W}{13} \left( \frac{12}{13} \right)$$

$$R_A = \frac{73W}{169} //$$

$$R(\leftrightarrow): F = R_c \sin \theta = \frac{8W}{13} \times \frac{5}{13} = \frac{40W}{169} //$$

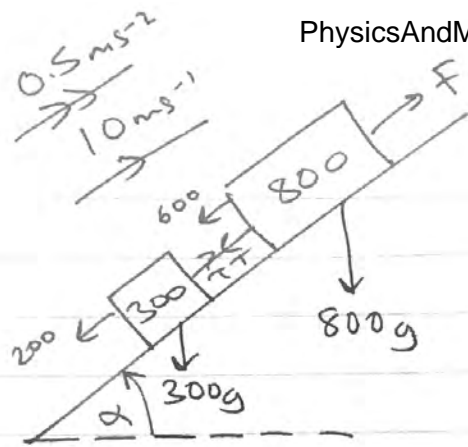


$$\tan \theta = \frac{\frac{40W}{169}}{\frac{73W}{169}}$$

$$= \boxed{\frac{40}{73} //}$$



Q6a)



$$\sin \alpha = \frac{1}{14}$$

$$P = Fv$$

$$P = F(10)$$

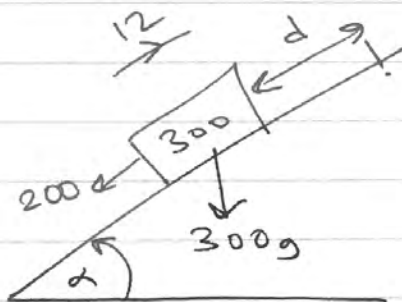
$$\text{N2L (system)}: F - 600 - 200 - 1100g \sin \alpha = 1100 \left(\frac{1}{2}\right)$$

$$F = 800 + \frac{1100g}{14} + 550 = \underline{2120}$$

$$P = F \times 10 = 21200 \text{ W} = 21.2 \text{ kW}$$

$$\text{so } \boxed{P = 21.2}$$

b)



$$\text{Initial energy: } KE = \frac{1}{2}(300)(12^2) = 21600$$

$$GPE = 0$$

$$\text{Final energy: } KE = 0$$

$$GPE = 300gd \sin \alpha = 210d$$

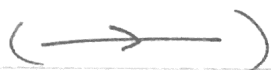
$$\text{\$ W.D by resistance} = -200 \times d$$

$$\text{so... } 21600 = 210d + 200d$$

$$d = \frac{21600}{210 + 200} = \boxed{52.7\text{m}}$$



Q7a)



$$\left. \begin{aligned} S &= x \\ u &= u \cos \alpha \\ v &= \\ a &= 0 \\ t &= t \end{aligned} \right\}$$

$$\left. \begin{aligned} S &= ut \\ x &= u \cos \alpha \times t \end{aligned} \right\}$$

$$\begin{aligned} \text{so } x &= ut \cos \alpha \\ \therefore t &= \frac{x}{u \cos \alpha} \end{aligned}$$

$$\left. \begin{aligned} S &= y \\ y &= u \sin \alpha \\ v &= \\ a &= -g \\ t &= t \end{aligned} \right\}$$

$$S = ut + \frac{1}{2} at^2$$

$$y = ut \sin \alpha - \frac{g}{2} t^2$$

$$\text{but } t = \frac{x}{u \cos \alpha}$$

$$\text{so } y = u \sin \alpha \left( \frac{x}{u \cos \alpha} \right) - \frac{g}{2} \left( \frac{x}{u \cos \alpha} \right)^2$$

$$y = x \tan \alpha - \frac{g x^2}{2 u^2 \cos^2 \alpha}$$

$$y = x \tan \alpha - \frac{g x^2 (\sec^2 \alpha)}{2 u^2}$$

$$(\sec^2 \alpha = 1 + \tan^2 \alpha) : y = x \tan \alpha - \frac{g x^2 (1 + \tan^2 \alpha)}{2 u^2}$$

$$b) \vec{a} = 0 \quad \text{so} \quad \vec{v} = u$$

$$\left( \begin{array}{l} \downarrow \\ \downarrow \end{array} \right) \left. \begin{array}{l} s = h \\ u = 0 \\ v = v \\ a = g \\ t = t \end{array} \right\} \begin{array}{l} v = u + at \\ v = gt \\ \therefore \downarrow v = gT \end{array} \quad (t = T) \text{ here}$$

$$\begin{aligned} \therefore \text{speed at B} &= \sqrt{\downarrow v^2 + \vec{v}^2} \\ &= \sqrt{(gt)^2 + u^2} \\ &= \boxed{\sqrt{g^2 T^2 + u^2}} \end{aligned}$$

$$c) \vec{s} = ut; \quad \underline{\underline{d = uT}}$$

$$\text{from a, } y = x \tan \alpha - \frac{gx^2}{2u^2} (1 + \tan^2 \alpha)$$

$$\underline{\underline{y = -h, x = d}} : -h = d \tan \alpha - \frac{gd^2}{2u^2} (1 + \tan^2 \alpha)$$

We do not want  $h$  or  $u$  in our equation.

$$\text{from (b): } \left( \begin{array}{l} \downarrow \\ \downarrow \end{array} \right) \left. \begin{array}{l} s = h \\ u = 0 \\ v = v \\ a = g \\ t = t \end{array} \right\} \begin{array}{l} s = ut + \frac{1}{2} at^2 \\ h = \frac{1}{2} g t^2 \end{array}$$

now we know  $d = uT$  and  
 $h = \frac{1}{2} gT^2$

So sub these in ...

$$-\frac{1}{2} gT^2 = uT \tan \alpha - \frac{g}{2u^2} (u^2 T^2) (\tan^2 \alpha + 1)$$

$\left. \begin{array}{l} \nearrow \\ \searrow \end{array} \right\} \text{leave as } d.$

$$-\frac{1}{2} gT^2 = d \tan \alpha - \frac{gT^2}{2} (\tan^2 \alpha + 1)$$

$$-\cancel{\frac{1}{2}} g \cancel{T^2} = d \tan \alpha - \cancel{\frac{gT^2}{2}} - \frac{gT^2 \tan^2 \alpha}{2}$$

$$\therefore d \tan \alpha = \frac{gT^2 \tan^2 \alpha}{2}$$
