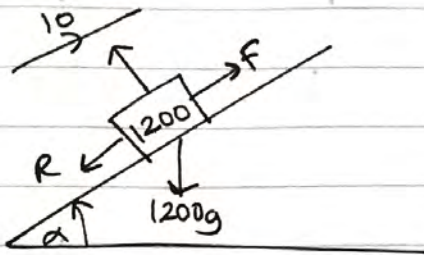


M2 Jan 17 IAL (MA)

Q1a)



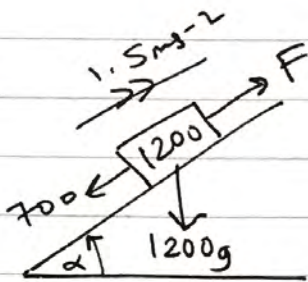
$$P = Fv$$

$$11760 = F(10)$$

$$F = \underline{\underline{1176 \text{ N}}}$$

N2L \nearrow (Parallel to slope): $F - R - 1200g \sin \alpha = 1200(0)$
 $R = 1176 - \frac{1200g}{15} = \underline{\underline{392 \text{ N}}}$

b)



$$P = Fv$$

$$50000 = Fv$$

$$F = \frac{50000}{v}$$

N2L \nearrow (Parallel to slope): $F - 1200g \sin \alpha - 700 = 1200(1.5)$

$$\frac{50000}{v} = 1200(1.5) + 700 + \frac{1200g}{15}$$

$$\frac{50000}{v} = 3284$$

$$\therefore v = \frac{50000}{3284} = \underline{\underline{15.2 \text{ m/s}}}$$

Q2ai) /ii)

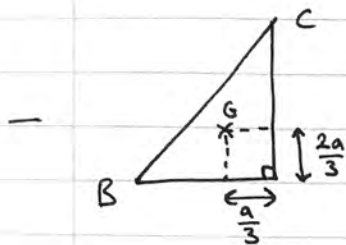
Shape	Area	AD	AB
+	$4a^2$	a	a
-	$\frac{1}{2} \times a \times 2a = a^2$	$2a - \frac{a}{3} = \frac{5a}{3}$	$\frac{2a}{3}$
=	$3a^2$	\bar{x}	\bar{y}



$$4a^2$$

$$a$$

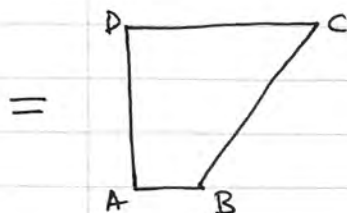
$$a$$



$$\frac{1}{2} \times a \times 2a = a^2$$

$$2a - \frac{a}{3} = \frac{5a}{3}$$

$$\frac{2a}{3}$$



$$3a^2$$

$$\bar{x}$$

$$\bar{y}$$

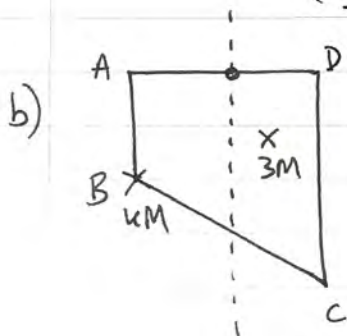
$$\sum m_i x_i = \bar{x} \sum m$$

$$\Rightarrow 4a^2 \begin{pmatrix} a \\ a \end{pmatrix} - a^2 \begin{pmatrix} 5a/3 \\ 2a/3 \end{pmatrix} = 3a^2 \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 4a - 5a/3 \\ 4a - 2a/3 \end{pmatrix} = 3 \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = \begin{pmatrix} 7a/3 \\ 10a/3 \end{pmatrix}$$

$$\therefore \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = \begin{pmatrix} 7a/9 \\ 10a/9 \end{pmatrix} \text{ so distance from...}$$

$$\boxed{\begin{matrix} AB = 10a/9 \\ AD = 7a/9 \end{matrix}}$$



taking moments about midpoint of AD

$$xM(a) = 3M \left(\frac{10a}{9} - a \right)$$

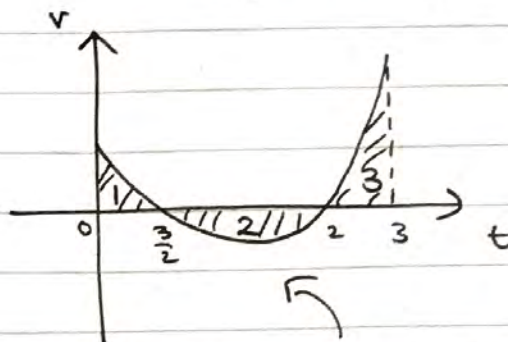
$$x(a) = 3M \left(\frac{1}{9} \right)$$

$$\therefore x = \frac{3}{9} = \frac{1}{3}$$

$$(03) \quad v = (2t-3)(t-2) \rightarrow \text{at } v=0 : t = \frac{3}{2}, t = 2 //$$

$$v = 2t^2 - 7t + 6$$

$$x = \int (v) dt$$



$$\text{Area 1} = \int_0^{\frac{3}{2}} (2t^2 - 7t + 6) dt$$

total distance = Area 1 + Area 2 + Area 3

$$= \left[\frac{2t^3}{3} - \frac{7t^2}{2} + 6t \right]_0^{\frac{3}{2}} = \frac{27}{8} - 0 = \frac{27}{8} //$$

$$\text{Area 2} = \int_{\frac{3}{2}}^2 (2t^2 - 7t + 6) dt = \left[\frac{2t^3}{3} - \frac{7t^2}{2} + 6t \right]_{\frac{3}{2}}^2$$

$$= \left[\frac{10}{3} \right] - \left[\frac{27}{8} \right] = -\frac{1}{24} // \rightarrow \text{distance} = \frac{1}{24} //$$

$$\text{Area 3} = \int_{-2}^3 [2t^2 - 7t + 6] dt = \left[\frac{2t^3}{3} - \frac{7t^2}{2} + 6t \right]_{-2}^3$$

$$= \left[\frac{9}{2} \right] - \left[\frac{10}{3} \right] = \frac{7}{6} //$$

$$\text{Total distance} = \frac{27}{8} + \left(\frac{1}{24} \right) + \frac{7}{6} = \boxed{\frac{55}{12} \text{ m}}$$

$$Q4a) \quad I = m(v-u)$$

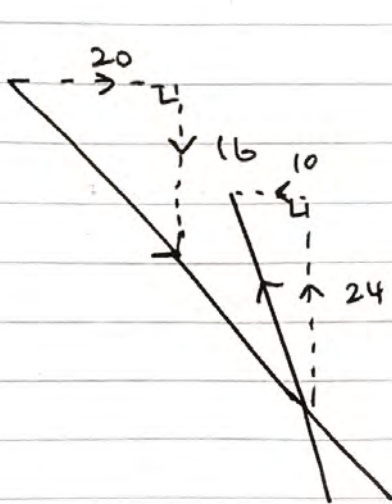
$$-6\mathbf{i} + 8\mathbf{j} = 0.2(v - (20\mathbf{i} - 16\mathbf{j}))$$

$$-6\mathbf{i} + 8\mathbf{j} = 0.2(v - 20\mathbf{i} + 16\mathbf{j})$$

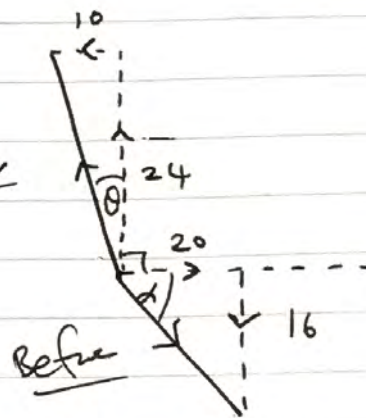
$$\textcircled{\times 5} : -30\mathbf{i} + 40\mathbf{j} = v - 20\mathbf{i} + 16\mathbf{j}$$

$$v = -10\mathbf{i} + 24\mathbf{j} \quad \therefore \text{speed} = \sqrt{10^2 + 24^2} = \boxed{26 \text{ ms}^{-1}}$$

b)



After



Before

\therefore angle required

$$= \arctan \frac{4}{5} + \arctan \frac{5}{12} + 90^\circ$$

$$= \boxed{151.3^\circ}$$

$$= 151^\circ \text{ to 3 s.f.}$$

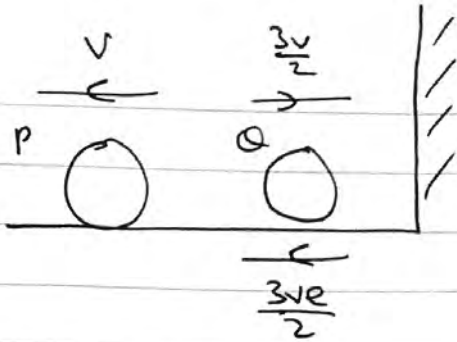
$$\tan \alpha = \frac{16}{20} = \frac{4}{5}$$

$$\alpha = \arctan \left(\frac{4}{5} \right)$$

$$\tan \theta = \frac{10}{24} = \frac{5}{12}$$

$$\theta = \arctan \left(\frac{5}{12} \right)$$

b)



By N.I.L, Q rebounds with speed $\frac{3v}{2} \times e$ //

Because there is a further collision, $\frac{3ve}{2} > v$

$$\Rightarrow \frac{3e}{2} > 1 \quad \therefore e > \frac{2}{3} //$$

$$\Rightarrow \boxed{1 \geq e > \frac{2}{3}}$$

Q6)

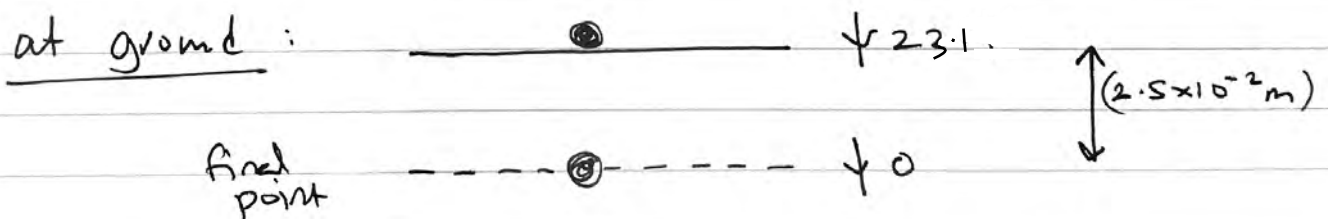
from point of projection to ground,

at point of projection: $KE = \frac{1}{2}(0.6)(22.4^2) = 150.53 \text{ J}$
 $GPE = 0.6g(1.5) = 8.82 \text{ J}$

at ground: $KE = \frac{1}{2}(0.6)v^2 = 0.3v^2$
 $GPE = 0$

(taking ground as 0 GPE level)

Conservation of Energy: $150.53 + 8.82 = 0.3v^2$
 $v^2 = 531.667 \dots$
 $v = 23.05 // = \text{speed at ground.}$



Q6
cont

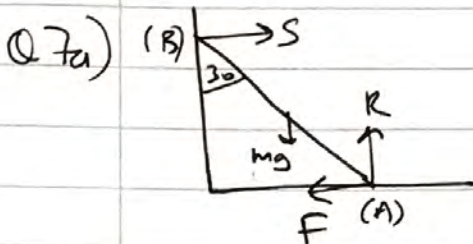
at ground: $KE = \frac{1}{2} (0.6) (23.65)^2 = 159.39$
 (not final position) $GPE = 0.6g (2.5 \times 10^{-2}) = 0.147$

\therefore Work Done by $R = 159.39 + 0.147$
 $= 159.537$ //

$\therefore R \times (2.5 \times 10^{-2}) = 159.537$

$\Rightarrow R = \frac{159.537}{0.025} = \boxed{6380\text{N}}$ (3s.f.)

[Work Done] = $F \times d$



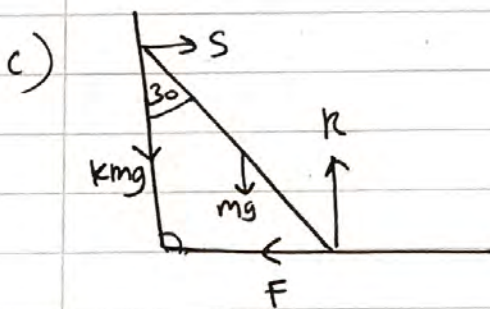
M(A): $mg \sin 30 (d) = S \sin 60 (2d)$

$S = \frac{mg \sin 30}{2 \sin 60} = \boxed{\frac{\sqrt{3} mg}{6}}$

b) $R_f(\updownarrow): R = mg$
 $R_f(\leftrightarrow): S = F$

$F \leq \mu R$
 $S \leq \mu mg$
 $\frac{mg\sqrt{3}}{6} \leq \mu mg$

$\therefore \mu \geq \frac{\sqrt{3}}{6}$



$R_f(\updownarrow): R = mg + kmg = mg(u+1)$

$R_f(\leftrightarrow): S = F$

M(A): $mg \sin 30 (d) + kmg \sin 30 (2d) = S \sin 60 (2d)$

$\xrightarrow{\times 2} : mg + 2kmg = (2\sqrt{3})F \quad (F=S)$

$$F = \mu R \quad (\text{limiting equilibrium})$$

$$\Rightarrow F = \frac{mg(1+2u)}{2\sqrt{3}} = \mu R$$

$$\text{but } R = mg(u+1) \text{ and } \mu = \frac{\sqrt{3}}{5},$$

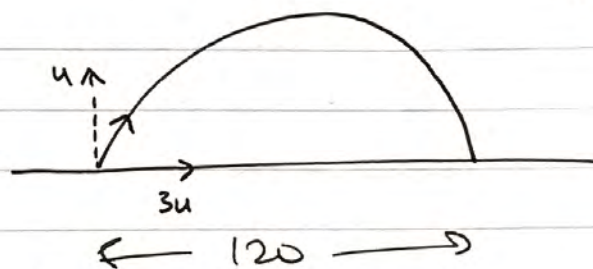
$$\Rightarrow \frac{mg(1+2u)}{2\sqrt{3}} = \frac{\sqrt{3}}{5} mg(1+u)$$

$$\underline{\underline{\times 2\sqrt{3}}} \Rightarrow 1+2u = \frac{6}{5}(1+u)$$

$$\Rightarrow 1+2u = \frac{6}{5} + \frac{6}{5}u$$

$$\Rightarrow \frac{4}{5}u = \frac{1}{5} \quad \therefore u = \frac{\frac{1}{5}}{\frac{4}{5}} = \boxed{\frac{1}{4}}$$

Q8a)



until
highest
point

$$\left(\begin{array}{l} \uparrow \\ \uparrow \end{array} \right) \left. \begin{array}{l} s = \\ u = u \\ v = 0 \\ a = -g \\ t = \frac{t}{2} \end{array} \right\}$$

$$\left. \begin{array}{l} v = u + at \\ 0 = u - \frac{gt}{2} \end{array} \right\}$$

$$\therefore t = \frac{2u}{g} = \text{time for total journey}$$

$$\underline{\underline{(\rightarrow)}} \quad \underline{\underline{s = ut}} : 120 = 3u \times \frac{2u}{g}$$

$$u^2 = \frac{120}{\frac{6}{9.8}} = 196$$

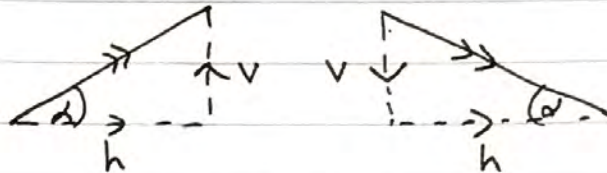
$$\therefore u = \sqrt{196} = \boxed{14}$$

$$b) \vec{v} = \vec{u} = 3u \quad (\text{no acceleration} \rightarrow)$$

$\downarrow v = u \uparrow$ due to symmetry of the motion

$$\therefore \text{Speed} = \sqrt{(u)^2 + (3u)^2} = 14\sqrt{10} = \boxed{44.3} \text{ m/s}$$

$$c) \tan d = \frac{1}{4}$$



$$\Rightarrow \frac{v}{h} = \frac{1}{4} \quad \therefore v = \frac{1}{4}h \quad \text{or} \quad 4v = h$$

So $\tan d = \frac{1}{4}$ means that the horizontal component of velocity must have a magnitude 4 times that of the vertical component.

expressing the velocity as a vector,

$$\vec{v} = \begin{pmatrix} 3u \\ u - gt \end{pmatrix} \quad \begin{array}{l} \sim \text{speed is constant} \\ \sim \text{initial speed } u \uparrow \\ \quad \text{under influence of } g. \end{array}$$

$$|u - gt| = \frac{1}{4}(3u)$$

$$u - gt = \frac{3u}{4} \quad \text{and} \quad -(u - gt) = \frac{3u}{4}$$

$$t = \frac{u}{4g} = \boxed{\frac{5}{14}}$$

$$-u + gt = \frac{3u}{4}$$

$$t = \frac{7u}{4g} = \boxed{\frac{5}{2}}$$