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1. A particle P of mass 0.6 kg is moving with velocity $(4\mathbf{i} - 2\mathbf{j})$ m s^{-1} when it receives an impulse $\mathbf{I}$ N s. Immediately after receiving the impulse, P has velocity $(2\mathbf{i} + 3\mathbf{j})$ m s^{-1}.

Find

(a) the magnitude of $\mathbf{I}$,

(4)

(b) the kinetic energy lost by P as a result of receiving the impulse.

(3)

$I = mv - mu$

$0.6(2i + 3j - (4i - 2j))$

$0.6(-2i + 5j)$

$-1.2i + 3j \qquad |I| = \sqrt{(1.2)^2 + (3)^2}$

$= 3.23$

b) $\frac{1}{2}mv^2$

$\frac{1}{2} \times 0.6 \times (|4i - 2j|^2 - |2i + 3j|^2)$

$\frac{1}{2} \times 0.6 \times (20 - 13)$

$2.1 J$

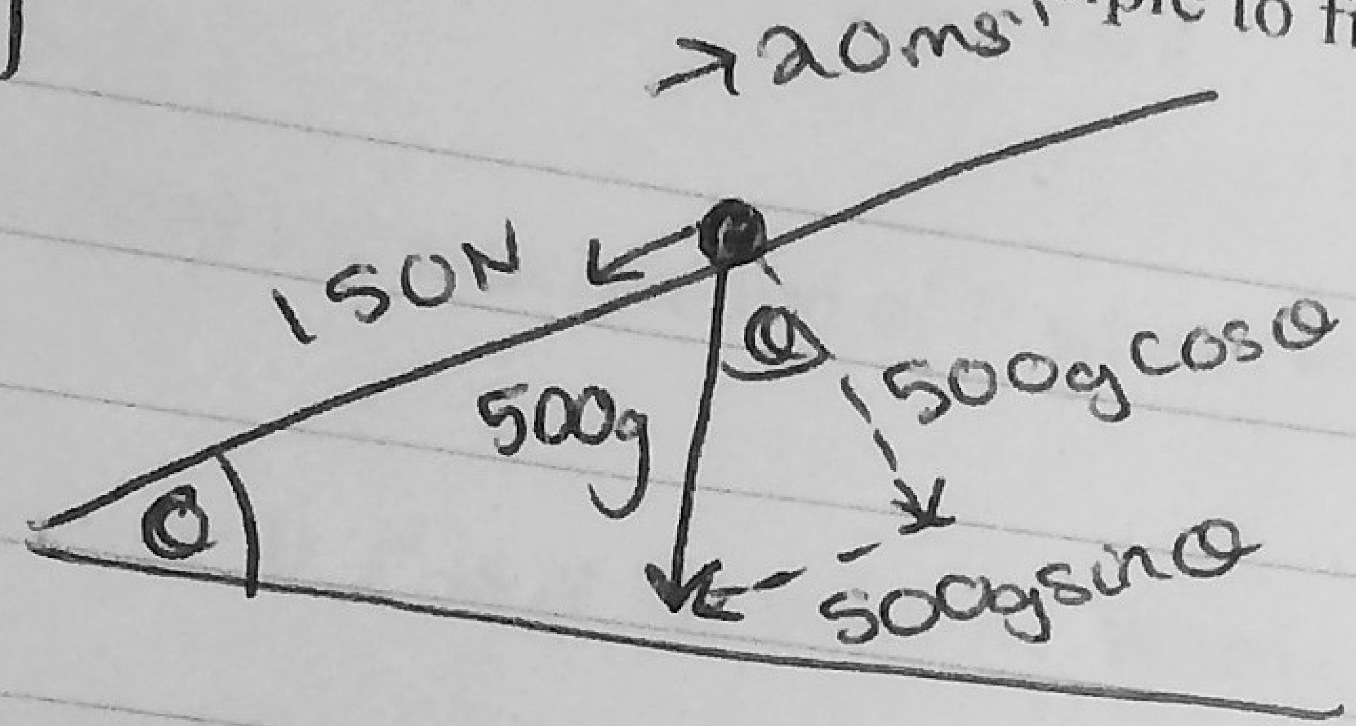
2. A car of mass 500 kg is moving at a constant speed of 20 m s^{-1} up a straight road inclined at an angle θ to the horizontal, where $\sin\theta = \frac{1}{20}$. The resistance to motion from non-gravitational forces is modelled as a constant force of magnitude 150 N.

(a) Find the rate of working of the engine of the car. **(5)**

When the car is travelling up the road at 20 m s^{-1}, the engine is switched off. The car then comes to instantaneous rest, without braking, having moved a distance d metres up the road from the point where the engine was switched off. The resistance to motion from non-gravitational forces is again modelled as a constant force of magnitude 150 N.

(b) Use the work-energy principle to find the value of d. **(4)**

2) a)



rate of working = Power

$P = Fv$

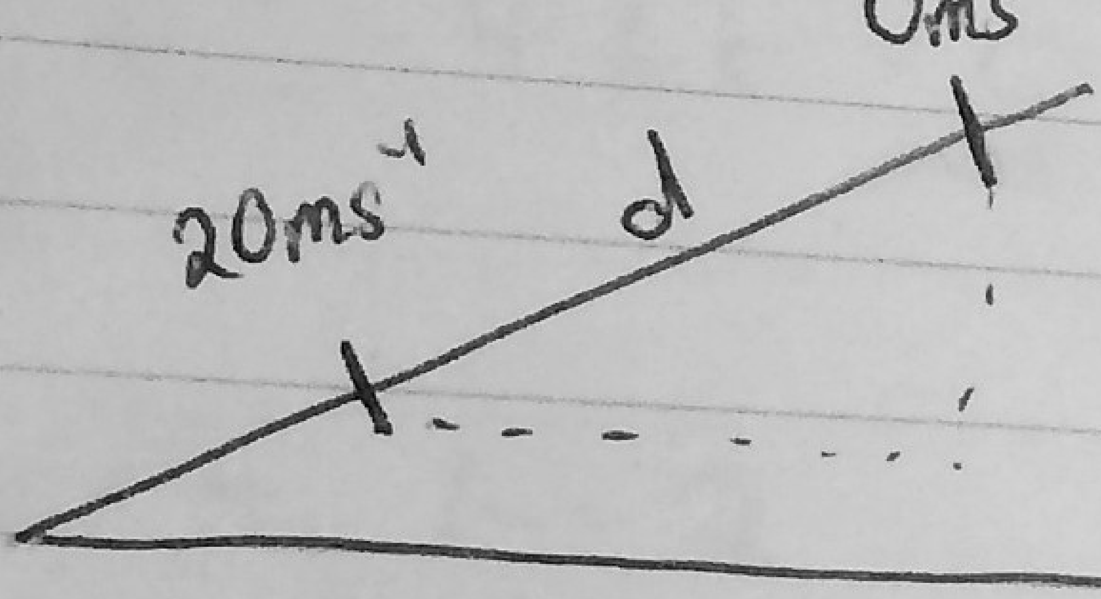
$P = 20F$

$500g\sin\theta + 150 = F$

$150 + 500(9.8)\left(\frac{1}{20}\right) = 395\text{N}$

$395 \times 20 = 7900\text{W} \quad 7.9\text{kW}$

b)



$\Delta KE = \frac{1}{2} \times 500 \times 20^2 \quad \frac{1}{2}mv^2$

$\Delta GPE = 500gd\sin\theta \quad mgh$

work done against Resistance = $150d$.

$150d + 500gd\sin\theta = \frac{1}{2} \times 500 \times 20^2$

$150d + 245d = 100000$

$395d = 100,000$

$d = 253.16\ldots$

$d = 250\text{m}$

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3. At time t seconds ($t \geqslant 0$) a particle P has position vector $\mathbf{r}$ metres, with respect to a fixed origin O, where

$$\mathbf{r} = \left(\frac{1}{8}t^4 - 2\lambda t^2 + 5\right)\mathbf{i} + (5t^2 - \lambda t)\mathbf{j}$$

and λ is a constant.

When $t = 4$, P is moving parallel to the vector $\mathbf{j}$.

(a) Show that $\lambda = 2$ **(5)**

(b) Find the speed of P when $t = 4$ **(1)**

(c) Find the acceleration of P when $t = 4$ **(2)**

When $t = 0$, P is at the point A. When $t = 4$, P is at the point B.

(d) Find the distance AB. **(4)**

3) a) $\frac{dr}{dt} = v = \left(\frac{1}{2}t^3 - 4\lambda t\right)i + (10t - \lambda)j$

$\frac{1}{2}t^3 - 4\lambda t = 0$ when $t = 4$

$32 - 16\lambda = 0$

$\lambda = 2$

b) $t = 4$ $\lambda = 2$

$\left(\frac{1}{2}(4)^3 - 4(2)(4)\right)i + (10(4) - 2)j$

$0i + 38j$ $38\ ms^{-1}$

c) $\frac{dv}{dt} = a = \left(\frac{3t^2}{2} - 8\right)i + 10j$ $t = 4$

$a = 16i + 10j$

d) $t = 0$ $r = 5i$

$t = 4$ $r = -27i + 72j$

$5 - (-27i + 72j)$ $32i - 72j$

$\sqrt{32^2 + 72^2} = 78.8\ m$

4.

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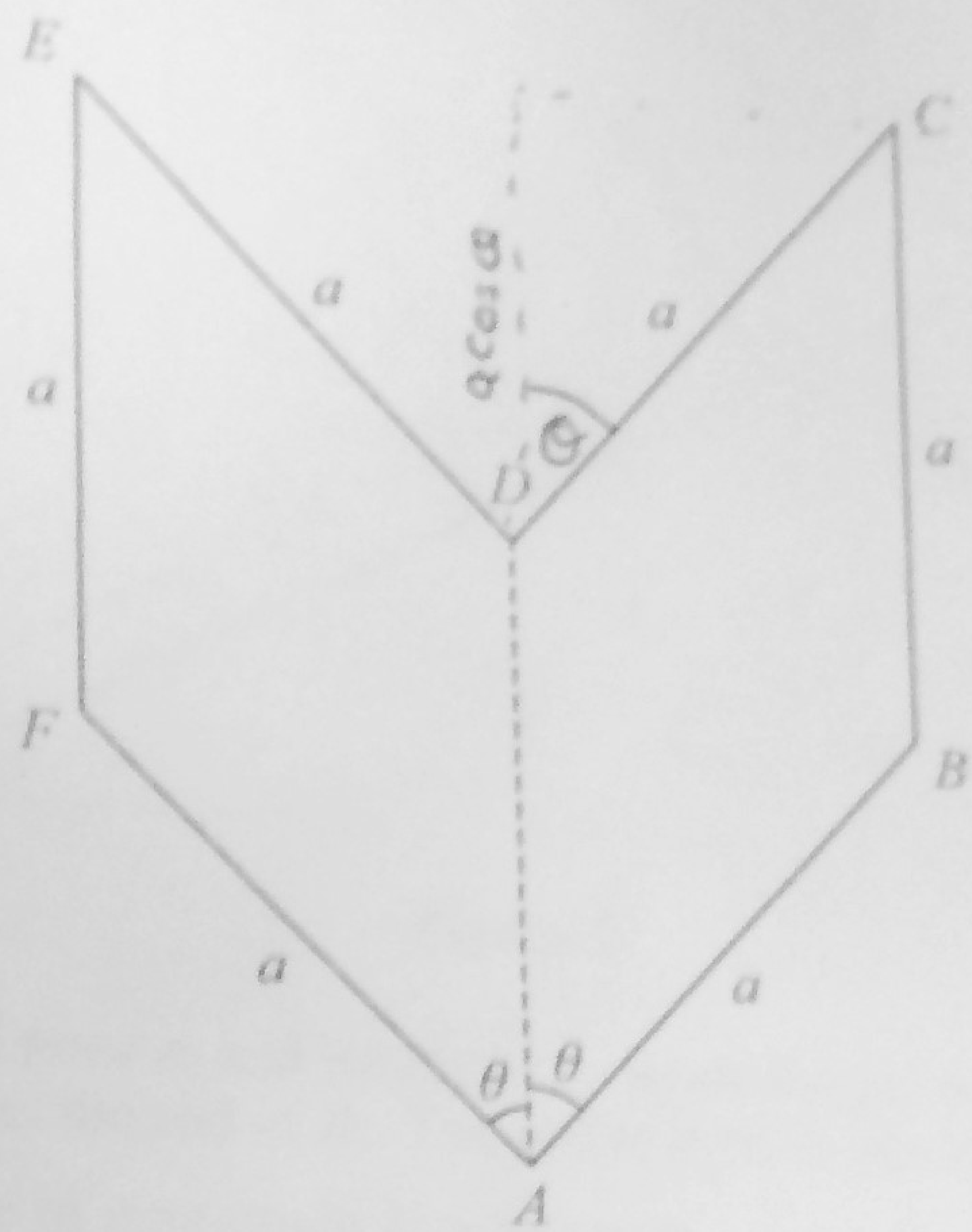

Figure 1

The uniform plane lamina $ABCDEF$ shown in Figure 1 is made from two identical rhombuses. Each rhombus has sides of length a and angle BAD = angle $DAF = \theta$. The centre of mass of the lamina is $0.9a$ from A.

(a) Show that $\cos\theta = 0.8$

(5)

The weight of the lamina is W. A particle of weight kW is fixed to the lamina at the point A. The lamina is freely suspended from B and hangs in equilibrium with DA horizontal.

(b) Find the value of k.

(4)

4 a) height from A to C $= a + a\cos\theta$

$0.9a = \frac{1}{2}(a + a\cos\theta)$

$\cos\theta = 0.8$

b) take Moments

$k\cancel{W}(a\cos\theta) = \cancel{W}(0.9a - a\cos\theta)$

$0.8k = 0.1 \qquad k = \frac{1}{8}$

5.

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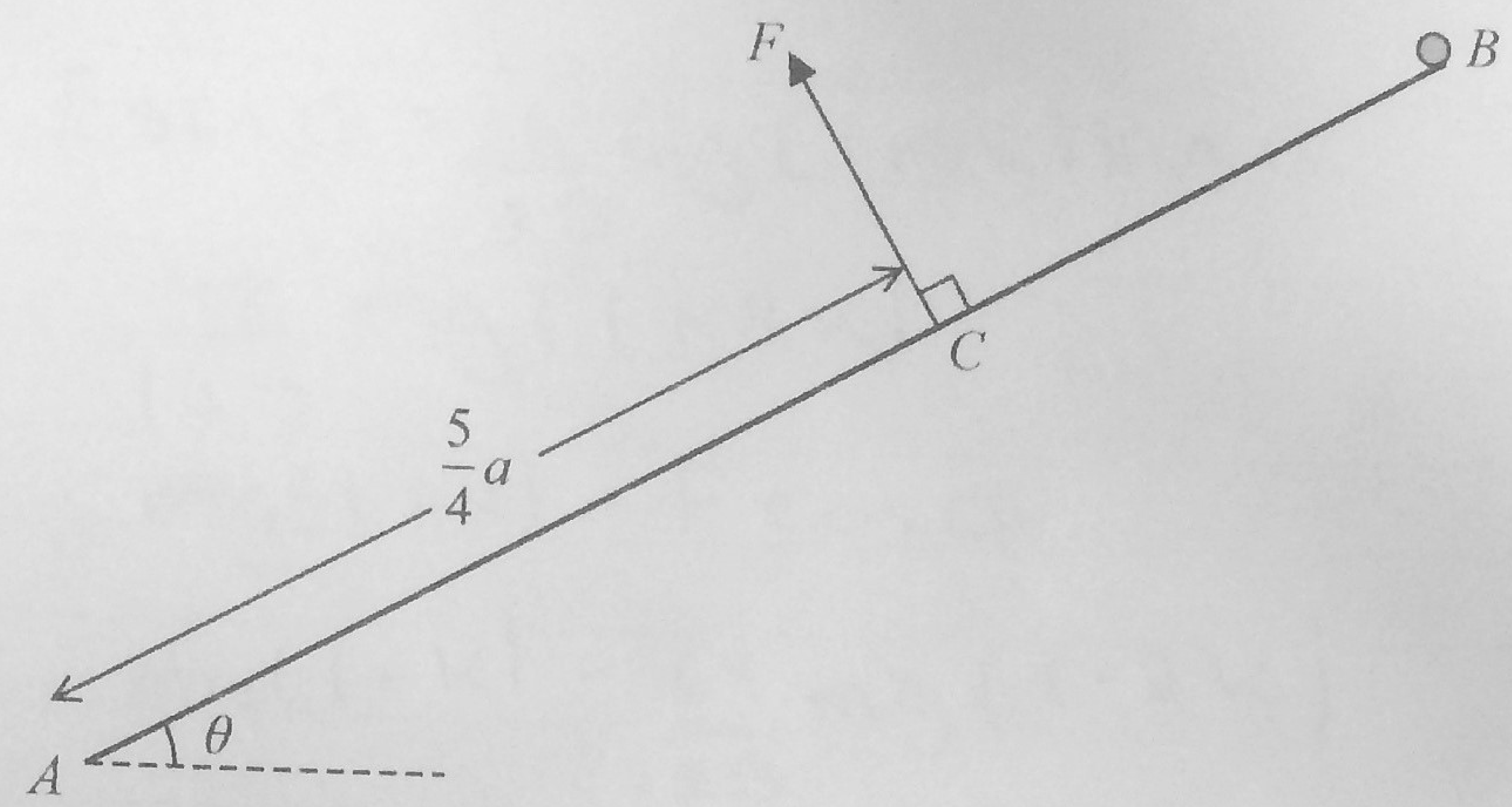

Figure 2

A uniform rod AB, of mass m and length $2a$, is freely hinged to a fixed point A. A particle of mass km is fixed to the rod at B. The rod is held in equilibrium, at an angle θ to the horizontal, by a force of magnitude F acting at the point C on the rod, where $AC = \frac{5}{4}a$, as shown in Figure 2. The line of action of the force at C is at right angles to AB and in the vertical plane containing AB.

Given that $\tan\theta = \frac{3}{4}$

(a) show that $F = \frac{16}{25}mg(1 + 2k)$,

(4)

(b) find, in terms of m, g and k,

(i) the horizontal component of the force exerted by the hinge on the rod at A,

(ii) the vertical component of the force exerted by the hinge on the rod at A.

(5)

Given also that the force acting on the rod at A acts at 45° above the horizontal,

(c) find the value of k.

(3)

5a) M(A) $F\left(\frac{5a}{4}\right) = mga\cos\theta + 2kmga\cos\theta$

$F = \frac{4mg\cos\theta}{5}(1+2k) = \frac{16}{25}mg(1+2k)$.

M2

5)b) $\rightarrow$ $F\sin\theta = \frac{16}{25}mg(1+2k)\sin\alpha$

$\frac{48}{125}mg(1+2k)$

$\uparrow$ $mg(1+k) - F\cos\alpha$

$mg(1+k) - \frac{64}{125}mg(1+2k)$

$= \frac{mg}{125}(61-3k)$

5)c) $\uparrow = \rightarrow$

$\frac{48}{125}mg(1+2k) = mg(1+k) - \frac{64}{125}mg(1+2k)$

$k = \frac{13}{99}$

6.

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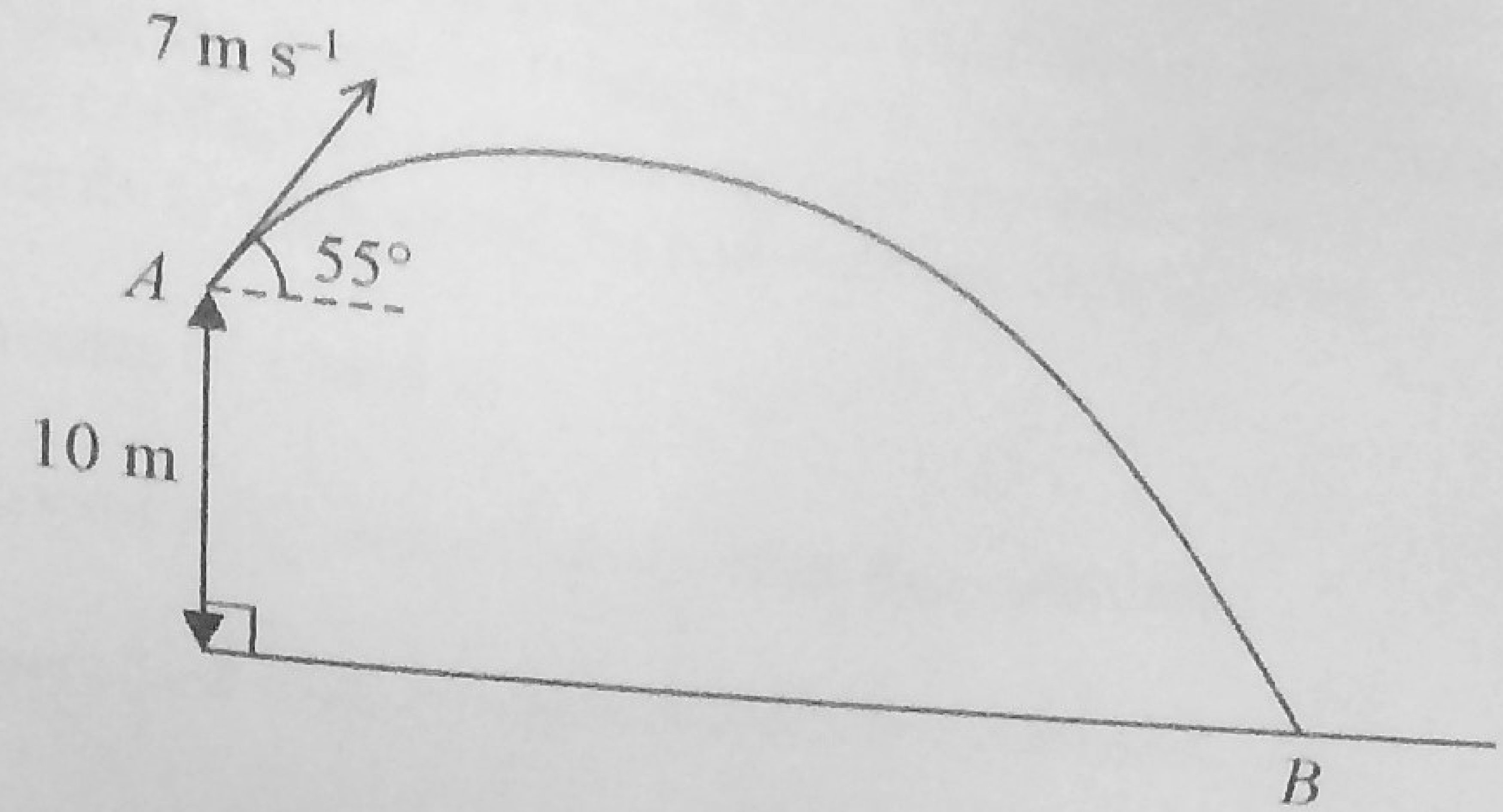

Figure 3

A small ball P is projected with speed 7 m s^{-1} from a point A 10 m above horizontal ground. The angle of projection is $55°$ above the horizontal. The ball moves freely under gravity and hits the ground at the point B, as shown in Figure 3.

Find

(a) the speed of P as it hits the ground at B,

(4)

(b) the direction of motion of P as it hits the ground at B,

(3)

(c) the time taken for P to move from A to B.

(5)

6a) Initial KE $= \frac{1}{2} \times m \times 7^2$ final KE $= \frac{1}{2}mv^2$

initial PE $= 10mg$ final PE $= 0$

$\frac{49}{2}\cancel{m} + 10\cancel{m}g = \frac{1}{2}\cancel{m}v^2$ $v = 15.7\text{ms}^{-1}$

b) $\cos\theta = \frac{7\cos 55}{15.7}$ $\theta = 75.2°$

c) $\uparrow$

s -10

u $7\sin 55$

v

a -9.8

t t

$-10 = 7\sin 55t - 4.9t^2$

$s = ut + \frac{1}{2}at^2$

$t = \frac{7\sin 55 + \sqrt{(7\sin 55)^2 + 40(4.9)}}{9.8}$

$= 2.13\text{s}$

Three particles P, Q and R lie at rest in a straight line on a smooth horizontal surface with Q between P and R. Particle P has mass m, particle Q has mass $2m$ and particle R has mass $3m$. The coefficient of restitution between each pair of particles is e. Particle P is projected towards Q with speed $3u$ and collides directly with Q.

(a) Find, in terms of u and e,

(i) the speed of Q immediately after the collision,

(ii) the speed of P immediately after the collision.

(6)

(b) Find the range of values of e for which the direction of motion of P is reversed as a result of the collision with Q.

(2)

Immediately after the collision between P and Q, particle R is projected towards Q with speed u so that R and Q collide directly. Given that $e = \frac{2}{3}$

(c) show that there will be a second collision between P and Q.

(6)

7)a) P: (m), $3u \rightarrow$ Q: $(2m)$, 0 P: (m), V_1 Q: $(2m)$, V_2

$3mu = mV_1 + 2mV_2$

$3u = V_1 + 2V_2$

$e = \frac{V_2 - V_1}{u_1 - u_2}$

$e = \frac{V_2 - V_1}{3u}$

$3u = V_1 + 6eu + 2V_1$

$3eu = V_2 - V_1$

$3u - 6eu = 3V_1$

$3eu + V_1 = V_2$

$u - 2eu = V_1$ $V_1 = u(1-2e)$

$V_2 = 3eu + u - 2eu = eu + u$

$V_2 = u(1+e)$

b) $1 - 2e < 0$ $e > \frac{1}{2}$

c) $V_1 = -\frac{u}{3}$ $V_2 = \frac{5}{3}u$. Q: $(2m)$, $\frac{5}{3}u \rightarrow$ R: $(3m)$, $\leftarrow u$

$\frac{2}{3} = \frac{V_2 - V_1}{\frac{5}{3}u - -u}$

$2m\left(\frac{5}{3}u\right) - 3m(u) = 2mV_1 + 3mV_2$

$\frac{16}{9}u = V_2 - V_1$

$\frac{1}{3}u = 2V_1 + 3V_2$ $\frac{1}{3}u = 2V_1 + \frac{16}{3}u + 3V_1$

$\frac{16}{9}u + V_1 = V_2$

$V_1 = -u$ $u > \frac{1}{3}u$ ∴ Q collide with P again.