M2 IAL January 2014

solutions by Ayman Zayed Mannan

1(a) $I = mv - mu$

$= m(v-u)$

$= 2(2\underset{\sim}{i} - 3\underset{\sim}{j} - 3\underset{\sim}{i} - 4\underset{\sim}{j})$

$= 2(-\underset{\sim}{i} - 7\underset{\sim}{j})$

$= -2\underset{\sim}{i} - 14\underset{\sim}{j}$

$|I| = \sqrt{(-2)^2 + (-14)^2}$

$= \sqrt{200}$

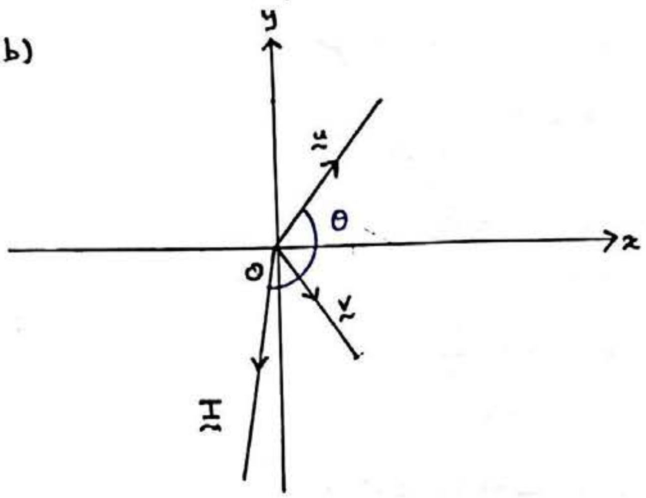
$= \sqrt{2 \times 10^2}$

$= 10\sqrt{2}$

$= 14.142\ldots$

$\therefore |\underset{\sim}{I}| \approx 14.1$ Ns (3 SF)

(b)



$|\underset{\sim}{u}| = \sqrt{3^2 + 4^2}$

$= \sqrt{25}$

$= 5$

scalar product of $\underset{\sim}{I}$ and $\underset{\sim}{u}$:

$\underset{\sim}{I} \cdot \underset{\sim}{u} = \begin{pmatrix} -2 \\ -14 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 4 \end{pmatrix}$

$= -2(3) + 4(-14)$

$= -62$

$\cos\theta = \dfrac{\underset{\sim}{I} \cdot \underset{\sim}{u}}{|\underset{\sim}{I}||\underset{\sim}{u}|}$

$= \dfrac{-62}{10\sqrt{2}(5)}$

$= -\dfrac{31\sqrt{2}}{50}$

$\theta = \arccos\left(-\dfrac{31\sqrt{2}}{50}\right)$

$= 151.26\ldots$

$\therefore \theta \approx 151°$ (3 SF)

2. $\underset{\sim}{v} = (t-2)(3t-10)$

$= 3t^2 - 10t - 6t + 20$

$= 3t^2 - 16t + 20$

(a) $\underset{\sim}{a} = \dfrac{dx}{dt} = 2(3)t - 16$

$= 6t - 16$

when $t = 3$,

$\underset{\sim}{a} = 6(3) - 16$

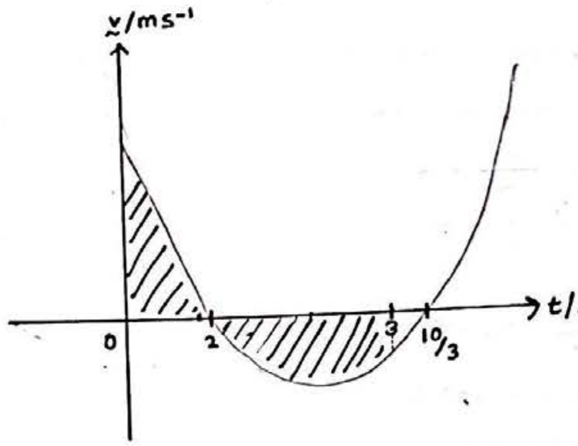
$= 18 - 16$

$\therefore \underset{\sim}{a} = 2$ ms^{-2}

(b) when $x = 0$,

$(t-2)(3t-10) = 0$

$t = 2$ or $t = \frac{10}{3}$



Distance travelled in the first 3 s

$= \int_0^2 (3t^2 - 16t + 20)\, dt + \int_2^3 |3t^2 - 16t + 20|\, dt$

$= \left[\dfrac{3t^3}{3} - \dfrac{16t^2}{2} + \dfrac{20t}{1}\right]_0^2 + \left[|t^3 - 8t^2 + 20t|\right]_2^3$

$= \left[2^3 - 8(2)^2 + 20(2) - 0\right] + \left[|3^3 - 8(3)^2 + 20(3) - 16|\right]$

$= 16 + |15 - 16|$

$= 16 + |-1|$

$= 16 + 1$

$= 17$ m

(c) $\underset{\sim}{x} = \int (3t^2 - 16t + 20)\, dt$

$= t^3 - 8t^2 + 20t + C$

when $t = 0$, $\underset{\sim}{x} = 0$,

$0 = C$

$\therefore \underset{\sim}{x} = t^3 - 8t^2 + 20t$

$\underset{\sim}{x} = t(t^2 - 8t + 20)$

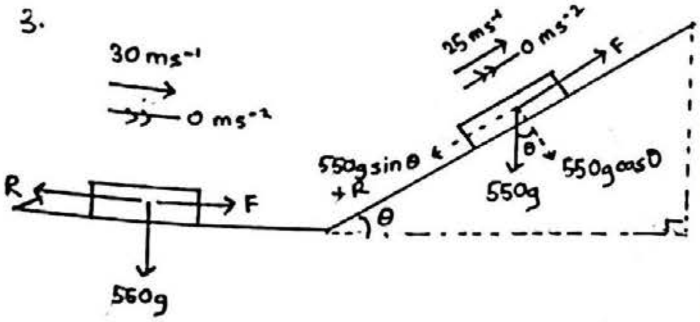
when $\underset{\sim}{x} = 0$,

$t = 0$ or $t^2 - 8t + 20 = 0$

$\Downarrow$

$b^2 - 4ac < 0 \Rightarrow$ roots are complex, hence, P does not return to O.

3.



(a) On horizontal road,

$R(\rightarrow): F - R = 0$

$F = R$

$P = Fv$

$P = R \times 30$

$= 30R$

On slope,

$R(\nearrow): F - 550g\sin\theta - R = 0$

$F = 550g\sin\theta + R$

$= \frac{550g}{14} + R$

$P = Fv$

$= \left(\frac{550g}{14} + R\right)(25)$

$= (385 + R)(25)$

$= 9625 + 25R$

(i) $30R = 9625 + 25R$

$5R = 9625$

$R = 1925$

$\therefore R \approx 1930\text{ N (3 SF)}$

(ii) $P = 30R$

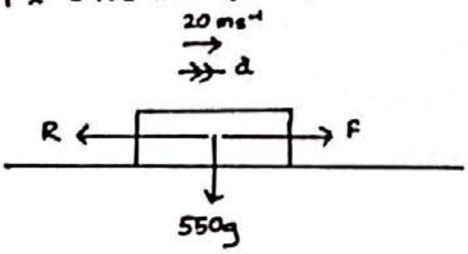
$= 30 \times 1925$

$= 57750\text{ W}$

$= 57.75\text{ kW}$

$\therefore P \approx 57.8\text{ kW (3 SF)}$

(b)



$P = Fv$

$50000 = F(20)$

$\therefore F = 2500\text{ N}$

$R(\rightarrow): F - R = 550a$

$2500 - 1925 = 550a$

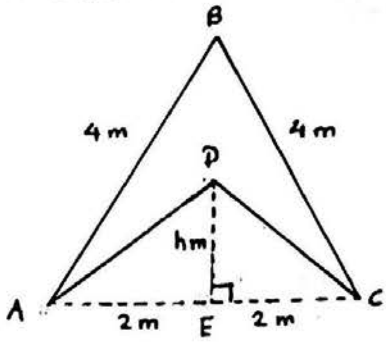
$a = \frac{575}{550}$

$= \frac{23}{22}$

$= 1.0454...$

$\therefore a \approx 1.05\text{ ms}^{-2}\text{ (3 SF)}$

4.



let mass/area be ρ

Area of $\triangle ACD = \frac{1}{2}(4)(h)$

$= 2h$

Considering $\triangle ABE$,

$\sin 60° = \frac{BE}{4}$

$BE = 4 \times \frac{\sqrt{3}}{2}$

$= 2\sqrt{3}\text{ m}$

Area of $\triangle ABC = \frac{1}{2}(4)(2\sqrt{3})$

$= 4\sqrt{3}\text{ m}^2$

Shape	Mass	Mass ratios	Distance of COM from AC
$\triangle ABC$	$4\rho\sqrt{3}$	$2\sqrt{3}$	$\frac{2\sqrt{3}}{3}$
$\triangle ACD$	$2\rho h$	h	$\frac{h}{3}$
ABCD	$4\rho\sqrt{3} - 2\rho h$	$2\sqrt{3} - h$	h

$M_{(A)}: (2\sqrt{3} - h)(h) = 2\sqrt{3}\left(\frac{2\sqrt{3}}{3}\right) - h\left(\frac{h}{3}\right)$

$2h\sqrt{3} - h^2 = 4 - \frac{h^2}{3}$

$0 = 4 - 2h\sqrt{3} + h^2 - \frac{h^2}{3}$

$\frac{2h^2}{3} - 2h\sqrt{3} + 4 = 0$

$2h^2 - 6h\sqrt{3} + 12 = 0$

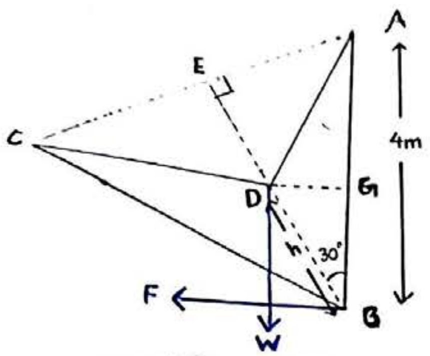
$h^2 - 3h\sqrt{3} + 6 = 0$

$h = \frac{3\sqrt{3} \pm \sqrt{(3\sqrt{3})^2 - 4(1)(6)}}{2(1)}$

$= \frac{3\sqrt{3} \pm \sqrt{3}}{2}$

$\therefore h = 2\sqrt{3}$ or $\sqrt{3}$ $\quad \therefore h = \sqrt{3}$ $[\because h < 2\sqrt{3}]$

(b)



considering ΔBDG,

$\sin 30° = \frac{DG}{h}$

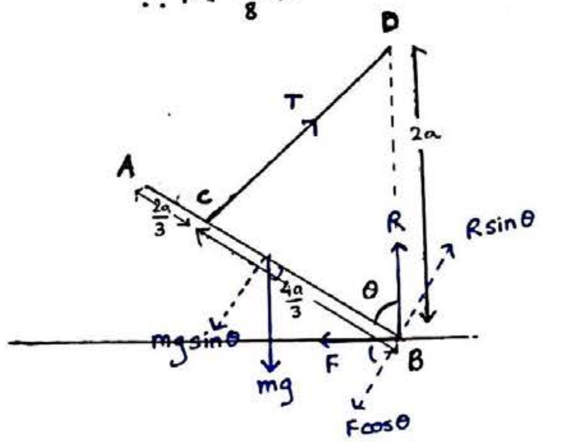
$DG = \frac{1}{2}\sqrt{3}$

$M_{(A)}$ +ve : $4F - \frac{W}{2}\sqrt{3} = 0$

$4F = \frac{W}{2}\sqrt{3}$

$\therefore F = \frac{W}{8}\sqrt{3}$ N

5.



(a) $M_{(D)}$ +ve : $2aF - mga\sin\theta = 0$

$2aF = mga\sin\theta$

$2F = mg\sin\theta$

$\therefore F = \frac{1}{2}mg\sin\theta$

(b) $M_{(C)}$ +ve : $\frac{4a}{3}R\sin\theta - \left(\frac{4a}{3} - a\right)mg\sin\theta - \frac{4a}{3}F\cos\theta = 0$

$\frac{4a}{3}R\sin\theta - \frac{a}{3}mg\sin\theta = \frac{4a}{3}F\cos\theta$

$\frac{\cancel{a}}{\cancel{3}}\cancel{\sin\theta}(4R - mg) = \frac{4\cancel{a}}{\cancel{3}} \times \frac{1}{2}mg\cancel{\sin\theta}\cos\theta$

$4R - mg = 2mg\cos\theta$

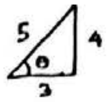
$4R = mg + 2mg\cos\theta$

$\therefore R = \frac{mg}{4}(1 + 2\cos\theta)$

(c) $F = \mu R$

$\frac{1}{2}\cancel{mg}\sin\theta = \frac{\mu \cancel{mg}}{4}(1 + 2\cos\theta)$

$2\sin\theta = \mu(1 + 2\cos\theta)$



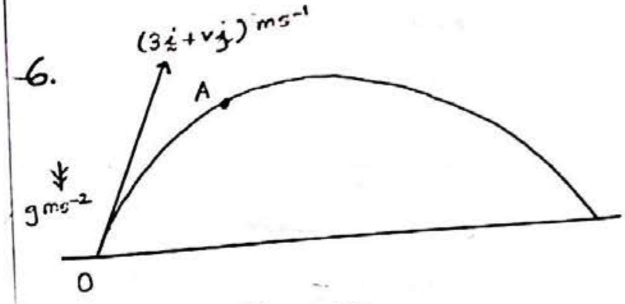
$2\left(\frac{4}{5}\right) = \mu\left[1 + 2\left(\frac{3}{5}\right)\right]$

$\frac{8}{5} = \frac{11\mu}{5}$

$11\mu = 8$

$\therefore \mu = \frac{8}{11}$

6.



(a) Conservation of energy:

$KE_O = KE_A + PE_A$

$E = \frac{1}{2}E + PE_A$

$\frac{1}{2}E = PE_A$

$R(\uparrow)$: $v = u + at$

$v_A = v - gt$

velocity at A is $3\underline{i} + (v - gt)\underline{j}$

$KE_O = 2KE_A$

$\frac{1}{2}m\left(\sqrt{3^2 + v^2}\right)^2 = \frac{1}{2}m(2)\left(\sqrt{3^2 + (v - gt)^2}\right)^2$

$3^2 + v^2 = 2\left(3^2 + (v - gt)^2\right)$

$9 + v^2 = 18 + 2(v^2 - 2gtv + g^2t^2)$

$9 + v^2 = 18 + 2\left(v^2 - 2gv\left(\frac{15}{49}\right) + \left(\frac{15}{49}g\right)^2\right)$

$9 + v^2 = 18 + 2v^2 - 12v + 18$

$\Rightarrow v^2 - 12v + 27 = 0$

$(v - 3)(v - 9) = 0$

$\therefore v = 9\ ms^{-1}$ $(\because v > 3)$

(b) $R(\uparrow)$: $v_A = u + at$

$= 9 - g\left(\frac{15}{49}\right)$

$= 6$

The ball moves downwards at B,

$\therefore\ v_B = -6\ ms^{-1}$

$R(\uparrow)$: $v = u + at$

$-6 = 9 - gt$

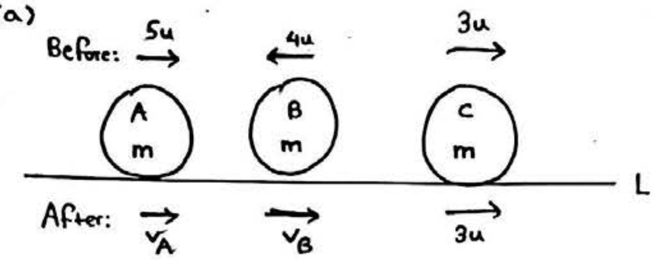
$-15 = -gt$

$t = \frac{15}{g}$

$t = \frac{75}{49} = 1.5306...$

$\therefore\ t \approx 1.53\ s$ (3 SF)

7. (a)



PCLM($\rightarrow$):

$\cancel{m}(5u) + \cancel{m}(-4u) = \cancel{m}v_A + \cancel{m}v_B$

$5u - 4u = v_A + v_B$

$\therefore\ v_A + v_B = u$ ——— (I)

NEL ($\rightarrow$):

$e = \frac{v_B - v_A}{5u + 4u}$

$\therefore\ -v_A + v_B = 9ue$ ——— (II)

(i) (I) $\Rightarrow$ $v_A + \cancel{v_B} = u$

(II) $\Rightarrow$ $-v_A + \cancel{v_B} = 9ue$

(−) $2v_A = u - 9ue$

$v_A = \frac{u}{2}(1 - 9e)$

$\therefore\ |v_A| = \left|\frac{u}{2}(1 - 9e)\right|\ ms^{-1}$

(ii) (I) $\Rightarrow$ $\cancel{v_A} + vb = u$

(II) $\Rightarrow$ $-\cancel{v_A} + vb = 9ue$

(+) $2v_B = u + 9ue$

$\therefore\ v_B = \frac{u}{2}(1 + 9e)\ ms^{-1}$

(b) dir^n of A is reversed $\Rightarrow$ $v_A < 0$

$\frac{u}{2}(1 - 9e) < 0$

$1 - 9e < 0$

$1 < 9e$

$\therefore\ \frac{1}{9} < e$

no collision between B and C $\Rightarrow$ $v_C \geq v_B$

$3u \geq \frac{u}{2}(1 + 9e)$

$6 \geq 1 + 9e$

$5 \geq 9e$

$\frac{5}{9} \geq e$

$\therefore\ e \leq \frac{5}{9}$

$\therefore$ Possible values of e:

$\frac{1}{9} < e \leq \frac{5}{9}$