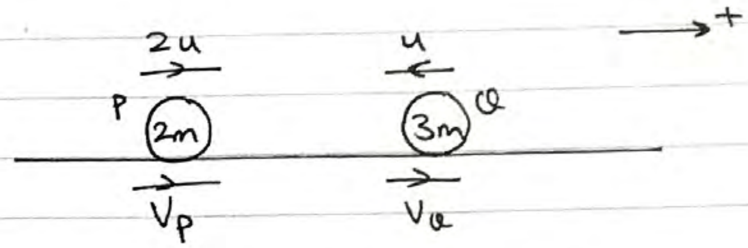
M1 October 2016 (IAL) (MA)

Q1(a)



$\overrightarrow{C.L.M}$: $2\cancel{m}(2u) + 3\cancel{m}(-u) = 2\cancel{m}(V_p) + 3\cancel{m}(V_Q)$

$\Rightarrow u = 2V_p + 3V_Q$

Impulse on P acts towards the left.

$\therefore -5mu = 2m(v-u) = -5\cancel{m}u = 2\cancel{m}(V_p - 2u)$

$\Rightarrow -5u = 2V_p - 4u$

$\Rightarrow 2V_p = -u \quad \therefore V_p = -\frac{u}{2}$

so speed of P $= \boxed{\frac{u}{2}}$

b) $V_p < 0$ so P is travelling in the opposite direction to that which we assumed in the diagram. So yes the dir. of P has been reversed.

c) from (a), $u = 2V_p + 3V_Q$

$\Rightarrow u = 2\left(-\frac{u}{2}\right) + 3V_Q$

$\Rightarrow 3V_Q = 2u \quad \therefore V_Q = \boxed{\frac{2u}{3}} = \text{speed}_Q$.

Q2a) $\sum F = ma$: $\overbrace{\begin{pmatrix} -10 \\ a \end{pmatrix} + \begin{pmatrix} b \\ -5 \end{pmatrix} + \begin{pmatrix} 2a \\ 7 \end{pmatrix}}^{\sum F} = \underbrace{3}_{m} \overbrace{\begin{pmatrix} 3 \\ 4 \end{pmatrix}}^{a}$

$\Rightarrow \begin{pmatrix} 2a + b - 10 \\ 2 + a \end{pmatrix} = \begin{pmatrix} 9 \\ 12 \end{pmatrix}$

equating $\underline{i}$: $2a + b - 10 = 9$

$\Rightarrow 2a + b = 19 \sim ①$

equating $\underline{j}$: $2 + a = 12$

$\Rightarrow \boxed{a = 10}$ //

① : $2(10) + b = 19$

so $\boxed{b = -1}$ //

b) $a = \frac{v-u}{t}$: $3\underline{i} + 4\underline{j} = \frac{(20\underline{i} + 20\underline{j}) - (\vec{u})}{(4)}$

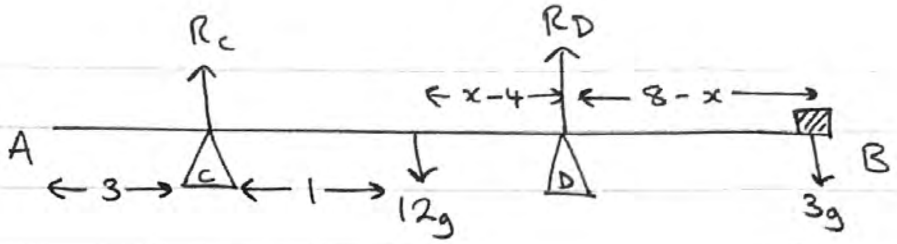
$\overset{\times 4}{\Rightarrow}$ $12\underline{i} + 16\underline{j} = 20\underline{i} + 20\underline{j} - (\vec{u})$

$\Rightarrow \vec{u} = 20\underline{i} - 12\underline{i} + 20\underline{j} - 16\underline{j}$

$\Rightarrow \vec{u} = 8\underline{i} + 4\underline{j}$ //

hence $u = |\vec{u}| = \sqrt{8^2 + 4^2} = \boxed{4\sqrt{5}}$ ms⁻

Q3)



we are told: $R_D = 2R_c$

$R(\updownarrow)$: $R_c + R_D = 12g + 3g$

$R_c + 2R_c = 15g$

$\therefore R_c = \frac{15g}{3} = 5g$ $\rightarrow R_D = 10g$

$M(A)$: $R_c(3) + R_D(x) = 12g(4) + 3g(8)$

$5\cancel{g}(3) + 10\cancel{g}(x) = 48\cancel{g} + 24\cancel{g}$

$10x + 15 = 72$

$10x = 57 \quad \therefore x = \frac{57}{10} = \boxed{5.7m}$

Q4a) $p = r_0 + vt$

$p = \begin{pmatrix} -5 \\ 9 \end{pmatrix} + t\begin{pmatrix} 1 \\ -2 \end{pmatrix} \Rightarrow \boxed{p = (-5+t)\underline{i} + (9-2t)\underline{j}}$

b) $\underline{j} = 2$ when P is due west of A.

$\therefore 9 - 2t = 2$

$\Rightarrow 7 = 2t \quad \therefore t = \frac{7}{2}$

at $t = \frac{7}{2}$, $p = (-5 + \frac{7}{2})\underline{i} + (9-7)\underline{j}$

$\boxed{p = -\frac{3}{2}\underline{i} + 2\underline{j}}$

c) if they move along parallel lines then their velocity vectors are parallel.

$\Rightarrow \begin{pmatrix} 2b-1 \\ 5-2b \end{pmatrix} = c\begin{pmatrix} 1 \\ -2 \end{pmatrix}$ for some c.

$\therefore \quad 2b - 1 = c \quad - ① \quad (\text{equating } \underline{i})$

$5 - 2b = -2c \quad - ② \quad (\text{equating } \underline{j})$

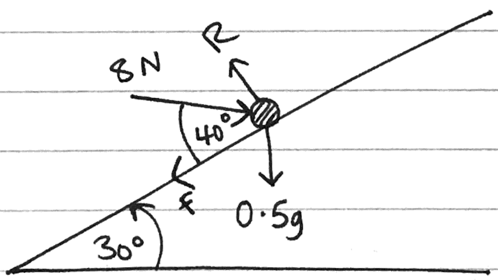
$\underline{① + ②}: \quad \cancel{2b} - 1 + 5 - \cancel{2b} = c - 2c$

$4 = -c \qquad \therefore c = -4$

and $2b - 1 = -4$

$\Rightarrow \quad b = \frac{-4+1}{2} = \boxed{-\frac{3}{2}}$

Q5)



$R(\updownarrow): \quad R = (8\sin 40 + 0.5g\cos 30)$

$R(\swarrow\nearrow): \quad 8\cos 40 = F + 0.5g\sin 30$

but $F = \mu R$ as P is on the point of sliding up the plane.

$\therefore \; 8\cos 40 = \mu R + 0.5g\sin 30$

$8\cos 40 = \mu(8\sin 40 + 0.5g\cos 30) + 0.5g\sin 30$

so $\mu = \dfrac{8\cos 40 - 0.5g\sin 30}{8\sin 40 + 0.5g\cos 30}$

$= \boxed{0.392}$

Q6) For A:

$s = s_A$
$u = 35$
$v = v_A$
$a = 0.4$
$t =$

For B:

$s = s_B$
$u = 44$
$v = v_B$
$a = 0.5$
$t =$

When B overtakes A, $s_B - 200 = s_A$ // (Since B starts 200m behind)

$s_A = 35t + 0.2t^2$

and $s_B = 44t + 0.25t^2$

but using $s_B - 200 = s_A$,

$\Rightarrow \; 44t + 0.25t^2 - 200 = 35t + 0.2t^2$

$\Rightarrow \; 0.05t^2 + 9t - 200 = 0$

$\times 20 \Rightarrow \; t^2 + 180t - 4000 = 0$

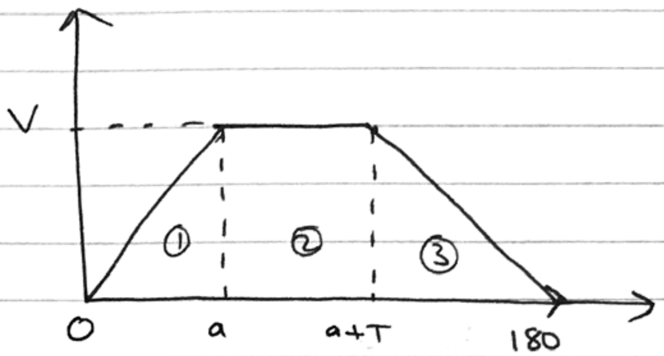
By Quadratic Formula:

$t = 20$ // or $\underbrace{t = -200}_{t > 0}$

So at $t = 20$ B overtakes A.

using $v = u + at$ with $B = V_B = 44 + 0.5(20)$
$= \boxed{54}$

(Q7a)



b) $a = \frac{v-u}{t}$ for ①: $1 = \frac{V-0}{a}$

$\therefore V = a.$

$a = \frac{v-u}{t}$ for ③: $-0.5 = \frac{0-V}{180-(V+T)}$

$-90 + 0.5(T+V) = -V$

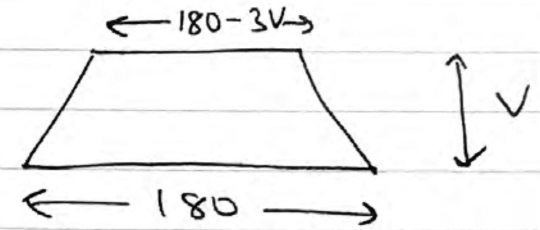
$90 - \frac{1}{2}(T+V) = V$

$180 - T - V = 2V$

$\therefore T = 180 - 3V$

c) Total area under graph $= 4800$ //

Area of trapezium $= \dfrac{(a+b)h}{2}$ $(= 4800)$



$\therefore\ 4800 = \frac{1}{2}(180 + 180 - 3v)V$

$9600 = 360V - 3V^2$

$(\div 3):\quad 3200 = 120V - V^2$

$V^2 - 120V + 3200 = 0$

$(V-40)(V-80) = 0$

so $\boxed{V = 40}$ or $V = 80$

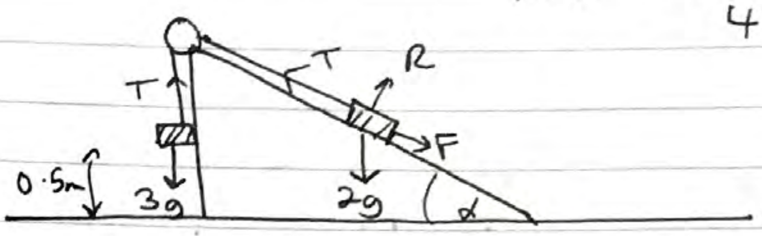
can't be $V=80$ as then area of (1) would be greater than 4800m which isn't possible.

Q8a)

$\cos\alpha = \frac{4}{5}$

$\sin\alpha = \frac{3}{5}$

$\tan\alpha = \frac{3}{4}$



N2L(Q): $3g - T = 3a$ ~ ①

N2L(P): $T - 2g\sin\alpha - F = 2a$ ~ ②

(Q)
$s = 0.5$
$u = 0$
$v = 1.4$
$a = a$
$t =$

$v^2 = u^2 + 2as$

$1.4^2 = 0^2 + 2a(0.5)$

$1.4^2 = a$ //

from ①: $T = 3g - 3a = 3g - 3(1.4^2)$

$= \boxed{23.5N}$

b) ②: $T - 2g\sin\alpha - \mu R = 2a$

$23.5 - 2g\left(\frac{3}{5}\right) - \mu\left(2g\cos\alpha\right) = 2(1.4^2)$

$23.5 - \frac{6g}{5} - \mu\left(2g \times \frac{4}{5}\right) = 2(1.4^2)$

$\therefore \mu = \dfrac{23.5 - \frac{6g}{5} - 2(1.4^2)}{2g \times \frac{4}{5}} = \boxed{0.5}$