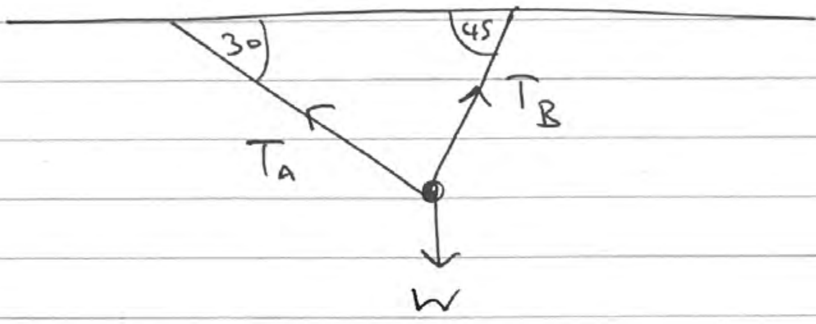
M1 January 2018 (IAL) (MA)

Q1 i)



$R(\updownarrow)$: $T_A \sin 30 + T_B \sin 45 = W$

$$\frac{T_A}{2} + T_B \frac{\sqrt{2}}{2} = W \sim ①$$

$R(\leftrightarrow)$: $T_A \cos 30 = T_B \cos 45$

$$\therefore T_A \cdot \frac{\sqrt{3}}{2} = T_B \cdot \frac{\sqrt{2}}{2}$$

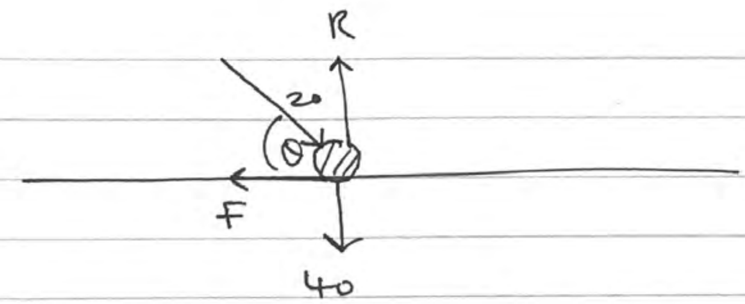
subbing into ① : $\frac{T_A}{2} + T_A \cdot \frac{\sqrt{3}}{2} = W$

$$T_A\left(\frac{\sqrt{3}+1}{2}\right) = W$$

so $\boxed{T_A = \frac{2W}{\sqrt{3}+1}} = 0.73W$

ii) $T_B = T_A \cdot \sqrt{\frac{3}{2}} = \boxed{\frac{2W\sqrt{3}}{\sqrt{6}+\sqrt{2}}} = 0.90W$

Q2)



$F \leq \mu R$. so $\mu \geq \frac{F}{R}$

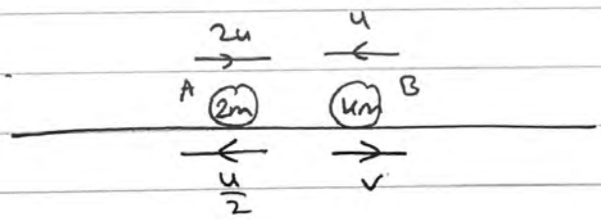
$R(\leftrightarrow)$: $20\cos\theta = F$

$R(\updownarrow)$: $20\sin\theta + 40 = R$

so $\mu \geq \dfrac{20\cos\theta}{20\sin\theta + 40}$

$\overset{\div 20}{\Longrightarrow}$ $\mu \geq \dfrac{\cos\theta}{\sin\theta + 2}$

Q3a)



Impulse (on A) $= 2m(v-u) = 2m\left(\frac{u}{2} - -2u\right)$

$\therefore I = \boxed{5mu}$

b) C.L.M : $2m(2u) - km(u) = 2m\left(-\frac{u}{2}\right) + km(v)$

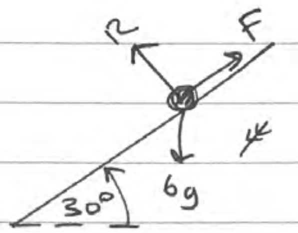
$5mu - kmu = kmv$

so $v = \frac{5u - uu}{u}$ //

but $v > 0 \rightarrow 5u - uu > 0$
(since B's dir is reversed)

$\therefore \boxed{u < 5}$

Q4a)



$+\swarrow$ N2L (package) : $6g\sin 30 - F = 6a$

$F = \mu R = \frac{1}{4} \times 6g\cos 30$ //

$\therefore a = \frac{\frac{6g}{2} - \frac{6g\cos 30}{4}}{6}$

$a \simeq \boxed{2.78 ms^{-2}}$

b)

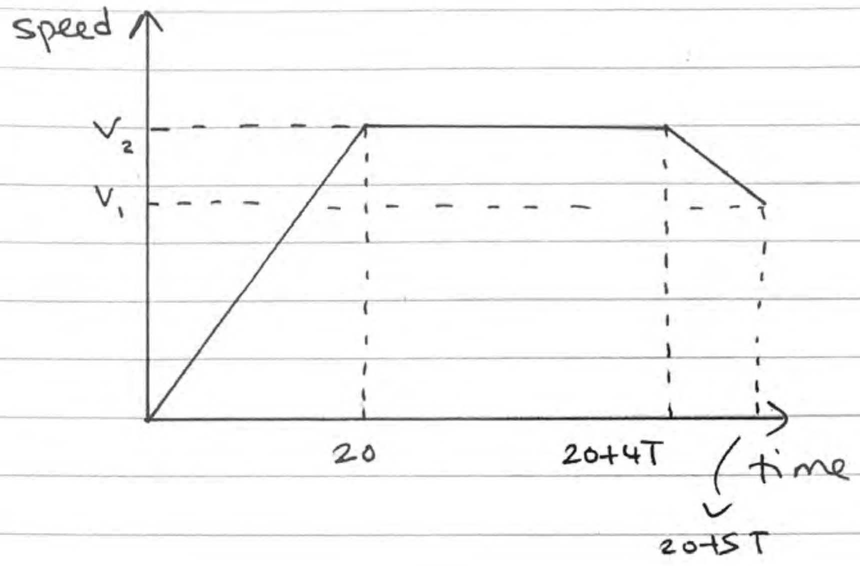
$s = 10$
$u = 0$
$v = v$
$a = 2.78$
$t =$

$v^2 = u^2 + 2as$

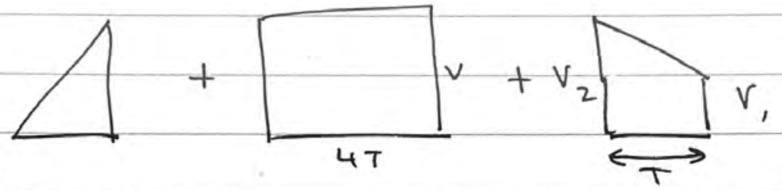
$v^2 = 2(10)(2.78)$

$\therefore \boxed{v \simeq 7.45 m/s}$

Q5a)



b) Total area = Total distance = 705m



$$\left(\tfrac{1}{2} \times 20 \times V_2\right) + (4T \times V_2) + \frac{(V_1 + V_2)(T)}{2} = 705$$

finding V_2 :

$s =$
$u = 0$
$v = V_2$
$a = 0.6$
$t = 20$

$V_2 = u + at$

$V_2 = 0.6 \times 20 = 12\,ms^{-1}$ //

finding V_1 :

$s =$
$u = 12$
$v = V_1$
$a = -0.3$
$t = T$

$V_1 = u + at$

$V_1 = 12 - 0.3T$ //

so $10(12) + 4T(12) + \frac{(24-0.3T)(T)}{2} = 705$

$120 + 48T + 12T - 0.15T^2 = 705$

$0.15T^2 - 60T + 585 = 0$

By Quadratic formula : $T = 390$
$T = 10$

$T < 20$ so $\boxed{T = 10}$

c) from B to C :

$$\left.\begin{aligned} s &= \\ u &= 9 \\ v &= 0 \\ a &= -0.3 \\ t &= t \end{aligned}\right\} \quad \begin{aligned} v &= u + at \\ 0 &= 9 - 0.3t \\ t &= \frac{9}{0.3} = 30s \end{aligned}$$

so total time from A to C $= 20 + 5(10) + 30$
$= \boxed{100s}$

Q6a) $\Sigma F = ma$: $\begin{pmatrix} 2 \\ 3 \end{pmatrix} + \begin{pmatrix} 4 \\ -5 \end{pmatrix} = \Sigma f = 6\underline{i} - 2\underline{j}$

$|F| = \sqrt{6^2 + 2^2} = 2\sqrt{10} = ma$

$2\sqrt{10} = 2a$ $\therefore \boxed{a = \sqrt{10}\ m/s^2}$

bi) $s =$

$u = -u\underline{i} + u\underline{j}$

$v = 10\underline{i} + 2\underline{j}$

$a = 3\underline{i} - \underline{j}$

$t = T$

$v = u + at$

$10\underline{i} + 2\underline{j} = -u\underline{i} + u\underline{j} + 3T\underline{i} - T\underline{j}$

$10\underline{i} + 2\underline{j} = (3T - u)\underline{i} + (u - T)\underline{j}$

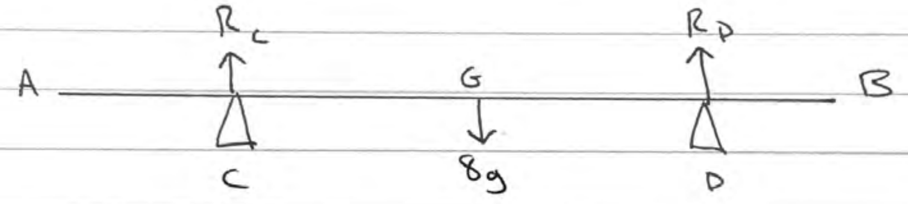
compare $\underline{i}$: $10 = 3T - u$ ~ ①

compare $\underline{j}$: $2 = u - T$ ~ ②

$\underline{① + ②}$: $12 = 2T$ ∴ $\boxed{T = 6}$ //

② : $2 = u - 6$ ∴ $\boxed{u = 8}$ //

Q7a)



$\underline{R_D = 2R_c}$

$R(\updownarrow)$: $R_c + R_D = 8g$ ∴ $3R_c = 8g$ //

$\hookrightarrow R_c = \frac{8g}{3}$

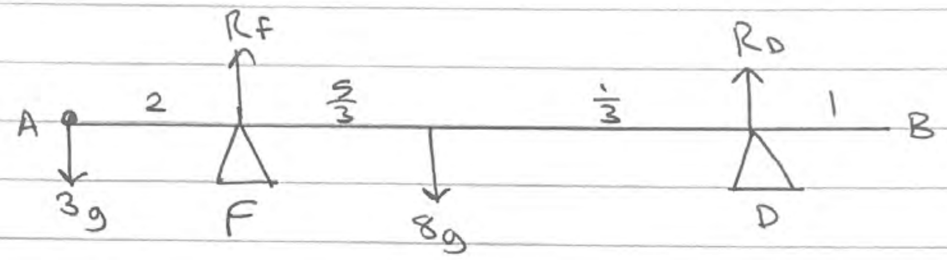
$M(C)$: $8g(x-1) = R_D(4)$

$8g(x-1) = \frac{64g}{3}$

$\therefore x - 1 = \frac{64g}{3 \times 8g} = 8/3$ //

$\therefore\ x = 1 + \frac{8}{3} = \boxed{\frac{11}{3}}$

b)



$R(\updownarrow):\quad R_F + R_D = 11g$

and $\quad R_D = kR_F$

$\therefore\quad (k+1)R_F = 11g$ //

$M(F):\quad 8g\left(\frac{5}{3}\right) = R_D(3) + 3g(2)$

$\frac{22g}{3} = 3R_D \quad \therefore R_D = \frac{22g}{9}$ //

so $\quad R_F = \frac{22g}{9k}$

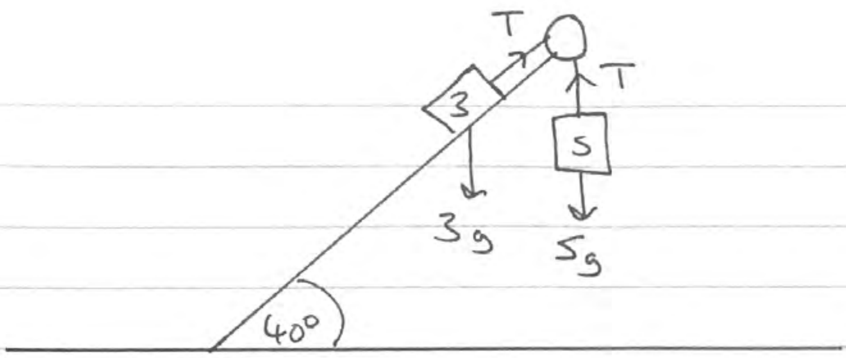
$\Rightarrow \quad \frac{11\cancel{g}}{k+1} = \frac{22\cancel{g}}{9k}$

$\frac{k+1}{11} = \frac{9k}{22}$

$\Rightarrow\ 2k + 2 = 9k$

$\Rightarrow\ 7k = 2 \qquad \therefore \boxed{k = \frac{2}{7}}$

Q8a)



$\nearrow^{+}$ N2L(A): $T - 3g\sin 40 = 3a$ ~ ①

$\downarrow_{+}$ N2L(B): $5g - T = 5a$ ~ ②

① + ②: $5g - 3g\sin 40 = 8a$

$\therefore a = 3.76..\ ms^{-2}$

from ② → $T = 5g - 5(3.76) \simeq \boxed{30.2N}$

b)
$s =$
$u = 0$
$v = v$
$a = 3.76...$
$t = 1.5$

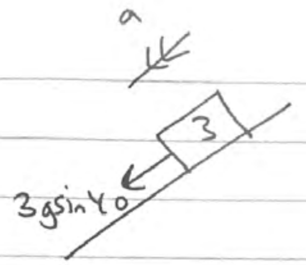
$v = u + at$
$v = 0 + (3.76...) \times 1.5 \simeq \boxed{5.6ms^{-1}}$

c) for first 1.5s:
$s = \cancel{d}$
$u = 0$
$v =$
$a = 3.76..$
$t = 1.5$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(3.76..)(1.5)^2$

$= 4.23$



New a : ↙+ N2L(A) : $3g\sin 40 = 3a$

$\therefore a = g\sin 40$

so for the rest of the motion:

$s = d$
$u = 5.6$
$v \quad 0$
$a = -g\sin 40$
$t =$

$v^2 = u^2 + 2as$

$0^2 = (5.6)^2 - 2gd\sin 40$

$d = \dfrac{5.6^2}{2g\sin 40} \simeq 2.52$

so total distance $= 2.52 + 4.23$

$\simeq \boxed{6.8m}$