MI JAN14 (INT)

1. A truck P of mass $2M$ is moving with speed U on smooth straight horizontal rails. It collides directly with another truck Q of mass $3M$ which is moving with speed $4U$ in the opposite direction on the same rails. The trucks join so that immediately after the collision they move together. By modelling the trucks as particles, find

(a) the speed of the trucks immediately after the collision, (3)

(b) the magnitude of the impulse exerted on P by Q in the collision. (3)
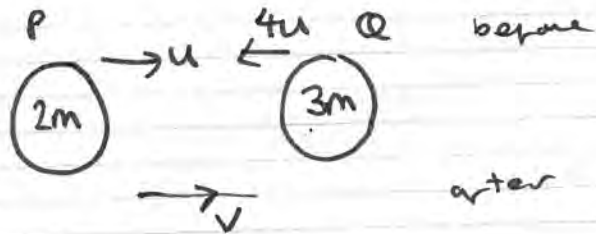
Total Mom before $= 2mu - 12mu$
$= -10mu.$

Total Mom after $= +5mv$

CLM $\Rightarrow$ $-10Mu = +5Mv$

$v = -2u$ $\therefore$ Speed $= 2u$

b) Mom P before $= 2Mu$ $\therefore$ Impulse
Mom P after $= -4Mu$ $= 6Mu$

2. A particle P is moving with constant velocity $(2i - 3j)$ m s^{-1}.

(a) Find the speed of P. (2)

The particle P passes through the point A and 4 seconds later passes through the point with position vector $(i - 4j)$ m.

(b) Find the position vector of A. (4)

a) speed $= \sqrt{2^2 + 3^2} = 3.6$ ms^{-1}

b) $\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 2 \\ -3 \end{pmatrix} \times 4 = \begin{pmatrix} 1 \\ -4 \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ -4 \end{pmatrix} - \begin{pmatrix} 8 \\ -12 \end{pmatrix} = \begin{pmatrix} -7 \\ 8 \end{pmatrix}$

3. A beam AB has length 15 m and mass 25 kg. The beam is smoothly supported at the point P, where $AP = 8$ m. A man of mass 100 kg stands on the beam at a distance of 2 m from A and another man stands on the beam at a distance of 1 m from B. The beam is modelled as a non-uniform rod and the men are modelled as particles. The beam is in equilibrium in a horizontal position with the reaction on the beam at P having magnitude 2009 N. Find the distance of the centre of mass of the beam from A. (5)
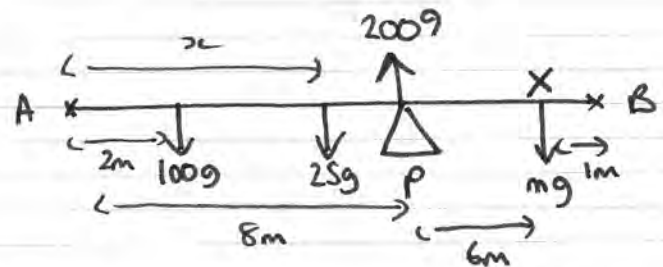
$\stackrel{\curvearrowright}{X}$ $\checkmark$ $2009 \times 6 = 25g(14-x) + 100g \times 12$

$12054 = 3430 - 245x + 11760$

$\therefore 245x = 3136$ $\Rightarrow x = 12.8$
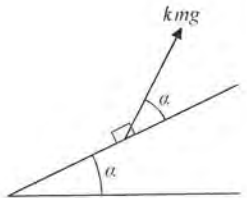
4.

kmg

Figure 1

A fixed rough plane is inclined to the horizontal at an angle α, where $\tan \alpha = \dfrac{3}{4}$.

A small box of mass m is at rest on the plane. A force of magnitude kmg, where k is a constant, is applied to the box. The line of action of the force is at angle α to the line of greatest slope of the plane through the box, as shown in Figure 1, and lies in the same vertical plane as this line of greatest slope. The coefficient of friction between the box and the plane is μ. The box is on the point of slipping up the plane. By modelling the box as a particle, find k in terms of μ.

(11)

$\tan\alpha = \dfrac{3}{4} \Rightarrow \sin\alpha = 0.6$

$\cos\alpha = 0.8$

$kmg \times 0.6$
$+NR$
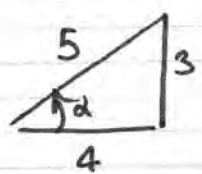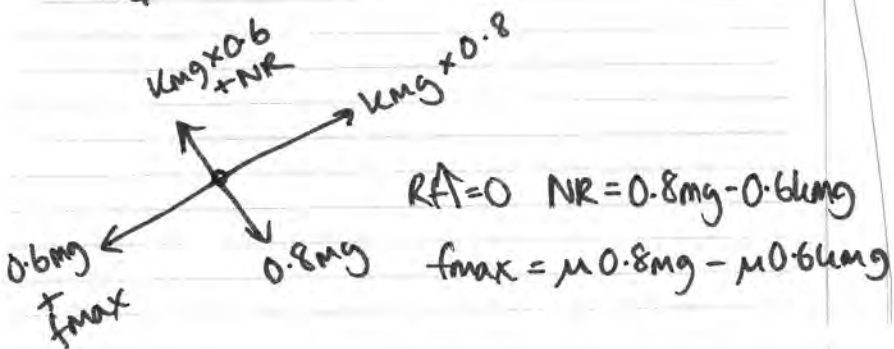
$kmg \times 0.8$

$R\uparrow = 0 \quad NR = 0.8mg - 0.6kmg$

$0.6mg$
f_{max}

$0.8mg$

$f_{max} = \mu 0.8mg - \mu 0.6kmg$

$R\nearrow = 0 \quad 0.8\mu mg - 0.6\mu kmg + 0.6mg = 0.8kmg$

$(\div mg) \quad 0.8\mu - 0.6\mu k + 0.6 = 0.8k$

$\Rightarrow \quad 0.8\mu + 0.6 = 0.8k + 0.6\mu k$

$\Rightarrow \quad 0.8\mu + 0.6 = k(0.8 + 0.6\mu)$

$\therefore \quad k = \dfrac{0.8\mu + 0.6}{0.8 + 0.6\mu}$

5. A racing car is moving along a straight horizontal track with constant acceleration. There are three checkpoints, P, Q and R, on the track, where $PQ = 48$ m and $QR = 200$ m. The car takes 3 s to travel from P to Q and 5 s to travel from Q to R. Find

(i) the acceleration of the car,

(ii) the speed of the car as it passes P.

(7)

$\overrightarrow{PQ}$	$\overrightarrow{PR}$
$S = 48$	$S = 248$
u *	u *
v	v
a *	a *
$t = 3$	$t = 8$

$S = ut + \tfrac{1}{2}at^2$

$\overrightarrow{PQ} \Rightarrow 48 = 3u + \dfrac{9}{2}a \Rightarrow 96 = 6u + 9a$

$\qquad\qquad\qquad\qquad\qquad\qquad 32 = 2u + 3a$

$\overrightarrow{PR} \quad 248 = 8u + 32a \Rightarrow 31 = u + 4a$

$\Rightarrow \quad 62 = 2u + 8a$

$\qquad\quad 32 = 2u + 3a$

$\qquad\overline{\quad 30 = 5a \quad} \quad \therefore \underline{a = 6}$

$\qquad\qquad\qquad \therefore u = 31 - 24 = \underline{7}$

6.

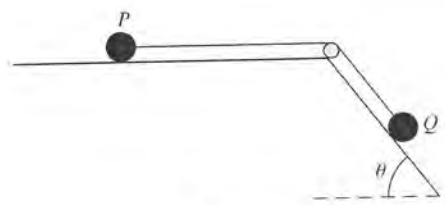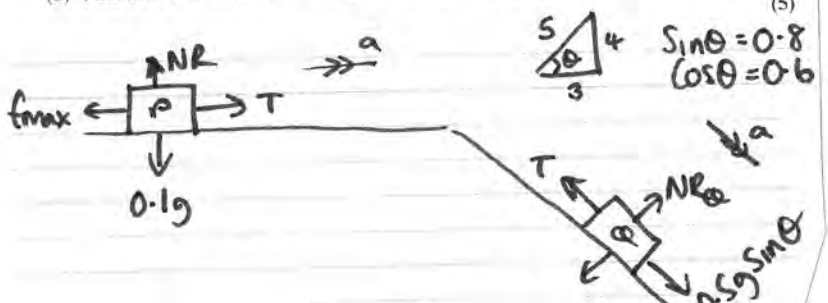**Figure 2**

Two particles P and Q have masses 0.1 kg and 0.5 kg respectively. The particles are attached to the ends of a light inextensible string. Particle P is held at rest on a rough horizontal table. The string lies along the table and passes over a small smooth pulley which is fixed to the edge of the table. Particle Q is at rest on a smooth plane which is inclined to the horizontal at an angle θ, where $\tan\theta = \dfrac{4}{3}$.

The string lies in the vertical plane which contains the pulley and a line of greatest slope of the inclined plane, as shown in Figure 2. Particle P is released from rest with the string taut. During the first 0.5 s of the motion P does not reach the pulley and Q moves 0.75 m down the plane.

(a) Find the tension in the string during the first 0.5 s of the motion.

(6)

(b) Find the coefficient of friction between P and the table.

(5)

$S_{in}\theta = 0.8$
$\cos\theta = 0.6$

$S = 0.75$
$u = 0$
v
a
$t = 0.5$

$v = u + at$

$S = \dfrac{(u+v)}{2}t$

$0.75 = \dfrac{V \times 0.5}{2}$

$\therefore V = 3$

$3 = 0 + \dfrac{1}{2}a \quad \therefore a = 6$

$0.5g\cos\theta$

Ⓠ $Rf \downarrow = ma$

$0.5g \times 0.8 - T = 0.5 \times 6$

$3.92 - T = 3 \qquad \therefore T = 0.92 \text{ N}$

Ⓟ $T - f_{max} = ma$

$\Rightarrow 0.92 - \mu \times 0.98 = 0.1 \times 6$

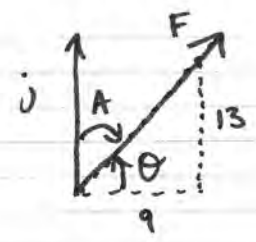$\Rightarrow 0.32 = \mu \times 0.98 \qquad \therefore \mu = 0.33$

7. A force **F** is given by $\mathbf{F} = (9\mathbf{i} + 13\mathbf{j})$ N.

(a) Find the size of the angle between the direction of **F** and the vector **j**.

(3)

The force **F** is the resultant of two forces **P** and **Q**. The line of action of **P** is parallel to the vector $(2\mathbf{i} - \mathbf{j})$. The line of action of **Q** is parallel to the vector $(\mathbf{i} + 3\mathbf{j})$.

(b) Find, in terms of **i** and **j**,

(i) the force **P**,

(ii) the force **Q**.

(9)

$A = 90 - \tan^{-1}\left(\dfrac{13}{9}\right)$

$A = 034.7 \qquad 035$

$\mu\begin{pmatrix}2\\-1\end{pmatrix} + \lambda\begin{pmatrix}1\\3\end{pmatrix} = \begin{pmatrix}9\\13\end{pmatrix}$

$2\mu + \lambda = 9$
$-\mu + 3\lambda = 13$

$\begin{array}{r} 2\mu + \lambda = 9 \\ + \ -2\mu + 6\lambda = 26 \\ \hline 7\lambda = 35 \\ \lambda = 5 \\ \Rightarrow \mu = 2 \end{array}$

$P\begin{pmatrix}4\\2\end{pmatrix} \qquad Q\begin{pmatrix}5\\15\end{pmatrix}$

8. Two trains, A and B, start together from rest, at time $t = 0$, at a station and move along parallel straight horizontal tracks. Both trains come to rest at the next station after 180 s.

Train A moves with constant acceleration $\frac{2}{3}$ m s^{-2} for 30 s, then moves at constant speed for 120 s and then moves with constant deceleration for the final 30 s. Train B moves with constant acceleration for 90 s and then moves with constant deceleration for the final 90 s.

(a) Sketch, on the same axes, the speed–time graphs for the motion of the two trains between the two stations.

(3)

(b) Find the acceleration of train B for the first half of its journey.

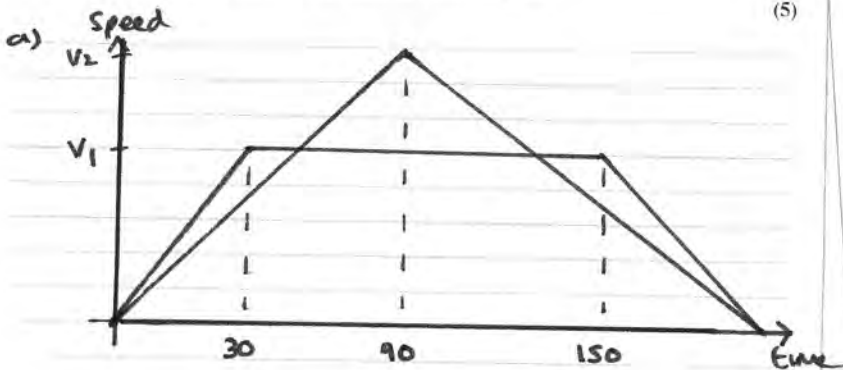(5)

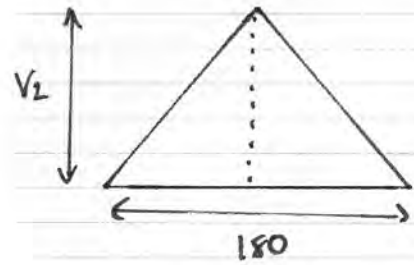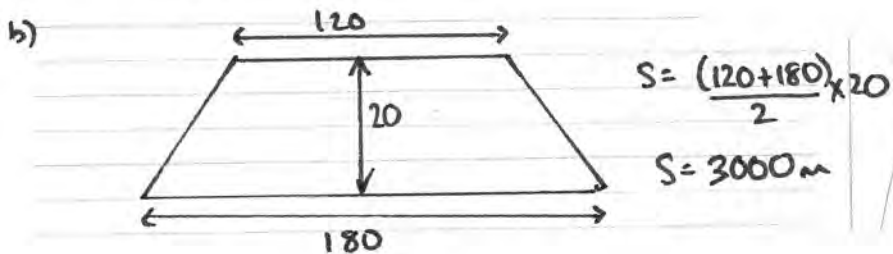(c) Find the times when the two trains are moving at the same speed.

(4)

(d) Find the distance between the trains 96 s after they start.

(5)

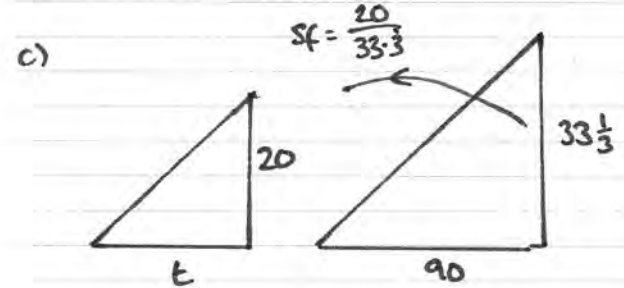a)

$V_1 = \frac{2}{3} \times 30 = 20$

b)

$S = \frac{(120+180)}{2} \times 20$

$S = 3000 \text{ m}$

$90 V_2 = 3000$

$\therefore V_2 = 33.\dot{3}$

$acc = \frac{33.\dot{3}}{90}$

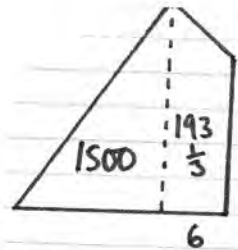$a = \frac{10}{27}$.

c)

$sf = \frac{20}{33.\dot{3}}$

$t = 90 \times \frac{20}{33.\dot{3}}$

$t = 54$

d)
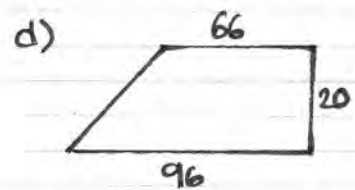
$S = \frac{(66+96)}{2} \times 20$

$= 1620 \text{ m}$

$\bar{u} = 33\frac{1}{3}$

v

$a = -\frac{10}{27}$

$t = 6$

$S = ut + \frac{1}{2}at^2$

$S = 200 - \frac{20}{3}$

$S = 193\frac{1}{3}$

$\Rightarrow 1693\frac{1}{3}$

difference $= 73\frac{1}{3}$ m