Surname	Other	names
Pearson Edexcel International Idvanced Level	Centre Number	Candidate Number
Further Pu	Ire	
Mathemate Advanced/Advanced	tics F3	
Mathemat	tics F3	Paper Reference WFM03/01

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer all questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
 there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for each question are shown in brackets
 use this as a guide as to how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶





$18\cosh x + 14\sin x$	$\sinh x = 11 + e^x$
10 COSH X + 143	$\min x - 11 + C$
Give your answers in the form ln a, where	
	1

	ì	e	ė	ď	ď
	٩	7	٠	4	ú
	1	Ŀ	ı		ı
٠	1	Ξ		3	2
]	c		c	_
۰	ľ	7	_	Ξ	ā
	J	ξ	S		
				7	7
	4	r	A	я	h
		ч	ę		ė
	3		3		2
	1	7		г	7
			ė		į
	Ì	ŀ		ė	
	J	۰			
	3	ė	٠	ġ	p
	4	ď		_	
	á	ė		Ė	i
	1	Ľ		Ľ	ı
	ŝ	7	-	7	7
	ł	P	ę	۰	ij
	i	i		ė	
	1	À	٩	è	ø
		Ŀ	d	b	
	į	3	3	Ė	h
	1	ε	Ę	S	á
	J	۹	7	7	Ξ
	1	Ĺ	٠	Ĺ	-
	1	۲	۰	1	9
	i	P	ø	۹	k
	-1	b	ė	ø	ď
	7	÷	ę	3	p
	ì	₫	ď		i
	•	ı,	•		٠
	j	Р		9	Š
	1	ø	ú	ø	7
	ŭ	ρ	۹	۹	Ň
	1	Ь	į		į

		_	ы	d		
×	j			r		
1		7	7	9	٩	
2			L	4		
ζ.	٩	7	₹	7	=	
2	1	C		2	2	
	8	7	₹		2	
-	4	'n	ø	ρ	Ŗ	
×	3	۹	ы	K	ü	
X	į	/		5		
×	Ì		ø	۹	h	•
Х	3	,	n		9	,
1	9	e	٠	P	8	,
ζ.	1	9		Ę		
2	í	ė	B,	è	ř	
	i	D				
٥	1	r	۰	7	×	
				*		
×	į	ź			ij.	4
×	ì		S	2	_	
X	ī	Ž	_	ä	=	,
×	5	×	٦	7	~	
	١	v	б		i	
	4	þ	ě	ĕ	В	
	ł	D		٩.		
N		۳	•	7	ĕ	
Τ	ŝ	4		6		
X	4	Z	S	ú	ä	,
				2	$\frac{x}{a}$	
1		4	S	Z	7	
	ς	=			R	
۲.	3	۰,	Ħ	ŝ	ĸ	
×	Į	P	7	7		
/	ì	К				
×		×	ì,	п	Ŕ	,
2	۹	ы	_	ż	×	
V	1		7	3		
	₹	٠	ú	d	R	
	ą	ì	ė	ģ	ø	
×	d	é	e	5	ż	^
	5	₹	7	7	7	
×		ż	1	K		,
	1	r	٦	d	ř	
	4	Ñ	ú	d	r	
	¢	á	ij	8	K	
X	4	r	Ź	4		

DO NOT WRITE IN THIS AREA

Question 1 continued	Leave blank
	Q1
(Total 5 m	iarks)



2.

$$\mathbf{A} = \begin{pmatrix} -1 & 3 & a \\ 2 & 0 & 1 \\ 1 & -2 & 1 \end{pmatrix}, \qquad \mathbf{B} = \begin{pmatrix} 2 & 0 & 4 \\ 3 & -2 & 3 \\ 1 & 2 & b \end{pmatrix}$$

where a and b are constants.

(a) Write down A^T in terms of a.

(1)

(b) Calculate AB, giving your answer in terms of a and b.

(2)

(c) Hence show that

$$(\mathbf{A}\mathbf{B})^{\mathrm{T}} = \mathbf{B}^{\mathrm{T}}\mathbf{A}^{\mathrm{T}}$$

(3)

vestion 2 continued	
nestion 2 continued	



estion 2 continued	

١,	ä	d	ø
٩	9	4	i
ì	Н		
3	Ξ	7	:
1		r	1
3	7	Ξ	ì
Ĵ	٤	π	1
		7	9
ì	r	я	ĺ.
3	u	y.,	Į
9	=		Ė
3	٩	r	8
j	4		
1	Ŀ	-	
J	г	7	
ì	÷	2	ì
ī	d	-	1
E	Ξ	Ξ	3
c	7	7	1
ì	ri	r I	i
9	н	н	
1	ч	-	ì
2	Г		_
e	Ξ	3	2
1		c	
ã	5		1
î	ē	2	•
ï	ä	₽	9
1	H	-	
8	L	×	
1	r		١
7	٩	0	
1	5	7	ı
Ĵ	Ę		ė
	ú	_	
Ĭ	r		į (
1	ù	d	7
Ĭ	Ρ	7	Ň
J	Н	-	ı

Question 2 continued	Leave blank
	Q2
(Total 6 marks)	



3. Given that

$$y = x - \operatorname{artanh}\left(\frac{2x}{1+x^2}\right)$$

(a) show that

$$1 - \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{k}{1 - x^2}$$

where k is a constant to be found.

(4)

(b) Hence, or otherwise, show that

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + x \left(1 - \frac{\mathrm{d}y}{\mathrm{d}x} \right)^2 = 0$$

(4)

4		9		7
	7	7	٩	٩
1	Ŀ	1		
Ĵ	ä		ī	3
1	E	3	ľ	
	1		à	á
Ĵ	ξ	S		
1	r.	d	P	١
	٩	7		4
þ				
3	۰		۲	
j		ē		ė
1	Ŀ	_	٠	_
1	r			
ź	ė		ė	
	d	ø	p	-
1	P	•	Ŧ	•
4	ŧ		į	
٠.		1		ż
ł	Ĺ	J		J
	L			
1	r	۰		۰
1	ŧ		į	
1	ř	١	ġ	ø
î				•
	3			b
1	Ę	ξ	Ę	è
J	P	۴		:
1	L			
1	۲	۰		ą
-	à	ø	ή	h
.[٤	_	i	J
7	3	Ξ	2	_
	3	3	7	7
j	É	÷		ė
	5	_		
ï	Р	4	9	١
٩	b	ė	d	ø
٠,	å	á	í	ć
1	Ľ	ď	1	

	Leav
	blanl
Question 3 continued	



estion 3 continued	

a.	
796	
AREA	
-	
<u>-</u>	
-	
4	
10	
9.1	
=	
- 1	
-	
-	
F 11	
-	
~	
_	
-11	
ш	
-	
DO NOT WRITE IN THIS A	
=	
00	
-	
>	
\leq	
-	
100	
•	
\sim	
-	
~	
٠	
\sim	
1	
انه	

			2	
	í			
		1		
	Į			
			١,	
	z			
	á		ú	
	ĕ			
		ч		ŀ
2	4	4	V	
2	9	4	Ņ	k
2	š		Ņ	k
2	i	X	Ž	k
₹	i	X	2	k
₹	i	Ž	Ž	k
3	i	×	7	į
?	į	×	2	į
2	į	×	2	į
?	į	×	2	ķ
?		×	2	k
?		×	Ž	k
3		×	Ž	k
?		×) 2	k
\geq	1	×	2	k
\geq	1	×) 2	k
\geq	1	×	Ž	
\geq	1	×	ž Ž	k
\geq	1	×	Y Z Z	k
\geq	1	× .		k
\geq	1			
\geq	1			
\geq	1			
?				
?				
?				
?				
?				
?				
?				k i k r k r k r k
?				k i kriji
) > >				K CYCL
) > >				K I K P K Y K Y K Y K
) > >				k in the control of
) > >				k in the control
) > >				k it received
) > >				k it received
) > >				k i k k k k k k k k k
) > >				比 ・
) > >				k it receives
>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>				k iteria
>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>				k i ke ke kara
) > >				K I CP C C C C C C C C C C C C C C C C C
>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>				K CYCL CYCL
> > > > >				k i k k k k k k k k k k k
> > > > >				k i k k k k k k k k k k
> > > > >				k it key key key
> > > > >				k iteration
> > > > >				k iver cross contract
> > > > >				k iver crystal crystal
> > > > >				k iver converse
> > > > >				k i krist Krist Krist Krist
> > > > >				k in the contract of the contr
> > > > >				k : 1 < 1 < 1 < 1 < 1 < 1 < 1 < 1 < 1 < 1
> > > > >				k : 1 < 1 < 1 < 1 < 1 < 1 < 1 < 1 < 1 < 1

\vee		2	9		١	2	۲,	
		7		١,		7		
×								
		>		2		>		
$^{\wedge}$	/	٦	1	١,	/	١	/	
V	/							
		>						
X	2	К	2		2		2	
V	١,				١,			
		/				`\		
×			>		2	S	ò	
	Κ	2		è		7		
Δ		١,	я	۹		L		
×			>			7	٩,	
	≺	>	ŝ	a	ĸ	а	ĸ	
\mathcal{L}	2	┖					ν	
V	`\		2	Z	2	Z		
		\rightarrow		2		7	۲	
×				ú		ú	ıν	
\vee	۱	2	7	7	2	2		
	/	7	ú			В	r	
×			9	6		ĸ		
	Κ	>		2		7		
\triangle	/	٦	1	\	/	١	/	
V	/		'n	2				
		>			ĸ	1	r	
×	>		2				>	
V	Υ.	/	۹	,	٠	,	٩.	
	C	/						
×				a	Б			
	<	2	2		ь	×	Ŕ.	
$^{\prime}$	7	╰	į		7			
×	`			4	4	4		
		>		7	₹	7	₹	
X	>				>			
\vee	\	/		ä	×	×		
		\rightarrow		3		9	۲	
×					2	÷	v	
V	۱	2						
		`	۳	-	۰	9	ν	
×								
	Κ	>	ú	À	ś.	à	ĸ.	
\triangle	/	٦		ч				
×	>			₹	7	₹		
		>		_	۷	۵		
	2	К						
							ь	
V			ę		⟨		?	
×	>	ς	ŝ	2	$\stackrel{\leq}{=}$	2	Ş	
Š	ζ	5	Š	2	S	2	3	
8	3	5	Š	2	S	2	}	
8	3	8	Š	2	Š	2		
8	3	5		2		2		
8	3	3		2				
×	3	<						
8	?	\ \ \						
	?	\ \ \						
	?	\ \ \						
	?	\ \ \						
	?	\ \ \						
	?	\ \ \ \						
	?	\ \ \ \						
		\ \ \						
		\ \ \ \						

		Leave blank
Question 3 continued		
		Q3
	(Total 8 marks)	
	()	



4.

$$\mathbf{M} = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$$

(a) Show that 6 is an eigenvalue of the matrix **M** and find the other two eigenvalues of M.

(4)

(b) Find a normalised eigenvector corresponding to the eigenvalue 6

(4)

ostion A continued	
estion 4 continued	



estion 4 continued	

1		à	d	ø	
4	Ş	ä		ċ	
٩	i	í		ì	
111111111	Ŀ	ä	i	ı	
d	ė	b	à	ø	
4	Ŀ	a	Ē	ı	
	1		à	ø	
J	ξ	5			
			7	9	
1	r	À	A	ń	
٦	b	g		1	
ì		Ė		Ė	
į	ŧ	ė	ø	ú	
ï			L	÷	
î	ī			=	
1	P		Ŧ		
ď	۰				
J	ė	9		þ	
111			-		
ŝ	Ė		Ě	8	
J	r	I		ı	
3	4	8		8	
1	L			i	
1	Г			-	
1	ŧ		Ŧ	5	
1	۲	١	ρ	P	
4	Ħ	4	Ħ		
ě	3		3	h	
A	Ę	ξ	Ę	á	
J	β	ę	-	7	
'n	ï				
J	þ	Ŗ		ij	
å	à	a	é	ı.	
J	ľ		g	9	
7	۹	۳	P		
7	7			μ	
Ί	É	5		ė	
				•	
7	ρ	=	9	ĸ	
٩	b	ė	d	p	
٠,	ä	ā	ē	ċ	
1	Ľ	ď	1	ì	
ě		7		7	

\ ·	7	98
- >4	6.	
×Ι		и
. 4	_	_
1		20
-/1	n	r
	Ę	
	Ž	
K		
٧j		
Zi.		
V		
K		
*3	-	-
ij	۰,	۰.
: ×	78	w.
1. 3	7.	-
v	7	2
		7
1	↽	_
	'>	
×.	Sen	ĸ.
- 71	r	78
4.	Z.	all
v.	퀫	~
$^{\prime}$	24	nie.

×	×	×		>
				ĸ.
\triangle	↶	$^{\circ}$	Λ.	0
X	X			>
V	V	V	v	ς
		<u>_</u>		C
X	X	X	X	
V	V	v	V	١,
		<u></u>		C
×	×	X	Х	>
V	V	V	v	S
		≅	49	К
Δ	Λ	ж	d.	2
×	×	X	X	5
S	C)	1112	w	K
		ж	m	ν
×	×	26	2	ē
V	V	102	T/	2
		=	=	ч
X	Х	×	ďθ	p
X	X	46	38	
			7	۲.
\triangle	Δ	$^{\wedge}$	Δ	0
\times	X			D
\vee	v	w	ĸ	ĸ.
			-	c
×	×	x	\times	⋾
×	V	∇	K	5
		≅	-	ĸ
Δ	$^{\wedge}$	3		7
×	×	ш	-	ò
S	Q	10		۲,
			8	<
X	X	_		P
X	X	26	<	
			4	ĸ
\triangle	Δ	-	_	ν
×	×	X		
V	V	1L	М	Ŕ.
		æ		r
X	Х	яï\	х	2
×				١,
		40		<
\triangle	$^{\wedge}$	ø	,	ν
×	×	26	Sid.	Ď
\sim	O	102	10	S
$\langle \rangle$	△		d	۲
×	×	59	÷.	⊳
V	V	鰀	ರ	1
		c		К
	A		\sim	2
×		≥<		>
$^{\circ}$	V	16	-	ĸ
\Diamond	\Diamond	10	0	<
X	Х	М	m,	Ò
V	×	W	S	Ь
		ব্য	ø	۲
Λ	Λ	ĕ	×	ď
×	×	¥	er.	>
		够	-	ĸ,
\triangle		4		2
×	X	×	ni.	>
V	V	W.	1	b
		K	ات	₹
$^{\wedge}$	Χ	2	=5	2
		ĸ	14	c
\bigcirc	Q	S.	-	ĸ
\triangle	Δ	75		7

Question 4 continued		Leave blank
		Q4
	(T) (12)	דע
	(Total 8 marks)	



(4)

5.

$$I_n = \int \csc^n x \, \mathrm{d}x, \quad 0 < x < \frac{\pi}{2}, \quad n \geqslant 0$$

(a) Show that, for $n \ge 2$

$$I_n = \frac{n-2}{n-1} I_{n-2} - \frac{1}{n-1} \cot x \csc^{n-2} x$$
 (5)

(b) Hence, or otherwise, find

$$\int \csc^4 x \, \mathrm{d}x$$

giving your answer in terms of $\cot x$.



	L
uestion 5 continued	
acstron 5 continued	
	_



Question 5 continued	Leave
	_
	_
	_
	_
	_
	_
	_
	_
	_
	_
	_
	_
	_
	_
	_
	_
	_
	Q5
(Total 9 mark	(s)



6. The hyperbola *H* has equation $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

and the ellipse E has equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

where a > b > 0

The line *l* is a tangent to **hyperbola** *H* at the point $P(a \sec \theta, b \tan \theta)$, where $0 < \theta < \frac{\pi}{2}$

(a) Using calculus, show that an equation for l is

$$bx \sec \theta - ay \tan \theta = ab$$

(4)

Given that the point F is the focus of **ellipse** E for which x > 0 and that the line l passes through F,

(b) show that l is parallel to the line y = x

(5)



	Leave
	blank
Question 6 continued	



	Leave blank
Question 6 continued	

Question 6 continued	blank
	Q6
(Total 9 marks)	
(



7. (a) Find

$$\int \frac{5+x}{\sqrt{4-3x^2}} \, \mathrm{d}x$$

(5)

(b) Hence find the exact value of

$$\int_0^1 \frac{5+x}{\sqrt{4-3x^2}} \, \mathrm{d}x$$

giving your answer in the form $p\pi\sqrt{3}+q$, where p and q are rational numbers to be found.

- 1	14	١
	U	ı
	`	,



	Leave
	blank
Question 7 continued	



	Leave blank
Question 7 continued	Ulalik

Question 7 continued		Leave
		Q7
	(Total 8 marks)	



8. The curve C has parametric equations

$$x = \theta - \sin \theta$$
, $y = 1 - \cos \theta$, $0 \le \theta \le 2\pi$

The curve C is rotated through 2π radians about the x-axis. The area of the curved surface generated is given by S.

(a) Show that

$$S = 2\pi\sqrt{2} \int_0^{2\pi} \left(1 - \cos\theta\right)^{\frac{3}{2}} d\theta$$
 (4)

(b) Hence find the exact value of *S*.

•	•		
	h	, N	
	u ,		

estion 8 continued	



estion 8 continued	

ì	ė	ø
٦	7	Si.
1	Н	ŀΪ
1	-	ᆿ
1	0	E
ľ	7	Ξ
Ĵ	c	Г
		77
ì	rί	ĸ.
٦	v	u
ì	=	
1	9	Α,
ì	4	H
ì	Ŀ	÷
1	г	_
ì	-	÷
Ī	d	_
1	7	Ξ
G	7	=
ì	ΡÌ	ΡĤ
1	н	щ
1	ш	-
1	Г	Ξ
1	_	Ξ
j	0	⊏
j	5	Ξ
ì	ė	~
j	3	3
	3	٠.
1	н	=
1	ь	6
1	E.	3
	3	=
ľ	2	₹.
ũ	٠	=
	٠.	
j	r	٦
1	ė	0
ũ	P	ď
1	÷	٠

ΧII		T.
1.	7	94.
a	Ü	Йľ
< 4	7	_
N.	A	à
<u></u>	5	6
1	2	迪
74		ĸ
< -	7	_
\sim	٥.	\simeq
< 4	И	
- 54	_	\simeq
/:	⇆	Z
1	71	↸.
X	ø	ij.
< 1	ıΧ	
N.	•	-
Ζ:	١.	Λ.
Š	-	◛
12	d	2
⟨ 4	_	÷
×	_	7
c -	\sim	C =
N		N
< i		7.
K	媗	щ.
7	Ŀ	Z
⟨ #	7	-
· M	n	~
∕ J	×	جنا
Š	۰,	¥
71		z
V	ä	₽
X		X
СÆ	ß	\simeq
ZII	7	Ź
/3	鉤	160
XI	K	2
:/:	$\overline{}$	z
v	Z	₹
Oil		\simeq
42	×	7
×	ía.	ĸ
X	K	20
X3	76	٣.
×	ρ	W.
O.	\mathbb{R}	a

$\langle \times \rangle$	\times	X
$\langle \cdot \rangle \langle$	$\times \rangle$	\propto
ČΧ	S	Z×
$\times 2$	SX,	\times
\approx	QX.	$\times \!\!\!>$
\times	\propto	$\times \rangle$
	\propto	$\times \rangle$
X	O.	S
58	ČE,	10
SS	27	₹.
\propto	ЖI	ΩK
X	X	w.
\propto	₹2	×
\propto	X	-
\sim	São	e.
22	S	e.
\times	\propto	$\times \rangle$
>>	aka	m
\times	Œ	X5
50		\overline{x}
SÕ	ČΧ	abla
SX	Ø	¥,
VX.	202	\cong
\propto	ΝD	T.
\propto	\sim	≌
$\langle \times \rangle$		
X		È
82	S	
\sim	W.	22
\times	SМ	Ŷ.
>>	GEC.	∇
X		\Rightarrow
20	≃	\hookrightarrow
X		S
SX	212	4
S		Z.
\propto	x =	₹
$\langle \times \rangle$	\sim	\Rightarrow
	X	\propto
$\langle \times \rangle$	4110	\simeq
82	SIIS	\propto
ZV.	Set 1	3
\times	SK.	M)
\times	Œ	\sim
\times	\sim	\sim
\times		$\overline{\sim}$
X	04	×
SX	¢	SIL
X5	8	25
	Ĉ	
QX.	ШK	ŒK.

Question 8 continued		Leave blank
		Q8
	(Total 10 marks)	
	(1000110 1101110)	



9	With respect to a fixed origin O , the points $A(-1, 5, 1)$, $B(1, 0, 3)$, $C(2, -1, 2)$ and $D(3, 6, -1)$ are the vertices of a tetrahedron.
	(a) Find the volume of the tetrahedron <i>ABCD</i> . (4)
	The plane Π contains the points A , B and C .
	(b) Find a cartesian equation of Π . (4)
	The point T lies on the plane Π .
	The line DT is perpendicular to Π .
	(c) Find the exact coordinates of the point T. (4)



estion 9 continued	



estion 9 continued	

	Leave
	blank
Question 9 continued	



	Leave blank
Question 9 continued	
	_
	_
	_
	_
	_
	_
	_
	_
	_
	_
	_
	_
	_
	_
	_
	_
	_
	_
	_
	_
	_
	_
	_
	Q9
(Total 12 marks	i)
TOTAL FOR PAPER: 75 MARK	\mathbf{s}
END	