edexcel

Mark Scheme (Results)

Summer 2016

Pearson Edexcel IAL Further Pure Mathematics 3 (WFM03/01)

ALWAYS LEARNING PEARSON

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL IAL MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- d… or dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $\ast$ The answer is printed on the paper or ag- answer given
- or d… The second mark is dependent on gaining the first mark

4. All A marks are ‘correct answer only’ (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Further Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to x = ...

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to x = ...

2. Formula

Attempt to use the correct formula (with values for a, b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0,\ \ q \neq 0$, leading to x = ...

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Question Number	Scheme	Notes	Marks
1.	$y = 9\cosh x + 3\sinh x + 7x$		
	$\frac{dy}{dx} = 9\sinh x + 3\cosh x + 7$	Correct derivative	B1
	$9\frac{\left(e^x - e^{-x}\right)}{2} + 3\frac{\left(e^x + e^{-x}\right)}{2} + 7 = 0$	Replaces sinh$x$ and cosh$x$ by the correct exponential forms	M1
	Note that the first 2 marks can score the other way round: M1: $y = 9\frac{\left(e^x + e^{-x}\right)}{2} + 3\frac{\left(e^x - e^{-x}\right)}{2} + 7x$ B1: $\frac{dy}{dx} = 9\frac{\left(e^x - e^{-x}\right)}{2} + 3\frac{\left(e^x + e^{-x}\right)}{2} + 7$		
	$12e^{2x} + 14e^x - 6 = 0$ oe	M1: Obtains a quadratic in e^x A1: Correct quadratic	M1A1
	$\left(3e^x - 1\right)\left(2e^x + 3\right) = 0 \Rightarrow e^x = ...$	Solves their quadratic as far as e^x = ...	M1
	$x = \ln\left(\frac{1}{3}\right)$	cso (Allow –ln3) $e^x = -\frac{3}{2}$ need not be seen. Extra answers, award A0	A1
			(6)
	Alternative		
	$\frac{dy}{dx} = 9\sinh x + 3\cosh x + 7$	Correct derivative	B1
	$9\sinh x = -3\cosh x - 7 \Rightarrow 81\sinh^2 x = 9\cosh^2 x + 42\cosh x + 49$		
	$72\cosh^2 x - 42\cosh x - 130 = 0$	Squares and attempts quadratic in cosh$x$	M1
	$\left(3\cosh x - 5\right)\left(12\cosh x + 13\right) = 0 \Rightarrow \cosh x = \frac{5}{3}$	M1: Solves quadratic A1: Correct value	M1A1
	$x = \ln\left(\frac{5}{3} \pm \sqrt{\left(\frac{5}{3}\right)^2 - 1}\right)$	Use of ln form of arcosh	M1
	$x = \ln\left(\frac{1}{3}\right)$	cso (Allow – ln3)	A1

NB: Ignore any attempts to find the y coordinate

Question Number	Scheme	Notes	Marks
2	$\frac{x^2}{25}+\frac{y^2}{4}=1, \quad P(5\cos\theta, 2\sin\theta)$		
(a)	$\frac{dx}{d\theta}=-5\sin\theta, \ \frac{dy}{d\theta}=2\cos\theta$ or $\frac{2x}{25}+\frac{2y}{4}\frac{dy}{dx}=0$	Correct derivatives or correct implicit differentiation	B1
	$\frac{dy}{dx}=\frac{2\cos\theta}{-5\sin\theta}$	Divides their derivatives correctly or substitutes and rearranges	M1
	$M_N=\frac{5\sin\theta}{2\cos\theta}$	Correct perpendicular gradient rule	M1
	$y-2\sin\theta=\frac{5\sin\theta}{2\cos\theta}(x-5\cos\theta)$	Correct straight line method (any complete method) **Must** use their gradient of the normal.	M1
	$5x\sin\theta-2y\cos\theta=21\sin\theta\cos\theta*$	cso	A1*
			(5)
(b)	At Q, $x=0 \Rightarrow y=-\frac{21}{2}\sin\theta$		B1
	M is $\left(\frac{0+5\cos\theta}{2}, \frac{2\sin\theta-\frac{21}{2}\sin\theta}{2}\right)$ $\left(=\left(\frac{5}{2}\cos\theta, -\frac{17}{4}\sin\theta\right)\right)$	Correct mid-point method for at least one coordinate Can be implied by a correct x coordinate	M1
	L cuts x-axis at $\frac{21}{5}\cos\theta$		B1
	Area $OPM = OLP + OLM$ $\frac{1}{2}.\frac{21}{5}\cos\theta.2\sin\theta+\frac{1}{2}.\frac{21}{5}\cos\theta.\frac{17}{4}\sin\theta$	M1: Correct triangle area method using their coordinates A1: Correct expression	M1A1
	$=\frac{105}{16}\sin 2\theta$	Or $6.5625\sin 2\theta$ must be positive	A1**(6)**
			Total 11
	ALTs for (b)		
1	**Using Area *OPM***		
	See above for B1M1		B1M1
	Area $\Delta OPM=\frac{1}{2}\begin{vmatrix}0 & 5\cos\theta & \frac{5}{2}\cos\theta & 0\\ 0 & 2\sin\theta & -\frac{17}{4}\sin\theta & 0\end{vmatrix}$	M1: Correct determinant with their coords, with 2 or 3 points. $\begin{matrix}0\\0\end{matrix}$ should be at both or neither end. A1: Correct determinant (There are more complicated determinants using the 3 points.)	M1A1

	$=\frac{1}{2}\left(0+5\sin\theta\cos\theta+0-0+\frac{85}{4}\sin\theta\cos\theta-0\right)$	A1	A1
	$=\frac{105}{4}\sin\theta\cos\theta$		
	$=\frac{105}{16}\sin 2\theta$		A1
2	**Using Area *OPQ*:**		
	At Q, $x=0 \Rightarrow y=-\frac{21}{2}\sin\theta$		B1
	Area $\Delta OPQ=\frac{1}{2}\begin{vmatrix}5\cos\theta & 0\\ 2\sin\theta & -\frac{21}{2}\sin\theta\end{vmatrix}$	Can be implied by the following line	M1A1
	$=\frac{1}{2}\times\frac{105}{2}\sin\theta\cos\theta$	*OQ* is base, *x* coord of *P* is height	A1
	$=\frac{105}{8}\sin 2\theta$		
	Area *OPM* = $\frac{1}{2}$ Area *OPQ*		M1
	$=\frac{105}{16}\sin 2\theta$		A1
3	At Q, $x=0 \Rightarrow y=-\frac{21}{2}\sin\theta$		B1
	M is $\left(\frac{0+5\cos\theta}{2},\frac{2\sin\theta-\frac{21}{2}\sin\theta}{2}\right)$ $\left(=\left(\frac{5}{2}\cos\theta,-\frac{17}{4}\sin\theta\right)\right)$		M1
	$OP=\sqrt{4\sin^2\theta+25\cos^2\theta}$ $\left(=\sqrt{4+21\cos^2\theta}\right)$		B1
	$d=\dfrac{\frac{5}{2}\cos\theta\times\frac{2\sin\theta}{5\cos\theta}+\frac{17}{4}\sin\theta}{\sqrt{\frac{4\sin^2\theta}{25\cos^2\theta}+1}}=\dfrac{\frac{21}{4}\sin\theta}{\sqrt{\frac{4+21\cos^2\theta}{25\cos^2\theta}}}$		
	Area $=\frac{1}{2}\times\dfrac{\frac{21}{4}\sin\theta}{\sqrt{\frac{4+21\cos^2\theta}{25\cos^2\theta}}}\times\sqrt{4+21\cos^2\theta}$		M1A1
	$=\frac{105}{16}\sin 2\theta$		A1

Question Number	Scheme	Notes	Marks
3(a)	$x^2+4x+13\equiv(x+2)^2+9$		B1
	$\displaystyle\int\frac{1}{(x+2)^2+9}\,\mathrm{d}x=\frac{1}{3}\arctan\left(\frac{x+2}{3}\right)$	M1: $k\arctan\mathrm{f}(x)$. A1: Correct expression	M1A1
	$\left[\frac{1}{3}\arctan\left(\frac{x+2}{3}\right)\right]_{-2}^{1}=\frac{1}{3}(\arctan 1-\arctan 0)$	Correct use of limits arctan0 need not be shown	M1
	$\frac{\pi}{12}$	cao	A1
			(5)
ALT:	**Sub** $x+2=3\tan t$		
	$x^2+4x+13\equiv(x+2)^2+9$		B1
	$\frac{\mathrm{d}x}{\mathrm{d}t}=3\sec^2 t\qquad x=-2,\ \tan t=0,\ t=0;\ x=1,\tan t=1,\ t=\frac{\pi}{4}$		
	$\displaystyle\int\frac{3\sec^2 t}{9\tan^2 t+9}\,\mathrm{d}t=\frac{1}{3}\int\,\mathrm{d}t=\frac{1}{3}t$	M1 sub and integrate inc use of $\tan^2+1=\sec^2$ A1 Correct expression Ignore limits	M1A1
	$..\left[\frac{\pi}{12}\right]_0^{\frac{\pi}{4}}.$	Either change limits and substitute Or reverse substitution and substitute original imits	M1
	$\frac{\pi}{12}$	cao	A1
(b)	$4x^2-12x+34=4\left(x-\frac{3}{2}\right)^2+25$ or $(2x-3)^2+25$	M1: $4(x\pm p)^2\pm q,\ (p,q\neq 0)$ A1: $4\left(x-\frac{3}{2}\right)^2+25$	M1A1
	$\displaystyle\int\frac{1}{\sqrt{4\left(x-\frac{3}{2}\right)^2+25}}\,\mathrm{d}x=\frac{1}{2}\int\frac{1}{\sqrt{\left(x-\frac{3}{2}\right)^2+\frac{25}{4}}}\,\mathrm{d}x\ =\frac{1}{2}\mathrm{arsinh}\left(\frac{x-\frac{3}{2}}{\frac{5}{2}}\right)$ M1: $k\,\mathrm{arsinh}\,\mathrm{f}(x)$. A1: Correct expression		M1A1
	$\left[\frac{1}{2}\mathrm{arsinh}\left(\frac{x-\frac{3}{2}}{\frac{5}{2}}\right)\right]_{-1}^{4}=\frac{1}{2}(\mathrm{ar}\sinh(1)-\mathrm{ar}\sinh(-1))$	Correct use of limits	M1
	$=\frac{1}{2}\left(\ln\left(1+\sqrt{2}\right)-\ln\left(-1+\sqrt{2}\right)\right)$	Uses the logarithmic form of arsinh	M1
	$=\frac{1}{2}\ln\left(3+2\sqrt{2}\right)$ or $\ln\left(1+\sqrt{2}\right)$	cao	A1
			(7)
			Total 12

ALT:	Second M1A1		
	Sub $2x-3=u$ or $2x-3=5\sinh u$		
	$\int_{\text{arsinh}-1}^{\text{arsinh}1} \frac{1}{\sqrt{25\sinh^2 u+25}} 5\cosh u\,du = \left[\frac{1}{2}\text{arsinh}\left(\frac{u}{5}\right)\right]_{-5}^{5}$	$\int_{-5}^{5} \frac{1}{2\sqrt{u^2+25}} du = \left[\frac{1}{2}\text{arsinh}\left(\frac{u}{5}\right)\right]$	M1A1

Question Number	Scheme	Notes	Marks
4	$\mathbf{M}=\begin{pmatrix}1 & k & 0\\ -1 & 1 & 1\\ 1 & k & 3\end{pmatrix}$		
(a)	$\lvert\mathbf{M}\rvert = 3-k-k(-3-1)(=3k+3)$	Correct determinant in any form	B1
	$\mathbf{M}^{\mathrm{T}}=\begin{pmatrix}1 & -1 & 1\\ k & 1 & k\\ 0 & 1 & 3\end{pmatrix}$ or minors $\begin{pmatrix}3-k & -4 & -k-1\\ 3k & 3 & 0\\ k & 1 & 1+k\end{pmatrix}$ or cofactors $\begin{pmatrix}3-k & 4 & -k-1\\ -3k & 3 & 0\\ k & -1 & 1+k\end{pmatrix}$		B1
	$\mathbf{M}^{-1}=\dfrac{1}{3+3k}\begin{pmatrix}3-k & -3k & k\\ 4 & 3 & -1\\ -k-1 & 0 & 1+k\end{pmatrix}$	M1: Identifiable full attempt at inverse **including reciprocal of determinant**. Could be indicated by at least 6 correct elements. A1ft: Two rows or two columns correct (**follow through their determinant** but not incorrect entries in the matrices used) A1ft: Fully correct inverse (follow through as before)	M1A1ftA1ft
	NB: If every element is the negative of the correct element, allow M1A1A0		(5)
(b)	$\mathbf{MN}=\begin{pmatrix}3 & 5 & 6\\ 4 & -1 & 1\\ 3 & 2 & -3\end{pmatrix}\Rightarrow \mathbf{N}=\mathbf{M}^{-1}\begin{pmatrix}3 & 5 & 6\\ 4 & -1 & 1\\ 3 & 2 & -3\end{pmatrix}$	Correct statement	B1
	$\mathbf{N}=\frac{1}{3}\begin{pmatrix}3 & 0 & 0\\ 4 & 3 & -1\\ -1 & 0 & 1\end{pmatrix}\begin{pmatrix}3 & 5 & 6\\ 4 & -1 & 1\\ 3 & 2 & -3\end{pmatrix}=\begin{pmatrix}3 & 5 & 6\\ 7 & 5 & 10\\ 0 & -1 & -3\end{pmatrix}$	M1: Multiplies the given matrix by their $\mathbf{M}^{-1}$ in the correct order Must include the "$\frac{1}{3}$" A2: Correct matrix (-1 each error). If left with $\frac{1}{3}$ outside the matrix award A0	M1A(2, 1, 0)
			(4)
			Total 9

Question Number	Scheme	Notes	Marks
5(a)	$y = \text{artanh}(\cos x)$		
	$\dfrac{dy}{dx} = \dfrac{1}{1-\cos^2 x} \times -\sin x$	Correct use of the chain rule	M1
	$= \dfrac{-\sin x}{\sin^2 x} = \dfrac{-1}{\sin x} = -\text{cosec}\, x$ *	A1: Correct completion with no errors	A1
			(2)
	Alternative 1		
	$\tanh y = \cos x \Rightarrow \text{sech}^2 y \dfrac{dy}{dx} = -\sin x$		
	$\dfrac{dy}{dx} = \dfrac{-\sin x}{\text{sech}^2 y} = \dfrac{-\sin x}{1-\cos^2 x}$	Correct differentiation to obtain a function of x	M1
	$= \dfrac{-\sin x}{\sin^2 x} = \dfrac{-1}{\sin x} = -\text{cosec}\, x$ *	A1: Correct completion with no errors	A1
	Alternative 2		
	$\text{artanh}(\cos x) = \dfrac{1}{2}\ln\left(\dfrac{1+\cos x}{1-\cos x}\right)$		
	$\dfrac{dy}{dx} = \dfrac{1}{2} \times \dfrac{1-\cos x}{1+\cos x} \times \dfrac{-\sin x(1-\cos x) - \sin x(1+\cos x)}{(1-\cos x)^2}$	Correct differentiation to obtain a function of x	M1
	$= \dfrac{-2\sin x}{2(1-\cos^2 x)} = -\text{cosec}\, x$ *	A1: Correct completion with no errors	A1
(b)	$\displaystyle\int \cos x\, \text{artanh}(\cos x)\, dx = \sin x\, \text{artanh}(\cos x) - \int \sin x \times -\text{cosec}x\, dx$ M1: Parts in the correct direction A1: Correct expression		M1A1
	$\left[\sin x\, \text{artanh}(\cos x) + x\right]_0^{\frac{\pi}{6}} = \dfrac{1}{2}\text{artanh}\left(\dfrac{\sqrt{3}}{2}\right) + \dfrac{\pi}{6}(-(0))$ M1: Correct use of limits on either part (provided both parts are integrated). Lower limit need not be shown		M1
	$= \dfrac{1}{4}\ln\left(\dfrac{1+\frac{\sqrt{3}}{2}}{1-\frac{\sqrt{3}}{2}}\right) + \dfrac{\pi}{6}$	Use of the logarithmic form of artanh	M1
	$= \dfrac{1}{4}\ln(7+4\sqrt{3}) + \dfrac{\pi}{6}$ or $\dfrac{1}{2}\ln(2+\sqrt{3}) + \dfrac{\pi}{6}$	Cao (oe)	A1
	The last 2 M marks may be gained in reverse order.		**(5)**
			Total 7

Question Number	Scheme	Notes	Marks
6(a)	$\overrightarrow{AB}=\begin{pmatrix}-2\\1\\1\end{pmatrix},\ \overrightarrow{AC}=\begin{pmatrix}1\\-1\\3\end{pmatrix},\ \overrightarrow{BC}=\begin{pmatrix}3\\-2\\2\end{pmatrix}$	Two correct vectors in Π Can be negatives of those shown	B1
	$\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\-2 & 1 & 1\\1 & -1 & 3\end{vmatrix}=\begin{pmatrix}4\\7\\1\end{pmatrix}$	M1: Attempt cross product of two vectors lying in Π (At least one no. to be correct.) A1: Correct normal vector	M1A1
	$\begin{pmatrix}4\\7\\1\end{pmatrix}\bullet\begin{pmatrix}1\\2\\3\end{pmatrix}=4+14+3$	Attempt scalar product with their normal and a point in the plane	dM1
	$4x+7y+z=21$	Cao (oe)	A1
	(a) Alternative		
	$a+2b+3c=d$ $-a+3b+4c=d$ $2a+b+6c=d$	Correct equations	B1
	$a=\dfrac{4}{21}d,\ b=\dfrac{1}{3}d,\ c=\dfrac{1}{21}d$	M1: Solve for a, b and c in terms of d A1: Correct equations	M1A1
	$d=21\Rightarrow a=...,\ b=...,\ c=...$	Obtains values for a, b, c and d	M1
	$4x+7y+z=21$	Cao (oe)	A1
			(5)
	Alternative: Using $\mathbf{r}=\mathbf{a}+s\mathbf{b}+t\mathbf{c}$ where $\mathbf{b}$ and $\mathbf{c}$ are vectors in Π		
	Two correct vectors in the plane	See main scheme	B1
	Eg $\mathbf{r}=\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}1\\2\\3\end{pmatrix}+s\begin{pmatrix}-2\\1\\1\end{pmatrix}+t\begin{pmatrix}1\\-1\\3\end{pmatrix}$		M1
	$x=1-2s+t$ $y=2+s-t$ $z=3+s+3t$	Deduce 3 correct equations	A1
	$4x+7y+z=21$	M1: Eliminate s, t A1: Cao	M1A1
(b)	$\text{AD}\square\text{AB}\times\text{AC}$	Attempt suitable triple product	M1
	$=\begin{pmatrix}4\\7\\1\end{pmatrix}\bullet\begin{pmatrix}k-1\\2\\11\end{pmatrix}=4k-4+14+11$		
	$\therefore\dfrac{1}{6}(4k+21)=6$	M1: Set $\dfrac{1}{6}$(their triple product) = 6 A1: Correct equation	dM1A1
	$k=\dfrac{15}{4}$	Cao (oe)	A1

	(b) Alternative		
	Area ABC $=\frac{1}{2}\left\|\overrightarrow{AB}\right\|\left\|\overrightarrow{AC}\right\|=\frac{1}{2}\sqrt{6}\sqrt{11}$	Attempt area *ABC* and distance between *D*and*Π*	M1
	D to *Π* is $\frac{4k+28+14-21}{\sqrt{16+49+1}}$		
	$\frac{1}{6}\sqrt{6}\sqrt{11}\frac{4k+28+14-21}{\sqrt{16+49+1}}=6$	M1: Set $\frac{1}{3}$(their area x their distance) = 6	dM1A1
		A1: Correct equation	
	$k=\frac{15}{4}$	Cao (oe)	A1
			(4)
			Total 9

Question Number	Scheme	Notes	Marks
7	$x = 3t^4, \quad y = 4t^3$		
(a)	$\frac{dx}{dt} = 12t^3, \quad \frac{dy}{dt} = 12t^2$	Correct derivatives	B1
	$S = (2\pi)\int y\left(\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2\right)^{\frac{1}{2}} dt = (2\pi)\int 4t^3\sqrt{(12t^3)^2 + (12t^2)^2}\, dt$ $\left(= (2\pi)\int 4t^3\left(144t^6 + 144t^4\right)^{\frac{1}{2}} dt\right)$ M1: Substitutes their derivatives into a correct formula (2π not required)		M1
	$S = (2\pi)\int 4t^3\left(144t^4\right)^{\frac{1}{2}}\left(t^2+1\right)^{\frac{1}{2}} dt$	Attempt to factor out at least t^4 - numerical factor may be left	M1
	$S = 96\pi\int_0^1 t^5\left(t^2+1\right)^{\frac{1}{2}} dt$	Correct completion	A1
			(4)
(b)	$u^2 = t^2 + 1 \Rightarrow 2u\frac{du}{dt} = 2t$ or $2u = 2t\frac{dt}{du}$	Correct differentiation	B1
	$t = 0 \Rightarrow u = 1, \ t = 1 \Rightarrow u = \sqrt{2}$	Correct limits **ALT:** reverse the substitution later. (Treat as M1 in this case and award later when work seen)	B1
	$S = (96\pi)\int t^5 \times u \times \frac{u}{t}\, du$		
	$S = (96\pi)\int \left(u^2 - 1\right)^2 \times u^2\, du$	M1: Complete substitution A1: Correct integral in terms of u. Ignore limits, need not be simplified	M1A1
	$S = (96\pi)\int \left(u^6 - 2u^4 + u^2\right)du = (96\pi)\left[\frac{u^7}{7} - \frac{2u^5}{5} + \frac{u^3}{3}\right]$ M1: Expands and attempts to integrate		dM1
	$S = 96\pi\left[\frac{u^7}{7} - \frac{2u^5}{5} + \frac{u^3}{3}\right]_1^{\sqrt{2}} = 96\pi\left\{\left(\frac{\sqrt{2}^7}{7} - \frac{2\sqrt{2}^5}{5} + \frac{\sqrt{2}^3}{3}\right) - \left(\frac{1}{7} - \frac{2}{5} + \frac{1}{3}\right)\right\}$ M1: Correct use of their changed limits (both to be changed) **ALT:** If sub reversed, substitute the original limits		ddM1
	$S = \frac{192\pi}{105}\left(11\sqrt{2} - 4\right)$	Cao eg $\frac{64\pi}{35}$	A1
			(7)
			Total 11

Question Number	Scheme	Notes	Marks
8.	$I_n = \int_0^{\ln 2} \tanh^{2n} x \, dx, \quad n \geq 0$		
(a)	$\tanh^{2n} x = \tanh^{2(n-1)} x \tanh^2 x$		B1
	$\tanh^{2n} x = \pm\tanh^{2(n-1)} x\left(1 - \text{sech}^2 x\right)$		M1
	$I_n = \int_0^{\ln 2} \tanh^{2(n-1)} x \, dx - \int_0^{\ln 2} \tanh^{2(n-1)} x \, \text{sech}^2 x \, dx$		
	$I_n = I_{n-1} - \left[\frac{1}{2n-1}\tanh^{2n-1} x\right]_0^{\ln 2}$	M1: Correctly substitutes for I_{n-1} and obtains $\int \tanh^{2(n-1)} x \, \text{sech}^2 x \, dx = k\tanh^{2n-1} x$ A1: Correct expression	M1A1
	$= I_{n-1} - \frac{1}{2n-1}\left(\frac{3}{5}\right)^{2n-1}$ *	Correct completion with no errors	A1*
			(5)
ALT:	$I_n - I_{n-1} = \int_0^{\ln 2} \left(\tanh^{2n} x - \tanh^{2(n-1)} x\right) dx$		
	$= \int_0^{\ln 2} \tanh^{2(n-1)} x\left(\tanh^2 x - 1\right) dx$		B1
	$= \int_0^{\ln 2} \tanh^{2(n-1)} x\left(-\text{sech}^2 x\right) dx$	$= \int_0^{\ln 2} \tanh^{2(n-1)} x\left(\pm\text{sech}^2 x\right) dx$	M1
	$I_n - I_{n-1} = -\left[\frac{1}{2n-1}\tanh^{2n-1} x\right]_0^{\ln 2}$	M1: Obtains $\int \tanh^{2(n-1)} x \, \text{sech}^2 x \, dx = k\tanh^{2n-1} x$ A1: Correct expression	M1A1
	$= I_{n-1} - \frac{1}{2n-1}\left(\frac{3}{5}\right)^{2n-1}$ *	Correct completion with no errors	A1*
(b)	$I_0 = \ln 2$	The integration must be seen.	B1
	$I_2 = I_1 - \frac{1}{3}\left(\frac{3}{5}\right)^3$	Applies the reduction formula once	M1
	$I_2 = I_0 - \frac{1}{1}\left(\frac{3}{5}\right)^1 - \frac{1}{3}\left(\frac{3}{5}\right)^3$	M1: Second application of the reduction formula A1: Correct expression	M1A1
	$I_2 = \ln 2 - \frac{84}{125}$	cao	A1
	Special Case:		
	If I_4 is found award B1 for I_0 or I_1 and M1M0A0A0		

	(b) Alternative		
	$I_1 = \int_0^{\ln 2} \tanh^2 x \, dx = \int_0^{\ln 2} \left(1 - \text{sech}^2 x\right) dx$		
	$I_1 = \left[x - \tanh x\right]_0^{\ln 2}$	Correct integration	B1
	$I_2 = I_1 - \frac{1}{3}\left(\frac{3}{5}\right)^3$	Applies the reduction formula once	M1
	$I_1 = \ln 2 - \tanh(\ln 2) = \ln 2 - \frac{3}{5}$	M1: Uses limits A1: Correct expression	M1A1
	$I_2 = \ln 2 - \frac{3}{5} - \frac{1}{3}\left(\frac{3}{5}\right)^3$		
	$= \ln 2 - \frac{84}{125}$		A1
			(5)
			Total 10