

## F3 IAL Specimen MA kprime2

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1. The line  $x=8$  is a directrix of the ellipse with equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \quad a > 0, b > 0,$$

and the point  $(2, 0)$  is the corresponding focus.

Find the value of  $a$  and the value of  $b$ .

(5)

$$\text{foci } (2, 0) \Rightarrow ae = 2$$

$$\text{Directrix } x=8 \Rightarrow \frac{a}{e} = 8$$

$$ae \times \frac{a}{e} = 2 \times 8$$

$$\Rightarrow a^2 = 16 \Rightarrow a = 4$$

$$e = \frac{1}{2}$$

$$b^2 = a^2(1 - e^2) = 16\left(1 - \frac{1}{4}\right)$$

$$= 16 \times \frac{3}{4} = 12$$

$$\Rightarrow b = \sqrt{12}$$

$$\therefore a = 4, b = \sqrt{12}$$

2. Use calculus to find the exact value of  $\int_{-2}^1 \frac{1}{x^2 + 4x + 13} dx$ .

(5)

$$\int_{-2}^1 \frac{1}{x^2 + 4x + 13} dx = \int_{-2}^1 \frac{1}{(x+2)^2 + 9} dx$$

$$= \left[ \frac{1}{3} \arctan\left(\frac{x+2}{3}\right) \right]_{-2}^1$$

$$= \frac{\pi}{12} - 0 = \frac{\pi}{12}$$

3. (a) Starting from the definitions of  $\sinh x$  and  $\cosh x$  in terms of exponentials, prove that

$$\cosh 2x = 1 + 2\sinh^2 x \tag{3}$$

(b) Solve the equation

$$\cosh 2x - 3\sinh x = 15,$$

giving your answers as exact logarithms.

(5)

(a)

$$\text{RHS} = 1 + 2\sinh^2 x = 1 + 2 \left( \frac{e^x - e^{-x}}{2} \right)^2$$

$$= 1 + \frac{(e^x - e^{-x})^2}{2}$$

$$= 1 + \frac{e^{2x} - 1 - 1 + e^{-2x}}{2}$$

$$= 1 + \frac{e^{2x} + e^{-2x} - 2}{2}$$

$$= 1 + \frac{e^{2x} + e^{-2x}}{2} - \frac{2}{2} = 1 + \frac{e^{2x} + e^{-2x}}{2} - 1$$

$$= \frac{e^{2x} + e^{-2x}}{2} = \cosh(2x) = \text{LHS}$$

as required.

## Question 3 continued

(b)

$$1 + 2\sinh^2 x - 3\sinh x = 15$$

$$\therefore 2\sinh^2 x - 3\sinh x - 14 = 0$$

$$(2\sinh x - 7)(\sinh x + 2) = 0$$

$$\therefore \sinh x = \frac{7}{2}$$

$$\Rightarrow \cancel{\text{arcsinh}} \quad x = \ln \left( \frac{7 + \sqrt{53}}{2} \right)$$

$$\cancel{\sinh x = -2} \quad \sinh x = -2$$

$$\Rightarrow x = \ln(\sqrt{5} - 2)$$

$$4. I_n = \int_0^a (a-x)^n \cos x \, dx, \quad a > 0, \quad n \geq 0$$

(a) Show that, for  $n \geq 2$ ,

$$I_n = na^{n-1} - n(n-1)I_{n-2} \quad (5)$$

(b) Hence evaluate  $\int_0^{\frac{\pi}{2}} \left(\frac{\pi}{2} - x\right)^2 \cos x \, dx.$  (3)

$$4. (a). I_n = \int_0^a (a-x)^n \cos x \, dx$$

$$\text{Let } u = (a-x)^n \quad u' = -n(a-x)^{n-1}$$

$$v' = \cos x \quad v = \sin x$$

$$\therefore I_n = \left[ \sin x (a-x)^n \right]_0^a + n \int_0^a (a-x)^{n-1} \sin x \, dx$$

$$\therefore \cancel{I_n = 0} \quad \therefore I_n = n \int_0^a (a-x)^{n-1} \sin x \, dx$$

$$\text{Let } u = (a-x)^{n-1} \quad u' = (1-n)(a-x)^{n-2}$$

$$v' = \sin x \quad v = -\cos x$$

$$\therefore I_n = n \left( \left[ -\cos x (a-x)^{n-1} \right]_0^a + \int_0^a (1-n) \cos x (a-x)^{n-2} \, dx \right)$$

$$\therefore I_n = n \left( 0 + a^{n-1} + (1-n) I_{n-2} \right)$$

## Question 4 continued

$$\Rightarrow I_n = n a^{n-1} + n(1-n) I_{n-2}$$

$$\therefore I_n = n a^{n-1} - n(n-1) I_{n-2}$$

as required.

$$(b) I_2 = 2 \left( \frac{\pi}{2} \right) - 2 I_0$$

$$= \pi - 2 \int_0^{\pi/2} (\frac{\pi}{2} - x) \cos x \, dx$$

~~$$= \pi - 2 \int_0^{\pi/2} (\frac{\pi}{2} - x) \cos x \, dx$$~~

$$u = (\frac{\pi}{2} - x) \quad v' = -1$$

$$v' = \cos x \quad v = \sin x$$

~~$$= \pi$$~~

$$= \pi - 2 \left( \left[ \sin x (\frac{\pi}{2} - x) \right]_0^{\pi/2} + \int_0^{\pi/2} \sin x \, dx \right)$$

$$= \pi - 2 \int_0^{\pi/2} \sin x \, dx$$

$$= \pi - 2 \left[ -\cos x \right]_0^{\pi/2}$$

~~$$= \pi - 2$$~~

5. Given that  $y = (\operatorname{arcosh} 3x)^2$ , where  $3x > 1$ , show that

$$(a) \quad (9x^2 - 1) \left( \frac{dy}{dx} \right)^2 = 36y, \quad (5)$$

$$(b) \quad (9x^2 - 1) \frac{d^2y}{dx^2} + 9x \frac{dy}{dx} = 18. \quad (4)$$

$$\text{5(a). } y = (\operatorname{arcosh} 3x)^2$$

$$\Rightarrow \frac{\partial y}{\partial x} = (2 \operatorname{arcosh} 3x) \times \frac{d}{dx} (\operatorname{arcosh} 3x)$$

consider  $\frac{d}{dx} (\operatorname{arcosh} 3x) :$

Let  $u = \operatorname{arcosh} 3x$

$$c^2 - s^2 = 1$$

$$\therefore s^2 = c^2 - 1$$

$$\Rightarrow \cosh u = 3x$$

$$\frac{\partial u}{\partial x} \sinh u = 3$$

$$\therefore \frac{\partial u}{\partial x} = \frac{3}{\sinh u} = \frac{3}{\sqrt{\cosh^2 u - 1}}$$

$$\therefore \frac{\partial u}{\partial x} = \frac{3}{\sqrt{9x^2 - 1}}$$

$$\therefore \frac{d}{dx} (\operatorname{arcosh} 3x) = \frac{3}{\sqrt{9x^2 - 1}}$$

Question 5 continued

$$\therefore \frac{\partial y}{\partial x} = (2 \operatorname{arcosh} 3x) \times \frac{d}{dx} (\operatorname{arcosh} (3x))$$

$$= 2 \operatorname{arcosh} 3x \times \frac{3}{\sqrt{9x^2 - 1}}$$

$$\therefore \frac{\partial y}{\partial x} = \frac{6 \operatorname{arcosh} 3x}{\sqrt{9x^2 - 1}}$$

$$\Rightarrow \left( \frac{\partial y}{\partial x} \right)^2 = \frac{36 (\operatorname{arcosh} 3x)^2}{9x^2 - 1}$$

$$\therefore (9x^2 - 1) \left( \frac{\partial y}{\partial x} \right)^2 = (9x^2 - 1) \frac{36 (\operatorname{arcosh} 3x)^2}{9x^2 - 1}$$

$$= 36 (\operatorname{arcosh} 3x)^2$$

$$= 36y^2 \quad \text{as required.}$$



Question 5 continued

$$(b) \frac{\partial y}{\partial x} = \frac{6 \operatorname{arccosh} 3x}{\sqrt{9x^2 - 1}}$$

~~$$\therefore \frac{\partial^2 y}{\partial x^2} = \frac{(\sqrt{9x^2 - 1})(18) - 6 \operatorname{arccosh} 3x}{(\sqrt{9x^2 - 1})^2}$$

Via quotient rule...~~

$$\therefore \frac{\partial^2 y}{\partial x^2} = \frac{(\sqrt{9x^2 - 1}) \left( \frac{18}{\sqrt{9x^2 - 1}} \right) - 6 \operatorname{arccosh} 3x \left( \frac{1}{2} (9x^2 - 1)^{-1/2} \cdot 18x \right)}{9x^2 - 1}$$

$$\therefore \frac{\partial^2 y}{\partial x^2} = \frac{18 - 54x \operatorname{arccosh} 3x (9x^2 - 1)^{-1/2}}{9x^2 - 1}$$

$$\Rightarrow (9x^2 - 1) \frac{\partial^2 y}{\partial x^2} = (9x^2 - 1) \frac{18 - 54x \operatorname{arccosh} 3x (9x^2 - 1)^{-1/2}}{9x^2 - 1}$$

$$(9x^2 - 1) \frac{\partial^2 y}{\partial x^2} = 18 - 54x \operatorname{arccosh} 3x \cdot (9x^2 - 1)^{-1/2}$$

$$= \frac{18 - 54x \operatorname{arccosh} 3x}{\sqrt{9x^2 - 1}}$$

Question 5 continued

$$\& \quad 9x \frac{dy}{dx} = 9x \cdot \frac{6 \operatorname{arccosh} 3x}{\sqrt{9x^2 - 1}}$$

$$\therefore 9x \frac{dy}{dx} = \frac{54x \operatorname{arccosh} 3x}{\sqrt{9x^2 - 1}}$$

$$\therefore \text{LHS} = (9x^2 - 1) \frac{d^2y}{dx^2} + 9x \frac{dy}{dx}$$

$$= \frac{18 - 54x \operatorname{arccosh} 3x}{\sqrt{9x^2 - 1}} + \frac{54x \operatorname{arccosh} 3x}{\sqrt{9x^2 - 1}}$$

$$= 18 + \frac{0}{\sqrt{9x^2 - 1}}$$

$$= 18 = \text{RHS}$$

as required.

(Total 9 marks)

6.  $M = \begin{pmatrix} 1 & 0 & 3 \\ 0 & -2 & 1 \\ k & 0 & 1 \end{pmatrix}$ , where  $k$  is a constant.

Given that  $\begin{pmatrix} 6 \\ 1 \\ 6 \end{pmatrix}$  is an eigenvector of  $M$ ,

(a) find the eigenvalue of  $M$  corresponding to  $\begin{pmatrix} 6 \\ 1 \\ 6 \end{pmatrix}$ , (2)

(b) show that  $k=3$ , (2)

(c) show that  $M$  has exactly two eigenvalues. (4)

A transformation  $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$  is represented by  $M$ .

The transformation  $T$  maps the line  $l_1$ , with cartesian equations  $\frac{x-2}{1} = \frac{y}{-3} = \frac{z+1}{4}$ , onto the line  $l_2$ .

(d) Taking  $k=3$ , find cartesian equations of  $l_2$ . (5)

S(a).  $Mx = \lambda x$

$$\therefore \begin{pmatrix} 1 & 0 & 3 \\ 0 & -2 & 1 \\ k & 0 & 1 \end{pmatrix} \begin{pmatrix} 6 \\ 1 \\ 6 \end{pmatrix} = \begin{pmatrix} 6\lambda \\ \lambda \\ 6\lambda \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 24 \\ 4 \\ 6k+6 \end{pmatrix} = \begin{pmatrix} 6\lambda \\ \lambda \\ 6\lambda \end{pmatrix} \Rightarrow \lambda = 4$$

E. Value = 4

(b)  $Mx = \begin{pmatrix} 1 & 0 & 3 \\ 0 & -2 & 1 \\ k & 0 & 1 \end{pmatrix} \begin{pmatrix} 6 \\ 1 \\ 6 \end{pmatrix} = \begin{pmatrix} 24 \\ 4 \\ 6k+6 \end{pmatrix} = \begin{pmatrix} 6\lambda \\ \lambda \\ 6\lambda \end{pmatrix} = \begin{pmatrix} 24 \\ 4 \\ 24 \end{pmatrix}$

Question 6 continued

$$\therefore 6k + 6 = 24 \Rightarrow k = \frac{24 - 6}{6}$$

$$\Rightarrow k = 3 \quad \text{as required}$$

(c)  $A - \lambda I = \begin{pmatrix} 1 - \lambda & 0 & 3 \\ 0 & -2 - \lambda & 1 \\ 3 & 0 & 1 - \lambda \end{pmatrix}$

$\det(A - \lambda I) = 0$

$$\Rightarrow \det(A - \lambda I) = (1 - \lambda)(-2 - \lambda)(1 - \lambda) + 3(-3(-2 - \lambda))$$

$$= (-2 - \lambda)[(1 - \lambda)^2 + 9] = 0$$

$$= (-2 - \lambda)(\lambda^2 - 2\lambda + 10) \Rightarrow -2 - \lambda = 0 \Rightarrow \lambda = -2$$

For an E. Value.

$$= -(\lambda + 2)(\lambda - 4) = \cancel{-(\lambda + 2)(\lambda - 4)} \quad \begin{matrix} (1 - \lambda)^2 + 9 = 0 \\ \Rightarrow (1 - \lambda)^2 = -9 \end{matrix}$$

$\Rightarrow \lambda = -2$   
 $\lambda = 4$   
 are only E. vals.

$\Rightarrow$   ~~$(\lambda + 2)$~~  no solution since discriminant  $< 0$ .

## Question 6 continued

$\therefore$  only eigen values are

$$\lambda = -2 \quad \text{and} \\ \lambda = 4$$

$$(d) \quad x - 2 = -\frac{1}{3}y \quad , \quad -\frac{1}{3}y = \frac{z}{4} + \frac{1}{4}$$

$$x = 2 - \frac{y}{3} \quad \leftarrow \text{and } x - 2 = \frac{z}{4} + \frac{1}{4}$$

$$z + 1 = -\frac{4}{3}y$$

$$\Rightarrow \frac{2 - \frac{y}{3} - 2}{3} = \frac{z}{4} + \frac{1}{4}$$

$$\therefore z = -1 - \frac{4}{3}y$$

$$\Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 - \frac{y}{3} \\ y \\ -1 - \frac{4}{3}y \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} + y \begin{pmatrix} -1/3 \\ 1 \\ -4/3 \end{pmatrix}$$

$$\text{Let } y = \lambda \Rightarrow \underline{r} = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} -1/3 \\ 1 \\ -4/3 \end{pmatrix}$$

$$\therefore \underline{r} = \begin{pmatrix} 2 - \lambda/3 \\ \lambda \\ -1 - 4\lambda/3 \end{pmatrix} \quad \text{is eqn of } l_1$$

$$x = -1 - \frac{13}{3}\lambda \Rightarrow -\frac{3(x+1)}{13} = \lambda$$

$$y = -\frac{10}{3}\lambda - 1 \Rightarrow \frac{3(y+1)}{-10} = \lambda$$

$$z = 5 - \frac{7\lambda}{3} \Rightarrow \frac{3(5-z)}{7} = \lambda$$

$$\therefore -\frac{3x+3}{13} = \frac{3y+3}{-10} = \frac{15-3z}{7}$$

~~3~~

$$-\frac{(x+1)}{13} = \frac{y+1}{-10} = \frac{5-z}{7}$$

7. The plane  $\Pi$  has vector equation

$$\mathbf{r} = 3\mathbf{i} + \mathbf{k} + \lambda(-4\mathbf{i} + \mathbf{j}) + \mu(6\mathbf{i} - 2\mathbf{j} + \mathbf{k})$$

- (a) Find an equation of  $\Pi$  in the form  $\mathbf{r} \cdot \mathbf{n} = p$ , where  $\mathbf{n}$  is a vector perpendicular to  $\Pi$  and  $p$  is a constant. (5)

The point  $P$  has coordinates  $(6, 13, 5)$ . The line  $l$  passes through  $P$  and is perpendicular to  $\Pi$ . The line  $l$  intersects  $\Pi$  at the point  $N$ .

- (b) Show that the coordinates of  $N$  are  $(3, 1, -1)$ . (4)

The point  $R$  lies on  $\Pi$  and has coordinates  $(1, 0, 2)$ .

- (c) Find the perpendicular distance from  $N$  to the line  $PR$ . Give your answer to 3 significant figures. (5)

Let:

(a)  $\vec{A} = \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix}$      ~~$\vec{B} = \begin{pmatrix} -4 \\ 1 \\ 0 \end{pmatrix}$~~      ~~$\vec{C} = \begin{pmatrix} 6 \\ -2 \\ 1 \end{pmatrix}$~~

~~$\vec{AB} = \begin{pmatrix} -4 \\ 1 \\ 0 \end{pmatrix}$~~      $\vec{AB} = \begin{pmatrix} -4 \\ 1 \\ 0 \end{pmatrix}$

$\vec{AC} = \begin{pmatrix} 6 \\ -2 \\ 1 \end{pmatrix}$

$\therefore \vec{AB} \times \vec{AC} = \begin{vmatrix} -4 & 1 & 0 \\ 6 & -2 & 1 \end{vmatrix} = \begin{vmatrix} -4 & 1 & 0 \\ 6 & -2 & 1 \end{vmatrix}$

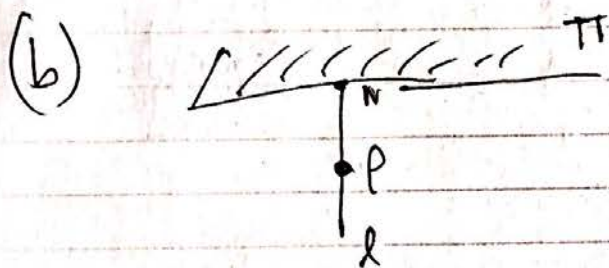
$\vec{AB} \times \vec{AC} = \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix} \Rightarrow \vec{n} = \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix}$

Question 7 continued

$$\therefore \underline{r} \cdot \underline{n} = A \cdot \underline{n}$$

$$\therefore \underline{r} \cdot \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix}$$

$$\underline{r} \cdot \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix} = 5$$



line  $l$  has direction vector  $\begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix}$

eqn:

$$\therefore \underline{r} = \begin{pmatrix} 6 \\ 13 \\ 5 \end{pmatrix} + t \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix}$$

$N$  lies on  $l \Rightarrow N = \begin{pmatrix} 6+t \\ 13+4t \\ 5+2t \end{pmatrix}$  for some  $t$ .

$$\underline{r} \cdot \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix} = 5 \Rightarrow \begin{pmatrix} 6+t \\ 13+4t \\ 5+2t \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix} = 5$$

$$\therefore 6+t + 52 + 16t + 10 + 4t = 5$$

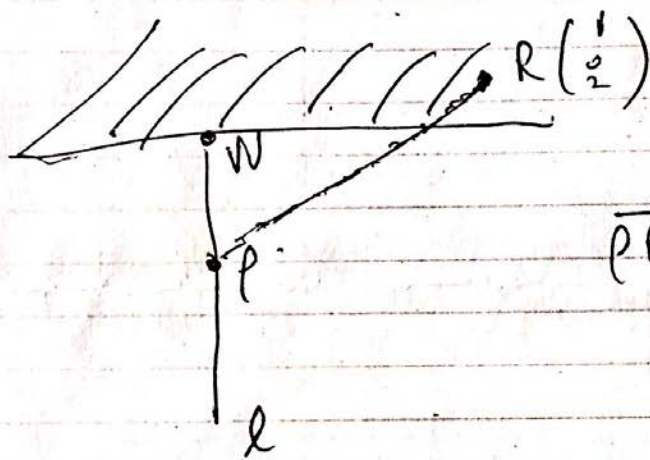
$$\Rightarrow 68 + 21t = 5 \Rightarrow t = -3$$



Question 7 continued

$$\therefore N = \begin{pmatrix} 6-3 \\ 13+4(-3) \\ 5+2(-3) \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \\ -1 \end{pmatrix} \quad \text{as required.}$$

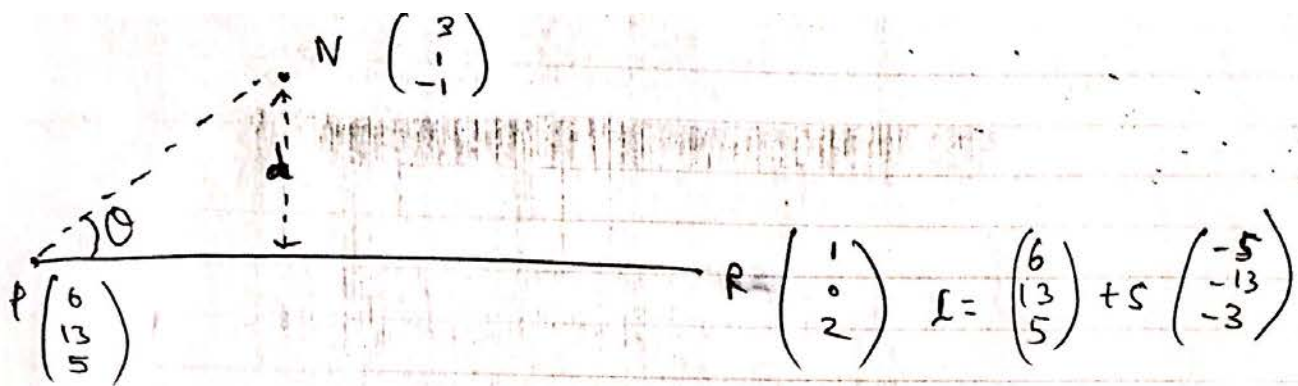
(c)



$$\vec{PR} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} - \begin{pmatrix} 6 \\ 13 \\ 5 \end{pmatrix} = \begin{pmatrix} -5 \\ -13 \\ -3 \end{pmatrix}$$

Line  $\vec{PR}$  has eqn  $\underline{r} = \begin{pmatrix} 6 \\ 13 \\ 5 \end{pmatrix} + s \begin{pmatrix} -5 \\ -13 \\ -3 \end{pmatrix}$

$$\begin{aligned} x &= 6 - 5s & \Rightarrow & \quad 6 - 2s \\ y &= 13 - 13s & \Rightarrow & \quad 13 - 8 = 5 \end{aligned}$$



$$\vec{PN} = \begin{pmatrix} -3 \\ -12 \\ -6 \end{pmatrix} \quad \wedge \quad \vec{PR} = \begin{pmatrix} -5 \\ -13 \\ -3 \end{pmatrix}$$

$$\therefore \cos \theta = \frac{\begin{pmatrix} -3 \\ -12 \\ -6 \end{pmatrix} \cdot \begin{pmatrix} -5 \\ -13 \\ -3 \end{pmatrix}}{\sqrt{3^2 + 12^2 + 6^2} \times \sqrt{5^2 + 13^2 + 3^2}}$$

$$\cos \theta = \frac{189}{29\sqrt{87}}$$

$$\Rightarrow \sin \theta = \frac{\sqrt{58}}{29} = \frac{d}{|PN|}$$

$$|PN| = 3\sqrt{21} \Rightarrow d = 3\sqrt{21} \sin \theta = 3.61$$

$$\therefore d = \underline{\underline{3.61}} \text{ units.}$$

(Total 5 marks)

Q1

8. The hyperbola  $H$  has equation  $\frac{x^2}{16} - \frac{y^2}{4} = 1$ .

The line  $l_1$  is the tangent to  $H$  at the point  $P(4 \sec t, 2 \tan t)$ .

(a) Use calculus to show that an equation of  $l_1$  is

$$2y \sin t = x - 4 \cos t \quad (5)$$

The line  $l_2$  passes through the origin and is perpendicular to  $l_1$ .

The lines  $l_1$  and  $l_2$  intersect at the point  $Q$ .

(b) Show that, as  $t$  varies, an equation of the locus of  $Q$  is

$$(x^2 + y^2)^2 = 16x^2 - 4y^2 \quad (8)$$

$$8(a). \quad \frac{x^2}{16} - \frac{y^2}{4} = 1$$

$$\frac{1}{8} x \Rightarrow \frac{1}{2} y \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{2x}{8y} = + \frac{x}{4y}$$

$$\therefore \frac{dy}{dx} = \frac{2x}{8y} = \frac{x}{4y}$$

$$\therefore \left( \frac{dy}{dx} \right)_P = \frac{4 \sec t}{8 \tan t} = \frac{\sec t}{2 \tan t}$$

$$= \frac{1/\cos t}{2 \frac{\sin t}{\cos t}}$$

$$= \frac{1}{2 \sin t}$$

$$m_T = -\frac{1}{2} \cos t$$

$$\therefore y - y_1 = m(x - x_1)$$

$$\begin{aligned} \therefore y - 2 \tan t &= \frac{1}{2} \operatorname{cosec} (x - 4 \operatorname{sect}) \\ &= \frac{x}{2} \operatorname{cosec} - 2 \operatorname{cosec} \operatorname{sect} \end{aligned}$$

$$2y \sin t - \frac{4 \sin^2 t}{\cos t} = x - 4 \operatorname{sect}$$

$$\therefore 2y \sin t = x - 4 \operatorname{sect} + \frac{4 \sin^2 t}{\cos t}$$

$$\therefore 2y \sin t = x + \frac{4 \sin^2 t - 4}{\cos t}$$

$$\therefore 2y \sin t = x + \frac{4(\sin^2 t - 1)}{\cos t}$$

$$\therefore 2y \sin t = x - \frac{4(1 - \sin^2 t)}{\cos t}$$

$$\therefore 2y \sin t = x - 4 \cdot \frac{\cos^2 t}{\cos t}$$

$$\therefore 2y \sin t = x - 4 \operatorname{sect}$$

as required.

Question 8 continued

$$\therefore 2y \sin t = \underline{\underline{x - 4 \cos t}} \quad \text{as required.}$$

$$(b) \text{ Gradient of } l_1 = \frac{-1}{\frac{1}{2} \operatorname{cosec} t} = -2 \sin t$$

$$\therefore l_1 \text{ has eqn: } y = -2 \sin t x$$

$$\& l_2 \text{ has eqn: } 2y \sin t = x - 4 \cos t$$

$$\Rightarrow -4 \sin^2 t x = x - 4 \cos t$$

$$\therefore (-4 \sin^2 t - 1)x = -4 \cos t$$

$$\therefore x = \frac{4 \cos t}{1 + 4 \sin^2 t}$$

$$\therefore y = \frac{-8 \sin t \cos t}{1 + 4 \sin^2 t}$$

$$\therefore Q \left( \frac{4 \cos t}{1 + 4 \sin^2 t}, \frac{-8 \sin t \cos t}{1 + 4 \sin^2 t} \right)$$

$$x^2 = \frac{16 \cos^2 t}{(1 + 4 \sin^2 t)^2} \quad \& \quad y^2 = \frac{64 \sin^2 t \cos^2 t}{(1 + 4 \sin^2 t)^2}$$

(Total 13 marks)

$$\text{LHS} = x^2 + y^2 = \frac{16\cos^2 t + 64\sin^2 t \cos^2 t}{(1+4\sin^2 t)^2}$$

$$\therefore x^2 + y^2 = \frac{16\cos^2 t (1 + 4\sin^2 t)}{(1+4\sin^2 t)^2}$$

$$\therefore x^2 + y^2 = \frac{16\cos^2 t}{1+4\sin^2 t}$$

$$\therefore (x^2 + y^2)^2 = \frac{256\cos^4 t}{(1+4\sin^2 t)^2}$$

$$\text{RHS} = 16x^2 - 4y^2 = \frac{256\cos^2 t}{(1+4\sin^2 t)^2} - \frac{256\sin^2 t \cos^2 t}{(1+4\sin^2 t)^2}$$

$$= \frac{256\cos^2 t (1 - \sin^2 t)}{(1+4\sin^2 t)^2}$$

$$= \frac{256\cos^4 t}{(1+4\sin^2 t)^2} = \underline{\underline{\text{LHS}}} \quad \text{as required.}$$