Surname	Other	names
Pearson Edexcel nternational Advanced Level	Centre Number	Candidate Number
Further Pu	ire	
Mathemat Advanced/Advanced	tics F2	
Mathemat	tics F2	Paper Reference WFM02/01

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer all questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
 there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for each question are shown in brackets
 use this as a guide as to how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶



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$z^5 = 32$	
Give your answers in the form $r(\cos \theta + i \sin \theta)$, where $r > 0$ and $0 \le \theta < 2\pi$	(

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3. (a) Show that $r^3 - (r-1)^3 \equiv 3r^2 - 3r + 1$

(1)

(b) Hence prove by the method of differences that, for $n \in \mathbb{Z}^+$

$$\sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}$$

[You may use
$$\sum_{r=1}^{n} r = \frac{n(n+1)}{2}$$
 without proof.]

(5)

Question 3 continued	



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4. $y = 3e^{-x}\cos 3x + Ae^{-x}\sin 3x$

is a particular integral of the differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 10y = 40e^{-x}\sin 3x$$

where *A* is a constant.

(a) Find the value of A.

(5)

(b) Hence find the general solution of this differential equation.

(4)

(c) Find the particular solution of this differential equation for which both y = 3 and

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 3 \text{ at } x = 0$$

(4)



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 $y = e^{\cos^2 x}$

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(a) Show that

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = \mathrm{e}^{\cos^2 x} \left(\sin^2 2x - 2\cos 2x \right)$$

(4)

(b) Hence find the Maclaurin series expansion of $e^{\cos^2 x}$ up to and including the term in x^2

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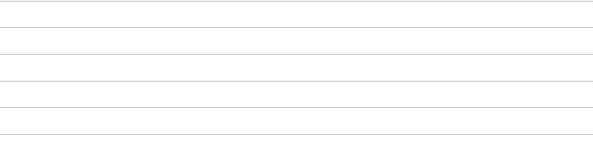
6.	Find the general	solution	of the	differential	equation

$$\cos x \, \frac{\mathrm{d}y}{\mathrm{d}x} + y \sin x = (\cos^2 x) \ln x, \qquad 0 < x < \frac{\pi}{2}$$

Give your answer	in the	form $y =$	f(x).
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(8)









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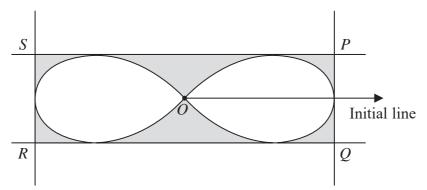


Figure 1

Figure 1 shows a sketch of the curve C with polar equation

$$r = 4\cos 2\theta$$
, $-\frac{\pi}{4} \leqslant \theta \leqslant \frac{\pi}{4}$ and $\frac{3\pi}{4} \leqslant \theta \leqslant \frac{5\pi}{4}$

The lines PQ, QR, RS and SP are tangents to C, where QR and SP are parallel to the initial line and PQ and RS are perpendicular to the initial line.

(a) Find the polar coordinates of the points where the tangent SP touches the curve. Give the values of θ to 3 significant figures.

(5)

(b) Find the exact area of the finite region bounded by the curve C, shown unshaded in Figure 1.

(5)

(c) Find the area enclosed by the rectangle *PQRS* but outside the curve *C*, shown shaded in Figure 1.

(5)



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8. (a) Use de Moivre's theorem to

(i) show that

$$\cos 5\theta \equiv \cos^5 \theta - 10\cos^3 \theta \sin^2 \theta + 5\cos \theta \sin^4 \theta$$

(ii) find an expression for $\sin 5\theta$ in terms of $\cos \theta$ and $\sin \theta$

(4)

(b) Hence show that

$$\tan 5\theta = \frac{t^5 - 10t^3 + 5t}{5t^4 - 10t^2 + 1}$$

where $t = \tan \theta$ and $\cos 5\theta \neq 0$

(2)

(c) Hence find a quadratic equation whose roots are $\tan^2 \frac{\pi}{5}$ and $\tan^2 \frac{2\pi}{5}$

Give your answer in the form $ax^2 + bx + c = 0$ where a, b and c are integers to be found.

(4)

(d) Deduce that $\tan \frac{\pi}{5} \tan \frac{2\pi}{5} = \sqrt{5}$

(2)



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