WFM02/01: Further Pure Mathematics F2

Question Number	Scheme	Marks
1(a)	$\frac{1}{3r-1}-\frac{1}{3r+2}$	M1 A1 (2)
(b)	$\sum_{r=1}^{n}\frac{3}{(3r-1)(3r+2)}=\frac{1}{2}-\frac{1}{5}+\frac{1}{5}-\frac{1}{8}+\frac{1}{8}-\frac{1}{11}+\ldots\frac{1}{3n-1}-\frac{1}{3n+2}$	M1 A1ft
	$=\frac{1}{2}-\frac{1}{3n+2}=\frac{3n}{2(3n+2)}$ *	A1 (3)
(c)	Sum = f(1000) – f(99) $\frac{3000}{6004}-\frac{297}{598}=0.00301$ or 3.01×10^{-3}	M1 A1 (2)
		7

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Question Number	Scheme	Marks
2	$f''(t) = -x - \cos x, \qquad f''(0) = -1$	B1
	$f'''(t) = (-1 + \sin x)\dfrac{dx}{dt}, \qquad f'''(0) = -0.5$	M1A1
	$f(t) = f(0) + tf'(0) + \dfrac{t^2}{2}f''(0) + \dfrac{t^3}{3!}f'''(0) + \ldots$	
	$= 0.5t - 0.5t^2 - \frac{1}{12}t^3 + \ldots$	M1 A1
		5

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Question Number	Scheme	Marks
3(a)	$(x+4)(x+3)^2-2(x+3)=0$, $(x+3)(x^2+7x+10)=0$ so $(x+2)(x+3)(x+5)=0$ or alternative method including calculator	M1
	Finds critical values –2 and -5	A1 A1
	Establishes $x > -2$	A1ft
	Finds and uses critical value –3 to give $-5 < x < -3$	M1A1 (6)
(b)	$x > -2$	B1ft (1)
		7

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Question Number	Scheme	Marks
4(a)	Modulus = 16	B1
	Argument = $\arctan(-\sqrt{3}) = \frac{2\pi}{3}$	M1 A1 (3)
(b)	$z^3 = 16^3(\cos(\frac{2\pi}{3}) + i\sin(\frac{2\pi}{3}))^3 = 16^3(\cos 2\pi + i\sin 2\pi)$ =4096 or 16^3	M1 A1 (2)
(c)	$w = 16^{\frac{1}{4}}(\cos(\frac{2\pi}{3}) + i\sin(\frac{2\pi}{3}))^{\frac{1}{4}} = 2(\cos\left(\frac{\pi}{6}\right) + i\sin\left(\frac{\pi}{6}\right))$ $\left(= \sqrt{3} + i\right)$	M1 A1ft
	OR $-1+\sqrt{3}i$ OR $-\sqrt{3}-i$ OR $1-\sqrt{3}i$	M1A2 (1,0) (5)
		10

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Question Number	Scheme	Marks
5(a)	$1.5+\sin 3\theta = 2 \quad \rightarrow \quad \sin 3\theta = 0.5 \quad \therefore 3\theta = \frac{\pi}{6}\left(\text{or } \frac{5\pi}{6}\right),$	M1 A1,
	and $\therefore \theta = \frac{\pi}{18}$ or $\frac{5\pi}{18}$	A1 (3)
(b)	Area $= \frac{1}{2}\left[\int_{\frac{\pi}{18}}^{\frac{5\pi}{18}} (1.5+\sin 3\theta)^2 \, d\theta\right], -\frac{1}{9}\pi\times 2^2$	M1, M1
	$= \frac{1}{2}\left[\int_{\frac{\pi}{18}}^{\frac{5\pi}{18}} (2.25+3\sin 3\theta+\frac{1}{2}(1-\cos 6\theta))d\theta\right] - \frac{1}{9}\pi\times 2^2$	M1
	$= \frac{1}{2}\left[(2.25\theta-\cos 3\theta+\frac{1}{2}(\theta-\frac{1}{6}\sin 6\theta))\right]_{\frac{\pi}{18}}^{\frac{5\pi}{18}} - \frac{1}{9}\pi\times 2^2$	M1 A1
	$= \frac{13\sqrt{3}}{24} - \frac{5\pi}{36}$	M1 A1 (7)
		10

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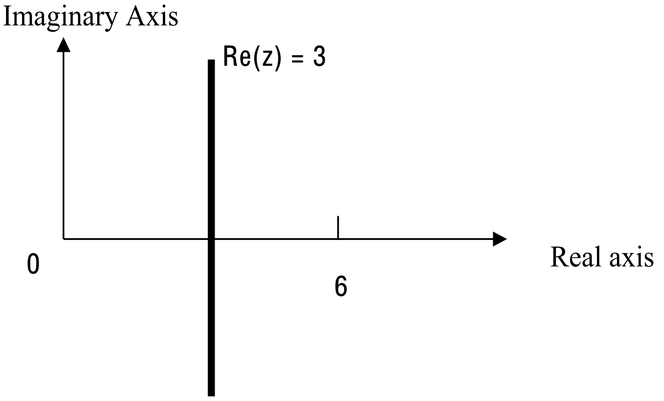
Question Number	Scheme	Marks
6(a)	 Vertical Straight line Through 3 on real axis	B1 B1 (2)
(b)	These are points where line $x = 3$ meets the circle centre (3, 4) with radius 5. The complex numbers are 3 + 9i and 3 – i.	M1 A1 A1 (3)
(c)	$\lvert z-6\rvert = \lvert z\rvert \Rightarrow \left\lvert \frac{30}{w} - 6\right\rvert = \left\lvert \frac{30}{w}\right\rvert$ $\therefore \lvert 30-6w\rvert = \lvert 30\rvert \;\; \Rightarrow \;\; \therefore \lvert 5-w\rvert = \lvert 5\rvert$ This is a circle with Cartesian equation $(u-5)^2 + v^2 = 25$	M1 M1 A1 M1 A1 (5) **10**

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Question Number	Scheme	Marks
7(a)	$\frac{dy}{dx} = \frac{dy}{dz}.\frac{dz}{dx}$ and $\frac{dy}{dz} = 2z$ so $\frac{dy}{dx} = 2z.\frac{dz}{dx}$	M1 M1 A1
	Substituting to get $2z.\frac{dz}{dx} - 4z^2 \tan x = 2z$ and thus $\frac{dz}{dx} - 2z \tan x = 1$ *	M1 A1 (5)
(b)	$\text{I.F.} = e^{\int -2\tan x dx} = e^{2\ln\cos x} = \cos^2 x$	M1 A1
	$\therefore \frac{d}{dx}\left(z\cos^2 x\right) = \cos^2 x \;\; \therefore z\cos^2 x = \int \cos^2 x dx$	M1
	$\therefore z\cos^2 x = \int \tfrac{1}{2}(\cos 2x + 1)\,dx = \tfrac{1}{4}\sin 2x + \tfrac{1}{2}x + c$	M1 A1
	$\therefore z = \tfrac{1}{2}\tan x + \tfrac{1}{2}x\sec^2 x + c\sec^2 x$	A1 (6)
(c)	$\therefore y = (\tfrac{1}{2}\tan x + \tfrac{1}{2}x\sec^2 x + c\sec^2 x)^2$	B1ft (1) **12**

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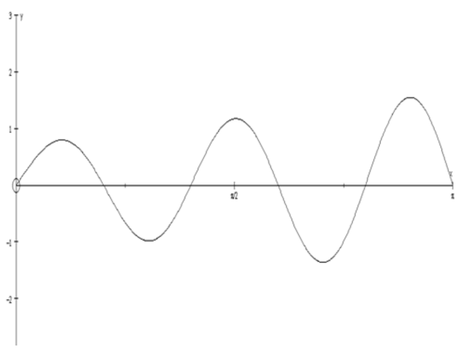
Question Number	Scheme	Marks
8(a)	Differentiate twice and obtaining $\frac{dy}{dx} = \lambda \sin 5x + 5\lambda x \cos 5x$ and $\frac{d^2y}{dx^2} = 10\lambda \cos 5x - 25\lambda x \sin 5x$	M1 A1
	Substitute to give $\lambda = \frac{3}{10}$	M1 A1 (4)
(b)	Complementary function is $y = A\cos 5x + B\sin 5x$ or $Pe^{5ix} + Qe^{-5ix}$	M1 A1
	So general solution is $y = A\cos 5x + B\sin 5x + \frac{3}{10}x\sin 5x$ or in exponential form	A1ft (3)
(c)	$y = 0$ when $x = 0$ means $A = 0$	B1
	$\frac{dy}{dx} = 5B\cos 5x + \frac{3}{10}\sin 5x + \frac{3}{2}x\cos 5x$ and at $x = 0$ $\frac{dy}{dx} = 5$ and so $5 = 5A$	M1 M1
	So $B = 1$	A1
	So $y = \sin 5x + \frac{3}{10}x\sin 5x$	A1 (5)
(d)	 "Sinusoidal" through O amplitude becoming larger	B1
	Crosses x axis at $\frac{\pi}{5}, \frac{2\pi}{5}, \frac{3\pi}{5}, \frac{4\pi}{5}$	B1 (2)
		14