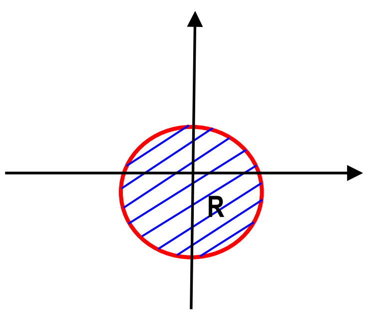
Further Pure Mathematics FP2 Mark scheme

Question	Scheme		Marks
1	$\dfrac{x}{x+2} < \dfrac{2}{x+5}$		
	Critical Values -2 and -5	Seen anywhere in solution Both correct B1B1; one correct B1B0	B1 B1
	$\dfrac{x}{x+2} - \dfrac{2}{x+5} < 0$		
	$\dfrac{x^2+3x-4}{(x+2)(x+5)} < 0$		
	$\dfrac{(x+4)(x-1)}{(x+2)(x+5)} < 0$	Attempt single fraction and factorise numerator or use quad formula	M1
	Critical values -4 and 1	Correct critical values May be seen on a graph or number line.	A1
	$-5 < x < -4,\ -2 < x < 1$ $(-5,-4)\cup(-2,1)$	dM1: Attempt an interval inequality using one of -2 or -5 with another cv A1, A1: Correct intervals Can be in set notation One correct scores A1A0 Award on basis of the inequalities seen - ignore any and/or between them Set notation answers do not need the union sign.	dM1 A1 A1
			(7)
	Alternative		
	Critical Values -2 and -5	Seen anywhere in solution	B1, B1
	$\dfrac{x}{x+2} < \dfrac{2}{x+5} \Rightarrow x(x+5)^2(x+2) < 2(x+2)^2(x+5)$		
	$\Rightarrow (x+5)(x+2)\left[x(x+5)-2(x+2)\right] < 0$		
	$\Rightarrow (x+5)(x+2)\left[(x-1)(x+4)\right] < 0$	Multiply by $(x+5)^2(x+2)^2$ and attempt to factorise a quartic or use quad formula	M1
	Critical values -4 and 1	Correct critical values	A1
	$-5 < x < -4,\ -2 < x < 1$ $(-5,-4)\cup(-2,1)$	dM1: Attempt an interval inequality using one of -2 or -5 with another cv A1, A1: Correct intervals Can be in set notation One correct scores A1A0	dM A1 A1
	Any solutions with no algebra (eg sketch graph followed by critical values with no working) scores max B1B1		
			(7 marks)

Question	Scheme		Marks
2(a)	$\frac{1}{(r+6)(r+8)}$		
	$\frac{1}{2(r+6)}-\frac{1}{2(r+8)}$ oe	Correct partial fractions, any equivalent form	B1
			(1)
(b)	$=\left(2\times\frac{1}{2}\right)\left(\frac{1}{7}-\frac{1}{9}+\frac{1}{8}-\frac{1}{10}+\frac{1}{9}-\frac{1}{11}\ldots+\frac{1}{n+5}-\frac{1}{n+7}+\frac{1}{n+6}-\frac{1}{n+8}\right)$ Expands at least 3 terms at start and 2 at end (may be implied) The partial fractions obtained in (a) can be used without multiplying by 2. Fractions may be $\frac{1}{2}\times\frac{1}{7}-\frac{1}{2}\times\frac{1}{9}$ etc These comments apply to both M1 and A1		M1
	$=\frac{1}{7}+\frac{1}{8}-\frac{1}{n+7}-\frac{1}{n+8}$	Identifies the terms that do not cancel	A1
	$=\frac{15(n+7)(n+8)-56(2n+15)}{56(n+7)(n+8)}$	Attempt common denominator Must have multiplied the fractions from (a) by 2 now	M1
	$=\frac{n(15n+113)}{56(n+7)(n+8)}$		A1 cso
			(4)
			(5 marks)

Question	Scheme		Marks
3(a)	$\frac{dy}{dx}+2xy = xe^{-x^2}y^3$		
	$z = y^{-2} \Rightarrow y = z^{-\frac{1}{2}}$		
	$\frac{dy}{dx} = -\frac{1}{2}z^{-\frac{3}{2}}\frac{dz}{dx}$	M1: $\frac{dy}{dx} = kz^{-\frac{3}{2}}\frac{dz}{dx}$ A1: Correct differentiation	M1 A1
	$-\frac{1}{2}z^{-\frac{3}{2}}\frac{dz}{dx}+\frac{2x}{z^{\frac{1}{2}}} = xe^{-x^2}z^{-\frac{3}{2}}$	Substitutes for dy/dx	M1
	$\frac{dz}{dx}-4xz = -2xe^{-x^2}$ *	Correct completion to printed answer with no errors seen	A1 cso
			(4)
	Alternative 1		
	$\frac{dz}{dy} = -2y^{-3}$ oe	M1: $\frac{dz}{dy} = ky^{-3}$ A1: Correct differentiation	M1 A1
	$-\frac{1}{2}y^3\frac{dz}{dx}+2xy = xe^{-x^2}y^3$	Substitutes for dy/dx	M1
	$\frac{dz}{dx}-4xz = -2xe^{-x^2}$ *	Correct completion to printed answer with no errors seen	A1
	Alternative 2		
	$\frac{dz}{dx} = -2y^{-3}\frac{dy}{dx}$	M1: $\frac{dz}{dx} = ky^{-3}\frac{dy}{dx}$ inc chain rule A1: Correct differentiation	M1 A1
	$-\frac{1}{2}y^3\frac{dz}{dx}+2xy = xe^{-x^2}y^3$	Substitutes for dy/dx	M1
	$\frac{dz}{dx}-4xz = -2xe^{-x^2}$ *	Correct completion to printed answer with no errors seen	A1
(b)	$I = e^{\int -4x\,dx} = e^{-2x^2}$	M1: $I = e^{\int \pm 4x\,dx}$ A1: e^{-2x^2}	M1 A1
	$ze^{-2x^2} = \int -2xe^{-3x^2}\,dx$	$z \times I = \int -2xe^{-x^2}I\,dx$	dM1
	$\frac{1}{3}e^{-3x^2}\ (+c)$	$\int xe^{qx^2}\,dx = pe^{qx^2}\ (+c)$	M1
	$z = ce^{2x^2}+\frac{1}{3}e^{-x^2}$	Or equivalent	A1
			(5)

Question	Scheme		Marks
3(c)	$\dfrac{1}{y^2} = c\mathrm{e}^{2x^2} + \dfrac{1}{3}\mathrm{e}^{-x^2} \Rightarrow y^2 = \dfrac{1}{c\mathrm{e}^{2x^2} + \frac{1}{3}\mathrm{e}^{-x^2}}$	$y^2 = \dfrac{1}{(b)}\left(= \dfrac{3\mathrm{e}^{x^2}}{1 + k\mathrm{e}^{3x^2}}\right)$	B1ft
			(1)
			(10 marks)

Question	Scheme		Marks
4(a)	$w = \dfrac{z-1}{z+1}$		
	$w = \dfrac{z-1}{z+1} \Rightarrow wz + w = z - 1 \Rightarrow z = \dots$	Attempt to make z the subject	M1
	$z = \dfrac{w+1}{1-w}$	Correct expression in terms of w	A1
	$= \dfrac{u+iv+1}{1-u-iv} \times \dfrac{1-u+iv}{1-u+iv}$	Introduces "$u + iv$" and multiplies top and bottom by the complex conjugate of the bottom	M1
	$x = \dfrac{-u^2 - v^2 + 1}{\dots\dots}, \quad y = \dfrac{2v}{\dots\dots}$		
	$y = 2x \Rightarrow 2v = -2u^2 - 2v^2 + 2$	Uses real and imaginary parts and $y = 2x$ to obtain an equation connecting "u" and "v" Can have the 2 on the wrong side.	M1
	$u^2 + \left(v + \frac{1}{2}\right)^2 - \frac{1}{4} = 1$	Processes their equation to a form that is recognisable as a circle ie coefficients of u^2 and v^2 are the same and no uv terms	M1
	Centre $(0, -\frac{1}{2})$, radius $\dfrac{\sqrt{5}}{2}$	A1: Correct centre (allow -½i) A1: Correct radius	A1,A1
			(7)
	Special Case:		
	$w = \dfrac{x+iy-1}{x+iy+1} = \dfrac{(x-1)+2xi}{(x+1)+2xi} \times \dfrac{(x+1)-2xi}{(x+1)-2xi}$	M1: rationalise the denominator, may have $2x$ or y	
	$= \dfrac{(x^2-1)+4x^2+2xi(x+1-(x-1))}{(x+1)^2+4x^2}$	A1: Correct result in terms of x only. Must have rational denominator shown, but no other simplification needed	
(b)		B1ft: Their circle correctly positioned provided their equation does give a circle B1: Completely correct sketch and shading	B1ft B1
			(2)
			(9 marks)

Question	Scheme		Marks
5(a)	$y=\cot x$		
	$\dfrac{\mathrm{d}y}{\mathrm{d}x}=-\mathrm{cosec}^2 x$		
	$\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2}=\left(-2\,\mathrm{cosec}\,x\right)\left(-\mathrm{cosec}\,x\cot x\right)$	M1: Differentiates using the chain rule or product/quotient rule A1: Correct derivative	M1A1
	$=2\,\mathrm{cosec}^2 x\cot x=2\cot x+2\cot^3 x$*	A1: Correct completion to printed answer $1+\cot^2 x=\mathrm{cosec}^2 x$ or $\cos^2 x+\sin^2 x=1$ must be used Full working must be shown	A1cso*
			(3)
	Alternative		
	$y=\dfrac{\cos x}{\sin x}\rightarrow\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{-\sin^2 x-\cos^2 x}{\sin^2 x}=-\dfrac{1}{\sin^2 x}$		
	$\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2}=-\left(-2\sin^{-3} x\cos x\right)=\ldots$		M1A1
	Correct completion to printed answer see above		A1
			(3)
(b)	$\dfrac{\mathrm{d}^3 y}{\mathrm{d}x^3}=-2\,\mathrm{cosec}^2 x-6\cot^2 x\,\mathrm{cosec}^2 x$	Correct third derivative	B1
	$=-2\left(1+\cot^2 x\right)-6\cot^2 x\left(1+\cot^2 x\right)$	Uses $1+\cot^2 x=\mathrm{cosec}^2 x$	M1
	$=-6\cot^4 x-8\cot^2 x-2$	cso	A1
			(3)
(c)	$\mathrm{f}\left(\dfrac{\pi}{3}\right)=\dfrac{1}{\sqrt{3}},\mathrm{f}'\left(\dfrac{\pi}{3}\right)=-\dfrac{4}{3},\mathrm{f}''\left(\dfrac{\pi}{3}\right)=\dfrac{8}{3\sqrt{3}},\mathrm{f}'''\left(\dfrac{\pi}{3}\right)=-\dfrac{16}{3}$ M1: Attempts all 4 values at $\dfrac{\pi}{3}$ No working need be shown		M1
	$(y=)\dfrac{1}{\sqrt{3}}-\dfrac{4}{3}\left(x-\dfrac{\pi}{3}\right)+\dfrac{4}{3\sqrt{3}}\left(x-\dfrac{\pi}{3}\right)^2-\dfrac{8}{9}\left(x-\dfrac{\pi}{3}\right)^3$ M1: Correct application of Taylor using their values. Must be up to and including $\left(x-\dfrac{\pi}{3}\right)^3$ A1: Correct expression Must start $y=\ldots.$ or $\cot x$ f(x) allowed provided defined here or above as $\mathrm{f}(x)=\cot x$ or y Decimal equivalents allowed (min 3 sf apart from 0.77), 0.578, 1.33, 0.770, (0.7698.., so accept 0.77) 0.889		M1A1
			(3)
			(9 marks)

Question	Scheme		Marks
6(a)	$$\frac{d^2y}{dx^2}-2\frac{dy}{dx}-3y=2\sin x$$		
	AE: $m^2-2m-3=0$		
	$m^2-2m-3=0\Rightarrow m=...(-1,3)$	Forms Auxiliary Equation and attempts to solve (usual rules)	M1
	$(y=)\,Ae^{3x}+Be^{-x}$	Cao	A1
	PI: $(y=)\,p\sin x+q\cos x$	Correct form for PI	B1
	$(y'=)\,p\cos x-q\sin x$ $(y''=)-p\sin x-q\cos x$		
	$-p\sin x-q\cos x-2(p\cos x-q\sin x)-3p\sin x-3q\cos x=2\sin x$ Differentiates twice and substitutes		M1
	$2q-4p=2,\ 4q+2p=0$	Correct equations	A1
	$p=-\frac{2}{5},\ q=\frac{1}{5}$	A1A1 both correct A1A0 one correct	A1 A1
	$y=\frac{1}{5}\cos x-\frac{2}{5}\sin x$		
	$y=Ae^{3x}+Be^{-x}+\frac{1}{5}\cos x-\frac{2}{5}\sin x$	Follow through their p and q and their CF	B1ft
			(8)
(b)	$y'=3Ae^{3x}-Be^{-x}-\frac{1}{5}\sin x-\frac{2}{5}\cos x$	Differentiates their GS	M1
	$0=A+B+\frac{1}{5},\ 1=3A-B-\frac{2}{5}$	M1: Uses the given conditions to give two equations in A and B A1: Correct equations	M1 A1
	$A=\frac{3}{10},\ B=-\frac{1}{2}$	Solves for A and B Both correct	A1
	$y=\frac{3}{10}e^{3x}-\frac{1}{2}e^{-x}+\frac{1}{5}\cos x-\frac{2}{5}\sin x$	Sub their values of A and B in their GS	A1ft
			(5)
			(13 marks)

Question	Scheme		Marks
7(a)	$\theta=\frac{\pi}{3}\Rightarrow r=\sqrt{3}\sin\left(\frac{\pi}{3}\right)=\frac{3}{2}$	Attempt to verify coordinates in at least one of the polar equations	M1
	$\theta=\frac{\pi}{3}\Rightarrow r=1+\cos\left(\frac{\pi}{3}\right)=\frac{3}{2}$	Coordinates verified in both curves (Coordinate brackets not needed)	A1
			(2)
	Alternative		
	Equate rs: $\sqrt{3}\sin\theta=1+\cos\theta$ and verify (by substitution) that $\theta=\frac{\pi}{3}$ is a solution **or** solve by using $t=\tan\frac{\theta}{2}$ **or** writing $\frac{\sqrt{3}}{2}\sin\theta-\frac{1}{2}\cos\theta=\frac{1}{2}$ $\quad\sin\left(\theta-\frac{\pi}{6}\right)=\frac{1}{2}$ $\quad\theta=\frac{\pi}{3}$ Squaring the original equation allowed as θ is known to be between 0 and π		M1
	Use $\theta=\frac{\pi}{3}$ in either equation to obtain $r=\frac{3}{2}$		A1
			(2)
(b)	$\frac{1}{2}\int(\sqrt{3}\sin\theta)^2\,\mathrm{d}\theta,\quad \frac{1}{2}\int(1+\cos\theta)^2\,\mathrm{d}\theta$	Correct formula used on at least one curve (1/2 may appear later) Integrals may be separate or added or subtracted.	M1
	$=\frac{1}{2}\int 3\sin^2\theta\,\mathrm{d}\theta,\quad \frac{1}{2}\int(1+2\cos\theta+\cos^2\theta)\,\mathrm{d}\theta$		
	$=\left(\frac{1}{2}\right)\int\frac{3}{2}(1-\cos 2\theta)\,\mathrm{d}\theta,\quad \left(\frac{1}{2}\right)\int\left(1+2\cos\theta+\frac{1}{2}(1+\cos 2\theta)\right)\mathrm{d}\theta$ Attempt to use $\sin^2\theta$ or $\cos^2\theta=\pm\frac{1}{2}\pm\frac{1}{2}\cos 2\theta$ on either integral **Not** dependent 1/2 may be missing		M1
	$=\frac{3}{4}\left[\theta-\frac{1}{2}\sin 2\theta\right]_{(0)}^{\left(\frac{\pi}{3}\right)},\quad \frac{1}{2}\left[\frac{3}{2}\theta+2\sin\theta+\frac{1}{4}\sin 2\theta\right]_{\left(\frac{\pi}{3}\right)}^{(\pi)}$ Correct integration (ignore limits) A1A1 or A1A0		A1, A1
	$R=\frac{3}{4}\left[\frac{\pi}{3}-\frac{\sqrt{3}}{4}(-0)\right]+\frac{1}{2}\left[\frac{3\pi}{2}-\left(\frac{\pi}{2}+\sqrt{3}+\frac{\sqrt{3}}{8}\right)\right]$	Correct use of limits for both integrals Integrals must be added. Dep on both previous M marks	**dd**M1
	$=\frac{3}{4}\left(\pi-\sqrt{3}\right)$	Cao No equivalents allowed	A1
			(6)
			(8 marks)

Question	Scheme		Marks
8(a)	$\left(z+\frac{1}{z}\right)^3\left(z-\frac{1}{z}\right)^3=\left(z^2-\frac{1}{z^2}\right)^3$		
	$=z^6-3z^2+\frac{3}{z^2}-z^{-6}$	M1: Attempt to expand A1: Correct expansion	M1A1
	$=z^6-\frac{1}{z^6}-3\left(z^2-\frac{1}{z^2}\right)$	Correct answer with no errors seen	A1
			(3)
	Alternative		
	$\left(z+\frac{1}{z}\right)^3=z^3+3z+\frac{3}{z}+\frac{1}{z^3}$, $\left(z-\frac{1}{z}\right)^3=z^3-3z+\frac{3}{z}-\frac{1}{z^3}$		M1A1
	M1: Attempt to expand both cubic brackets A1: Correct expansions		
	$=z^6-\frac{1}{z^6}-3\left(z^2-\frac{1}{z^2}\right)$	Correct answer with no errors	A1
			(3)
(b)(i)(ii)	$z^n=\cos n\theta+i\sin n\theta$	Correct application of de Moivre	B1
	$z^{-n}=\cos(-n\theta)+i\sin(-n\theta)=\pm\cos n\theta\pm\sin n\theta$ but must be different from their z^n	Attempt z^{-n}	M1
	$z^n+\frac{1}{z^n}=2\cos n\theta$*, $z^n-\frac{1}{z^n}=2i\sin n\theta$*	$z^{-n}=\cos n\theta-i\sin n\theta$ must be seen	A1*
			(3)
(c)	$\left(z+\frac{1}{z}\right)^3\left(z-\frac{1}{z}\right)^3=(2\cos\theta)^3(2i\sin\theta)^3$		B1
	$z^6-\frac{1}{z^6}-3\left(z^2-\frac{1}{z^2}\right)=2i\sin 6\theta-6i\sin 2\theta$	Follow through their k in place of 3	B1ft
	$-64i\sin^3\theta\cos^3\theta=2i\sin 6\theta-6i\sin 2\theta$	Equating right hand sides and simplifying $2^3\times(2\mathrm{i})^3$ (B mark needed for each side to gain M mark)	M1
	$\cos^3\theta\sin^3\theta=\frac{1}{32}(3\sin 2\theta-\sin 6\theta)$ *		A1cso
			(4)

Question	Scheme		Marks
8(d)	$\int_0^{\frac{\pi}{8}} \cos^3\theta \sin^3\theta \,d\theta = \int_0^{\frac{\pi}{8}} \frac{1}{32}(3\sin 2\theta - \sin 6\theta)\,d\theta$		
	$= \frac{1}{32}\left[-\frac{3}{2}\cos 2\theta + \frac{1}{6}\cos 6\theta\right]_0^{\frac{\pi}{8}}$	M1: $p\cos 2\theta + q\cos 6\theta$ A1: Correct integration Differentiation scores M0A0	M1 A1
	$= \frac{1}{32}\left[\left(-\frac{3}{2\sqrt{2}} - \frac{1}{6\sqrt{2}}\right) - \left(-\frac{3}{2} + \frac{1}{6}\right)\right] = \frac{1}{32}\left(\frac{4}{3} - \frac{5\sqrt{2}}{6}\right)$	dM1: Correct use of limits – lower limit to have non-zero result. Dep on previous M mark A1: Cao (oe) but must be exact	dM1 A1
			(4)
			(14 marks)