Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel
International Advanced Level

Centre Number | Candidate Number

Sample Assessment Materials for first teaching September 2018

(Time: 1 hour 30 minutes) | Paper Reference **WFM02/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Further Pure Mathematics FP2

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for each question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

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Answer ALL questions. Write your answers in the spaces provided.

1. Using algebra, find the set of values of x for which

$$\frac{x}{x+2} < \frac{2}{x+5}$$

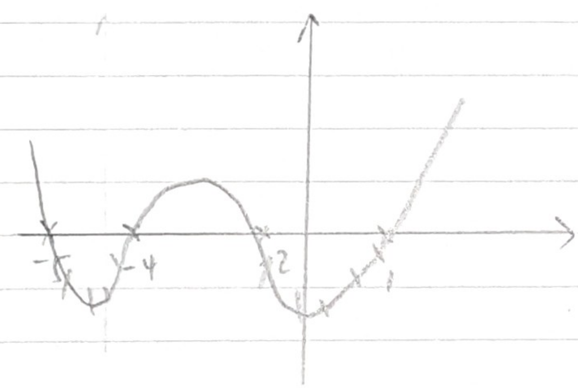
(7)

$x(x+5)^2(x+2) < 2(x+5)(x+2)^2$

$x(x+5)^2(x+2) - 2(x+5)(x+2)^2 < 0$

$(x+5)(x+2)(x^2+3x-4) < 0$

$(x+5)(x+2)(x+4)(x-1) < 0$



$-5 < x < -4$ and $-2 < x < 1$

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2. (a) Express $\dfrac{1}{(r+6)(r+8)}$ in partial fractions.

(1)

(b) Hence show that

$$\sum_{r=1}^{n} \frac{2}{(r+6)(r+8)} = \frac{n(an+b)}{56(n+7)(n+8)}$$

where a and b are integers to be found.

(4)

(a) $\dfrac{A}{r+6} + \dfrac{B}{r+8}$

$A(r+8) + B(r+6) = 1$

let $r = -8$

$-2B = 1$

$B = -\dfrac{1}{2}$

let $r = -6$

$2A = 1$

$A = \dfrac{1}{2}$

$\therefore = \dfrac{1}{2(r+6)} - \dfrac{1}{2(r+8)}$

(b) $\sum 2\left(\dfrac{1}{2(r+6)} - \dfrac{1}{2(r+8)}\right)$

$= \sum \dfrac{1}{r+6} - \dfrac{1}{r+8}$

let $r=1$ $\quad \dfrac{1}{7} \quad \cancel{-\dfrac{1}{9}}$

$r=2$ $\quad \dfrac{1}{8} \quad \cancel{-\dfrac{1}{10}}$

$r=3$ $\quad \cancel{\dfrac{1}{9}} \quad \cancel{-\dfrac{1}{11}}$

$r=n-1 = \cancel{\dfrac{1}{n+5}} \quad -\dfrac{1}{n+7}$

$r=n$ $\quad \cancel{\dfrac{1}{n+6}} \quad -\dfrac{1}{n+8}$

$\dfrac{1}{7} + \dfrac{1}{8} - \dfrac{1}{n+7} - \dfrac{1}{n+8}$.

$\dfrac{8(n+7)(n+8) + 7(n+7)(n+8) - 56(n+8) - 56(n}{56(n+7)(n+8)}$

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Question 2 continued

$8(n^2+15n+56)+7(n^2+15n+56)-56n-448-56n-392$

$=\dfrac{15n^2+113n}{56(n+7)(n+8)}$

$\dfrac{n(15n+113)}{56(n+7)(n+8)}$

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3. (a) Show that the substitution $z = y^{-2}$ transforms the differential equation

$$\frac{dy}{dx} + 2xy = xe^{-x^2}y^3 \quad \text{(I)}$$

into the differential equation

$$\frac{dz}{dx} - 4xz = -2xe^{-x^2} \quad \text{(II)}$$

(4)

(b) Solve differential equation (II) to find z as a function of x.

(5)

(c) Hence find the general solution of differential equation (I), giving your answer in the form $y^2 = f(x)$.

(1)

$z = \frac{1}{y^2} \qquad y^2 = \frac{1}{z}.$

$2y\frac{dy}{dx} = -\frac{1}{z^2}\frac{dz}{dx}.$

Multiplying the original eqn. by $2y$ gives

$\underline{2y\frac{dy}{dx}} + 4y^2x = 2y^4xe^{-x^2}$

$z^2 \times -\frac{1}{z^2} \cdot \frac{dz}{dx} + \frac{4x}{z} = \frac{2}{z^2} \cdot xe^{-x^2} \times z^2$

$\frac{dz}{dx} - 4xz = -2xe^{-x^2}.$

(b) ~~cf~~ $e^{\int p\,dx}$

$= e^{\int -4x\,dx}$

$= e^{-2x^2} \Rightarrow$ If.

If$\cdot z = \int$ If$\cdot Q\,dx + c.$

$e^{-2x^2} \cdot z = \int e^{-2x^2} \cdot -2xe^{-x^2}\,dx.$

$e^{-2x^2} \cdot z = \frac{1}{3}e^{-3x^2} + c.$

$z = \frac{1}{3}e^{-x^2} + ce^{2x^2}.$

(c) $y^2 = \frac{1}{z}. \qquad z = \frac{1}{y^2}.$

$\frac{1}{y^2} = \frac{1}{3}e^{-x^2} + ce^{2x^2}.$

$y^2 = \frac{1}{\frac{1}{3}e^{-x^2} + ce^{2x^2}}.$

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4. A transformation T from the z-plane to the w-plane is given by

$$w = \frac{z-1}{z+1}, \quad z \neq -1$$

The line in the z-plane with equation $y = 2x$ is mapped by T onto the curve C in the w-plane.

(a) Show that C is a circle and find its centre and radius. **(7)**

The region $y < 2x$ in the z-plane is mapped by T onto the region R in the w-plane.

(b) Sketch circle C on an Argand diagram and shade and label region R. **(2)**

$wz + w = z - 1$

$wz - z = -w - 1$

$z(w-1) = -1(w+1)$

$z = \frac{-1(w+1)}{(w-1)}$

$z = -\frac{(u+iv+1)}{((u-1)+iv)} \times \frac{((u-1)-iv)}{((u-1)-iv)}$

$= -1[(u+1)(u-1) - iv(u+1) + iv(u-1) - i^2v^2]$

$= -1[u^2 - 1 - iuv - iv + iuv - iv + v^2]$

$= \frac{-1 + v^2 + u^2 - 2iv}{(u-1)^2 + v^2}$

$x = \frac{u^2 + v^2 - 1}{(u-1)^2 + v^2}$

$2x = \frac{-2v}{(u-1)^2 + v^2}$

$\frac{2u^2 + 2v^2 - 2}{\cancel{(u-1)^2 + v^2}} = \frac{-2v}{\cancel{(u-1)^2 + v^2}}$

$2u^2 + 2v^2 + 2v - 2 = 0$

$u^2 + v^2 + v - 1 = 0$

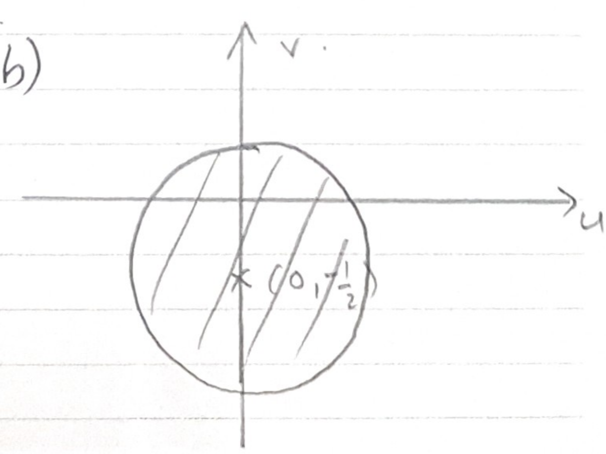
$u^2 + \left(v + \frac{1}{2}\right)^2 - \frac{1}{4} - 1 = 0$

$u^2 + \left(v + \frac{1}{2}\right)^2 = \frac{5}{4}$

circle centre $(0, -\frac{1}{2})$

radius $= \frac{\sqrt{5}}{2}$

(b)



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5. Given that $y = \cot x$,

(a) show that

$$\frac{d^2y}{dx^2} = 2\cot x + 2\cot^3 x$$

(3)

(b) Hence show that

$$\frac{d^3y}{dx^3} = p\cot^4 x + q\cot^2 x + r$$

where p, q and r are integers to be found.

(3)

(c) Find the Taylor series expansion of $\cot x$ in ascending powers of $\left(x - \frac{\pi}{3}\right)$ up to and including the term in $\left(x - \frac{\pi}{3}\right)^3$.

(3)

$y = \frac{\cos x \to u}{\sin x \to v}$

$\frac{dy}{dx} = \frac{vu' - uv'}{v^2}$

$= \frac{\sin x(-\sin x) - (\cos x \cdot \cos x)}{\sin^2 x}$

$= -\sin^2 x - \cos^2 x$

$= \frac{-1(\cos^2 x + \sin^2 x)}{\sin^2 x}$

$\frac{dy}{dx} = \frac{-1}{\sin^2 x}$

$\cancel{(b)}\ \frac{d^2y}{dx^2} = \frac{\sin^2 x(0) - (-1 \cdot 2\sin x\cos x)}{\sin^4 x}$

$= \frac{2\sin x\cos x}{\sin^4 x} = \frac{2\cos x}{\sin^3 x}$

$\frac{2\cos x}{\sin x} \cdot \frac{1}{\sin^2 x}$

$= 2\cot x \cdot \operatorname{cosec}^2 x$

$2\cot x(1 + \cot^2 x)$

$\frac{d^2y}{dx^2} = 2\cot x + 2\cot^3 x$ as req

(b) $\frac{d^2y}{dx^2} = \frac{2\cos x}{\sin^3 x}$

$= \frac{\sin^3 x \cdot -2\sin x - [2\cos x \cdot 3\sin^2 x\cos x]}{\sin^6 x}$

$= \frac{-2\sin^4 x - 6\sin^2 x\cos^2 x}{\sin^6 x}$

$= \frac{-2\sin^2 x - 6\cos^2 x}{\sin^4 x}$

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Question 5 continued

$= -2\operatorname{cosec}^2 x - 6\cot^2 x . \operatorname{cosec}^2 x$

$-2(1+\cot^2 x) - 6\cot^2 x(1+\cot^2 x)$

$= -2 - 2\cot^2 x - 6\cot^2 x + 6\cot^4 x$

$= -8\cot^2 x - 6\cot^4 x - 2.$

(c) $a = \pi/3.$

$f(x) = \cot x$

$f'(x) = \dfrac{-1}{\sin^2 x}$

$f''(x) = 2\cot x + 2\cot^3 x$

$f'''(x) = -6\cot^4 x - 8\cot^2 x - 2$

$f(\pi/3) = \dfrac{\sqrt{3}}{3}$

$f'(\pi/3) = -4/3$

$f''(\pi/3) = \dfrac{8\sqrt{3}}{9}$

$f'''(\pi/3) = \dfrac{-16}{3}.$

$f(x) = \dfrac{\sqrt{3}}{3} - \dfrac{4}{3}(x - \pi/3) + \dfrac{8\sqrt{3}}{9\times 2!}(x - \pi/3)^2 - \dfrac{16}{3\times 3!}(x - \pi/3)^3.$

$= \dfrac{\sqrt{3}}{3} - \dfrac{4}{3}\left(x - \dfrac{\pi}{3}\right) + \dfrac{4}{9}\sqrt{3}\left(x - \dfrac{\pi}{3}\right)^2 - \dfrac{8}{9}\left(x - \dfrac{\pi}{3}\right)^3$

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6. (a) Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 2\sin x \quad \text{(I)}$$

(8)

Given that $y = 0$ and $\frac{dy}{dx} = 1$ when $x = 0$

(b) find the particular solution of differential equation (I).

(5)

CF

$m^2 - 2m - 3 = 0$

$\frac{2 \pm \sqrt{2^2 - 4(-3)}}{2}$

$m = -1 \quad m = 3$

$Ae^{-x} + Be^{3x} \rightarrow CF$

PI

$y = \lambda \sin x + \mu \cos x$

$\frac{dy}{dx} = \lambda \cos x - \mu \sin x$

$\frac{d^2y}{dx^2} = -\lambda \sin x - \mu \cos x$

$-\lambda \sin x - \mu \cos x$

$+ \; +2\mu \sin x - 2\lambda \cos x$

$+ \; -3\lambda \sin x \quad -3\mu \cos x$

$= \sin x(-4\lambda + 2\mu) + \cos x(-4\mu - 2\lambda)$

$-4\lambda + 2\mu = 2$

$-4\mu - 2\lambda = 0$

$\lambda = -\frac{2}{5} \quad \mu = \frac{1}{5}$

$\therefore y = Ae^{-x} + Be^{3x} - \frac{2}{5}\sin x + \frac{1}{5}\cos x$

(b) $0 = Ae^0 + Be^0 - \frac{2}{5}(0) + \frac{1}{5}$

$A + B = -\frac{1}{5}$

$\frac{dy}{dx} = -Ae^{-x} + 3Be^{3x} - \frac{2}{5}\cos x - \frac{1}{5}\sin x$

$1 = -A + 3B - \frac{2}{5}$

$-A + 3B = \frac{7}{5}$

$A = -\frac{1}{2} \quad B = \frac{3}{10}$

$y = -\frac{1}{2}e^{-x} + \frac{3}{10}e^{3x} - \frac{2}{5}\sin x + \frac{1}{5}\cos x$

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7.

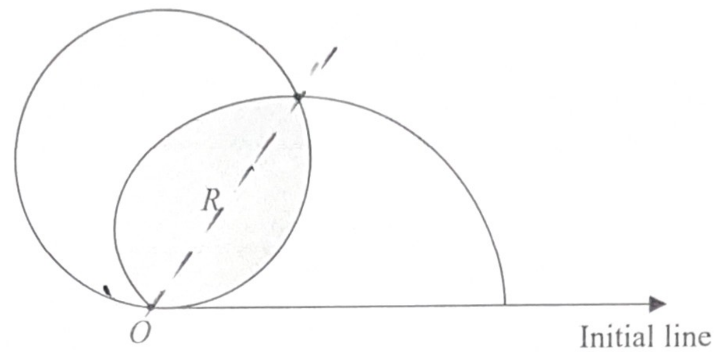

Figure 1

Figure 1 shows the two curves given by the polar equations

$$r = \sqrt{3}\sin\theta, \quad 0 \leqslant \theta \leqslant \pi$$

$$r = 1 + \cos\theta, \quad 0 \leqslant \theta \leqslant \pi$$

(a) Verify that the curves intersect at the point P with polar coordinates $\left(\frac{3}{2}, \frac{\pi}{3}\right)$. **(2)**

The region R, bounded by the two curves, is shown shaded in Figure 1.

(b) Use calculus to find the exact area of R, giving your answer in the form $a\left(\pi - \sqrt{3}\right)$, where a is a constant to be found. **(6)**

(a) $r = \sqrt{3}\sin\left(\frac{\pi}{3}\right)$

$= \frac{3}{2}$

$r = 1 + \cos\frac{\pi}{3}$

$= \frac{3}{2}$

$\therefore$ POI $= \left(\frac{3}{2}, \frac{\pi}{3}\right)$

(b) $\frac{1}{2}\int_0^{\frac{\pi}{3}} (\sqrt{3}\sin\theta)^2 \, d\theta$

$= \frac{1}{2}\int 3\sin^2\theta$

$= \frac{3}{2}\int \sin^2\theta$

$= \frac{3}{2}\int \frac{1-\cos 2\theta}{2}$

$= \frac{3}{4}\int 1 - \cos 2\theta \, d\theta$

$\frac{3}{4}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi/3}$

$= \left[\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right] \times \frac{3}{4}$ — ①

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Question 7 continued

$$\frac{1}{2}\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} (1+\cos\theta)^2 \, d\theta$$

$$\frac{1}{2}\int 1+2\cos\theta+\cos^2\theta \, d\theta$$

$$\frac{1}{2}\int 1+2\cos\theta + \frac{1}{2}\int \cos 2\theta+1$$

$$= \frac{1}{2}\left[\theta + 2\sin\theta + \frac{\sin 2\theta}{4} + \frac{1}{2}\theta\right]_{\frac{\pi}{3}}^{\pi}$$

$$= \frac{1}{2}\left[\frac{3\pi}{2} - \left(\frac{\pi}{2} + \sqrt{3} + \frac{\sqrt{3}}{8}\right)\right] \leftarrow ②$$

① + ②

$$= \frac{3}{4}(\pi - \sqrt{3})$$

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8. (a) Show that

$$\left(z+\frac{1}{z}\right)^3\left(z-\frac{1}{z}\right)^3 = z^6 - \frac{1}{z^6} - k\left(z^2 - \frac{1}{z^2}\right)$$

where k is a constant to be found.

(3)

Given that $z = \cos\theta + \mathrm{i}\sin\theta$, where θ is real,

(b) show that

(i) $z^n + \frac{1}{z^n} = 2\cos n\theta$

(ii) $z^n - \frac{1}{z^n} = 2\mathrm{i}\sin n\theta$

(3)

(c) Hence show that

$$\cos^3\theta\sin^3\theta = \frac{1}{32}\,(3\sin 2\theta - \sin 6\theta)$$

(4)

(d) Find the exact value of

$$\int_0^{\frac{\pi}{8}} \cos^3\theta\sin^3\theta\,\mathrm{d}\theta$$

(4)

(a) $\left(z+\frac{1}{z}\right)^3\left(z-\frac{1}{z}\right)^3$

is the same as

$\left(z^2-\frac{1}{z^2}\right)^3$

$= z^6 - 3z^2 + \frac{3}{z^2} - z^{-6}$

$= z^6 - \frac{1}{z^6} - 3\left(z^2 - \frac{1}{z^2}\right)$

where $\underline{\underline{k=2}}$

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Question 8 continued

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(b) $z^n + \frac{1}{z^n}$.

$z = \cos\theta + i\sin\theta$

z^n from De Moivre's Theorem

$= (\cos\theta + i\sin\theta)^n$

$= \cos n\theta + i\sin n\theta$

$z^{-n} = \cos n\theta - i\sin n\theta$

$\therefore z^n + z^{-n} = 2\cos n\theta$ as req

$z^n - z^{-n}$

$= \cos n\theta + i\sin n\theta - \cos n\theta + i\sin n\theta$

$= \underline{2i\sin n\theta}$

(c) $\left(z+\frac{1}{z}\right)^3\left(z-\frac{1}{z}\right)^3$

$=(2\cos\theta)^3(2i\sin\theta)^3$

$z^6 - \frac{1}{z^6} - 3\left(z^2 - \frac{1}{z^2}\right)$

$= 2i\sin 6\theta - 6i\sin 2\theta$.

$-64i\sin^3\theta\cos^3\theta = 2i\sin 6\theta$

$-6i\sin 2\theta$.

$\therefore \cos^3\theta\sin^3\theta = \frac{1}{32}\left[3\sin 2\theta - \sin 6\theta\right]$

(d) $\frac{1}{32}\int_0^{\pi/8} 3\sin 2\theta - \sin 6\theta \, d\theta$.

$= \left[-\frac{3\cos 2\theta}{2} + \frac{\cos 6\theta}{6}\right]_0^{\pi/8} \cdot \frac{1}{32}$

$= \frac{1}{32}\left[\left(-\frac{3}{2\sqrt{2}} - \frac{1}{6\sqrt{2}}\right) - \left(-\frac{3}{2} + \frac{1}{6}\right)\right]$

$= \frac{1}{32}\left(\frac{4}{3} - \frac{5\sqrt{2}}{6}\right)$