Write your name here

Surname	Other names

Pearson Edexcel
International
Advanced Level

Centre Number | | | | | |

Candidate Number | | | | |

Further Pure Mathematics F1

Advanced/Advanced Subsidiary

Friday 19 May 2017 – Morning
Time: 1 hour 30 minutes

Paper Reference
WFM01/01

You must have:
Mathematical Formulae and Statistical Tables (Blue)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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Pearson

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1. The quadratic equation

$$3x^2 - 5x + 1 = 0$$

has roots α and β.

Without solving the quadratic equation, find the exact value of

$$\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$$

(4)

Leave blank

Question 1 continued

Q1

(Total 4 marks)

Leave blank

2. Given that

$$\mathbf{A} = \begin{pmatrix} 3 & 1 & -2 \\ -1 & 0 & 5 \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} 2 & 4 \\ -k & 2k \\ 3 & 0 \end{pmatrix}, \text{ where } k \text{ is a constant}$$

(a) find the matrix **AB**,

(2)

(b) find the exact value of k for which $\det(\mathbf{AB}) = 0$

(2)

Leave blank

Question 2 continued

Q2

(Total 4 marks)

Leave blank

3. Prove by induction that for $n \in \mathbb{Z}^+$

$$\sum_{r=1}^{n} \frac{2}{r(r+1)(r+2)} = \frac{1}{2} - \frac{1}{(n+1)(n+2)}$$

(5)

Leave blank

Question 3 continued

Q3

(Total 5 marks)

Leave blank

4. The rectangular hyperbola H has parametric equations

$$x = 4t, \ y = \frac{4}{t}$$

The straight line with equation $3y - 2x = 10$ intersects H at the points A and B.

Given that the point A is above the x-axis,

(a) find the coordinates of the point A and the coordinates of the point B.

(5)

(b) Find the coordinates of the midpoint of AB.

(2)

Leave blank

Question 4 continued

Q4

(Total 7 marks)

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5. $$f(x) = 30 + \frac{7}{\sqrt{x}} - x^5, \qquad x > 0$$

The only real root, α, of the equation $f(x) = 0$ lies in the interval [2, 2.1].

(a) Starting with the interval [2, 2.1], use interval bisection twice to find an interval of width 0.025 that contains α.

(4)

(b) Taking 2 as a first approximation to α, apply the Newton-Raphson process once to $f(x)$ to find a second approximation to α, giving your answer to 2 decimal places.

(5)

Leave blank

Question 5 continued

Q5

(Total 9 marks)

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6. (a) Use the standard results for $\sum_{r=1}^{n} r^2$ and for $\sum_{r=1}^{n} r^3$ to show that, for all positive integers n,

$$\sum_{r=1}^{n} r^2(r+1) = \frac{n}{a}(n+1)(n+2)(3n+b)$$

where a and b are integers to be found.

(4)

(b) Hence find the value of

$$\sum_{r=25}^{49} (r^2(r+1)+2)$$

(4)

Leave blank

Question 6 continued

Q6

(Total 8 marks)

Leave blank

7.

$$f(z) = z^4 + 4z^3 + 6z^2 + 4z + a$$

where a is a real constant.

Given that $1 + 2i$ is a complex root of the equation $f(z) = 0$

(a) write down another complex root of this equation.

(1)

(b) (i) Hence, find the other roots of the equation $f(z) = 0$

(ii) State the value of a.

(7)

Leave blank

Question 7 continued

Q7

(Total 8 marks)

Leave blank

8. The parabola C has cartesian equation $y^2 = 36x$. The point $P(9p^2, 18p)$, where p is a positive constant, lies on C.

(a) Using calculus, show that an equation of the tangent to C at P is

$$py - x = 9p^2$$

(4)

This tangent cuts the directrix of C at the point $A(-a, 6)$, where a is a constant.

(b) Write down the value of a.

(1)

(c) Find the exact value of p.

(3)

(d) Hence find the exact coordinates of the point P, giving each coordinate as a simplified surd.

(3)

Leave blank

Question 8 continued

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Question 8 continued

Leave blank

Question 8 continued

Q8

(Total 11 marks)

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9. $$z = \frac{1}{5} - \frac{2}{5}\mathrm{i}$$

(a) Find the modulus and the argument of z, giving the modulus as an exact answer and giving the argument in radians to 2 decimal places.

(3)

Given that

$$zw = \lambda \mathrm{i}$$

where λ is a real constant,

(b) find w in the form $a + \mathrm{i}b$, where a and b are real. Give your answer in terms of λ.

(3)

(c) Given that $\lambda = \frac{1}{10}$

(i) find $\frac{4}{3}(z + w)$,

(ii) plot the points A, B, C and D, representing z, zw, w and $\frac{4}{3}(z + w)$ respectively, on a single Argand diagram.

(4)

Leave blank

Question 9 continued

Leave blank

Question 9 continued

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Question 9 continued

Q9

(Total 10 marks)

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10. *In your answers to this question, the elements of each matrix should be expressed in exact form in surds where necessary.*

The transformation U, represented by the 2×2 matrix **P**, is a rotation through 45° anticlockwise about the origin.

(a) Write down the matrix **P**.

(1)

The transformation V, represented by the 2×2 matrix **Q**, is a rotation through 60° anticlockwise about the origin.

(b) Write down the matrix **Q**.

(1)

The transformation U followed by the transformation V is the transformation T.
The transformation T is represented by the matrix **R**.

(c) Use your matrices from parts (a) and (b) to find the matrix **R**.

(3)

(d) Give a full geometric description of T as a single transformation.

(2)

(e) Deduce from your answers to parts (c) and (d) that $\sin 75° = \dfrac{1+\sqrt{3}}{2\sqrt{2}}$ and find the exact value of cos 75°, explaining your answers fully.

(2)

Leave blank

Question 10 continued

Leave blank

Question 10 continued

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Question 10 continued

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Question 10 continued

Q10

(Total 9 marks)

TOTAL FOR PAPER: 75 MARKS

END