Write your name here

Surname	Other names

Pearson Edexcel
International
Advanced Level

Centre Number | Candidate Number

Further Pure Mathematics F1

Advanced/Advanced Subsidiary

Thursday 14 May 2015 – Morning
Time: 1 hour 30 minutes

Paper Reference
WFM01/01

You must have:
Mathematical Formulae and Statistical Tables (Blue)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

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1. Given that

$$2z^3 - 5z^2 + 7z - 6 \equiv (2z - 3)(z^2 + az + b)$$

where a and b are real constants,

(a) find the value of a and the value of b.

(2)

(b) Given that z is a complex number, find the three exact roots of the equation

$$2z^3 - 5z^2 + 7z - 6 = 0$$

(3)

Leave blank

2. Use the standard results for $\sum_{r=1}^{n} r$ and for $\sum_{r=1}^{n} r^2$ to show that

$$\sum_{r=1}^{n} (3r - 2)^2 = \frac{n}{2}\left(an^2 + bn + c\right)$$

where a, b and c are integers to be found.

(5)

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3. It is given that α and β are roots of the equation

$$2x^2 - 7x + 4 = 0$$

(a) Find the exact value of $\alpha^2 + \beta^2$

(3)

(b) Find a quadratic equation which has roots $\dfrac{\alpha}{\beta}$ and $\dfrac{\beta}{\alpha}$, giving your answer in the form $ax^2 + bx + c = 0$, where a, b and c are integers.

(3)

Leave blank

4.

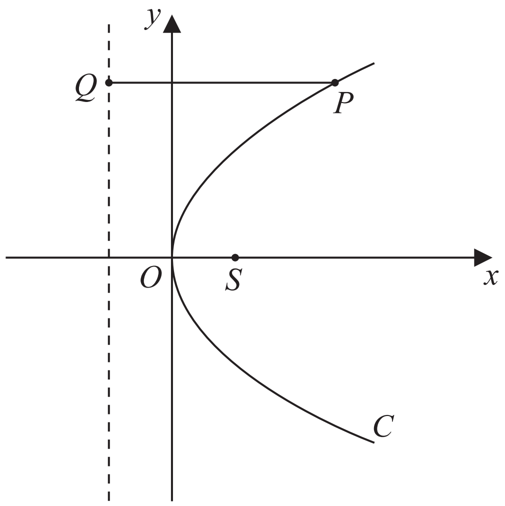

Figure 1

Figure 1 shows a sketch of the parabola C with equation $y^2 = 4ax$, where a is a positive constant. The point S is the focus of C and the point Q lies on the directrix of C. The point P lies on C where $y > 0$ and the line segment QP is parallel to the x-axis.

Given that the length of PS is 13

(a) write down the length of PQ.

(1)

Given that the point P has x coordinate 9

find

(b) the value of a,

(2)

(c) the area of triangle PSQ.

(3)

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Question 4 continued

(Total 6 marks)

Q4

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5. In the interval $2 < x < 3$, the equation

$$6 - x^2 \cos\left(\frac{x}{5}\right) = 0, \text{ where } x \text{ is measured in radians}$$

has exactly one root α.

(a) Starting with the interval [2, 3], use interval bisection twice to find an interval of width 0.25 which contains α.

(4)

(b) Use linear interpolation once on the interval [2, 3] to find an approximation to α. Give your answer to 2 decimal places.

(3)

Leave blank

6. The rectangular hyperbola, H, has cartesian equation

$$xy = 36$$

The three points $P\left(6p, \frac{6}{p}\right)$, $Q\left(6q, \frac{6}{q}\right)$ and $R\left(6r, \frac{6}{r}\right)$, where p, q and r are distinct, non-zero values, lie on the hyperbola H.

(a) Show that an equation of the line PQ is

$$pqy + x = 6(p + q)$$

(4)

Given that PR is perpendicular to QR,

(b) show that the normal to the curve H at the point R is parallel to the line PQ.

(6)

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Question 6 continued

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7.

$z = -3k - 2k\text{i}$, where k is a real, positive constant.

(a) Find the modulus and the argument of z, giving the argument in radians to 2 decimal places and giving the modulus as an exact answer in terms of k.

(3)

(b) Express in the form $a + \text{i}b$, where a and b are real and are given in terms of k where necessary,

(i) $\dfrac{4}{z + 3k}$

(ii) z^2

(5)

(c) Given that $k = 1$, plot the points A, B, C and D representing z, z^*, $\dfrac{4}{z + 3k}$ and z^2 respectively on a single Argand diagram.

(3)

Leave blank

Question 7 continued

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8.

$$\mathbf{P} = \begin{pmatrix} 3a & -4a \\ 4a & 3a \end{pmatrix}, \text{ where } a \text{ is a constant and } a > 0$$

(a) Find the matrix $\mathbf{P}^{-1}$ in terms of a.

(3)

The matrix **P** represents the transformation U which transforms a triangle T_1 onto the triangle T_2.
The triangle T_2 has vertices at the points $(-3a, -4a)$, $(6a, 8a)$, and $(-20a, 15a)$.

(b) Find the coordinates of the vertices of T_1

(3)

(c) Hence, or otherwise, find the area of triangle T_2 in terms of a.

(3)

The transformation V, represented by the 2×2 matrix **Q**, is a rotation through an angle α **clockwise** about the origin, where $\tan \alpha = \frac{4}{3}$ and $0 < \alpha < \frac{\pi}{2}$

(d) Write down the matrix **Q**, giving each element as an exact value.

(2)

The transformation U followed by the transformation V is the transformation W. The matrix **R** represents the transformation W.

(e) Find the matrix **R**.

(2)

Leave blank

Question 8 continued

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9. (i) Prove by induction that, for $n \in \mathbb{Z}^+$,

$$\sum_{r=1}^{n} r^2(2r-1) = \frac{1}{6}n(n+1)(3n^2+n-1)$$

(6)

(ii) Prove by induction that, for $n \in \mathbb{Z}^+$,

$$\begin{pmatrix} 7 & -12 \\ 3 & -5 \end{pmatrix}^n = \begin{pmatrix} 6n+1 & -12n \\ 3n & 1-6n \end{pmatrix}$$

(6)

Leave blank

Question 9 continued

P 4 4 9 7 1 R A 0 2 5 2 8

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