Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel International Advanced Level

Centre Number | Candidate Number

Monday 14 January 2019

Afternoon (Time: 1 hour 30 minutes) | Paper Reference **WFM01/01**

Further Pure Mathematics F1

Advanced/Advanced Subsidiary

You must have:
Mathematical Formulae and Statistical Tables (Blue)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need*.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

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1. The point A (12, 12) lies on the parabola with equation $y^2 = 12x$. The point S is the focus of this parabola. The line l passes through A and S.

(a) Find an equation of the line l.

(3)

The line l meets the directrix of the parabola at the point B.

(b) Find the coordinates of B.

(3)

(a) $S \Rightarrow \quad 4a = 12$

$a = 3.$

$(3, 0) \qquad (12, 12)$

$\frac{12-0}{12-3} = \frac{12}{9} = \frac{4}{3}$

$y = \frac{4}{3}x + c \qquad (3, 0)$

$0 = 4 + c$

$c = -4$

$y = \frac{4}{3}x - 4$

(b) $x = -3.$

$y = -4 - 4$

$y = -8.$

$(-3, -8).$

2.

$$f(z) = z^3 - 2z^2 + 16z - 32$$

(a) Show that f(2) = 0

(1)

(b) Use algebra to solve f(z) = 0 completely.

(3)

(c) Show, on a single Argand diagram, all three roots of the equation f(z) = 0

(2)

(a) $(2)^3 - 2(2)^2 + 16(2) - 32$

$= 0$

(b) $(z-2)(az^2 + bz + c)$

$az^3 = 1z^3 \quad \underline{a = 1}$

$-2c = -32 \quad \underline{c = 16}$

$-2az^2 + bz^2 = -2$

$-2 + b = -2$

$b = 0$

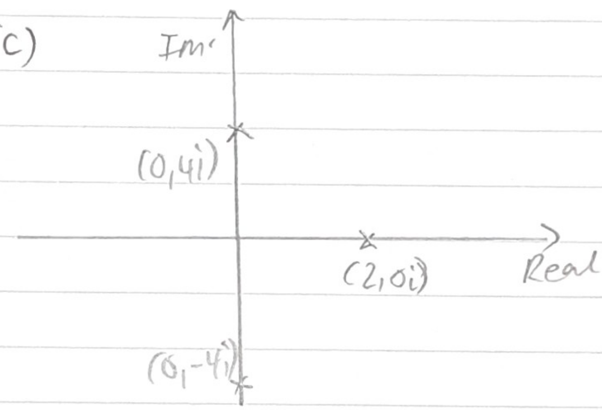
$\therefore f(z) = (z-2)(z^2+16)$

$z = 2$

$z^2 = -16$

$z = 0 \pm 4i$

(c)



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3. (a) Use the standard results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$ to show that, for all positive integers n,

$$\sum_{r=1}^{n} (2r+5)^2 = \frac{n}{3}\left[(an+b)^2 + c\right]$$

where a, b and c are integers to be found.

(5)

(b) Use the answer to part (a) to evaluate $\sum_{r=0}^{100} (2r+5)^2$

(2)

$\sum(2r+5)^2$

$= 4r^2 + 20r + 25.$

$4\sum r^2 + 20\sum r + 25\sum 1$

$= 4 \times \frac{1}{6} n(n+1)(2n+1) + 20\frac{n(n+1)}{2}$

$+25n.$

$= \frac{2}{3} n(n+1)(2n+1) + 10n(n+1)$

$+25n.$

$\frac{n}{3}\Big[2(n+1)(2n+1) + 30(n+1)$

$+75\Big].$

$\frac{n}{3}\left[2(2n^2+3n+1) + 30n + 105\right]$

$\frac{n}{3}\left[4n^2 + 36n + 107\right].$

$\frac{n}{3}\left[(2n+9)^2 + 26\right].$

(b) $\sum_{r=1}^{100} (2r+5)^2$

$= \frac{100}{3}\left[(2(100)+9)^2 + 26\right].$

$= 1456900.$

$1456900 + 25 =$

$\underline{\underline{1456925}}$

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4.

$$f(x) = 2x^3 - \frac{7}{x^2} + 16, \qquad x \neq 0$$

The equation $f(x) = 0$ has a single root α between $x = -2$ and $x = -1$

(a) Starting with the interval $[-2, -1]$, use interval bisection twice to find an interval of width 0.25 that contains α.

(3)

The equation $f(x) = 0$ also has a single root β in the interval $[0.6, 0.7]$.

(b) Taking 0.65 as a first approximation to β, apply the Newton-Raphson procedure once to $f(x)$ to obtain a second approximation to β, giving your answer to 4 decimal places.

(4)

(a)

$-2 \quad -1.5 \quad -1$

$f(-2) = \cancel{25/4} \; -7/4$

$f(-1) = 7$

$f(-1.5) = 221/36$

$-2 \quad -1.75 \quad -1.5$

$f(-1.75) = 671/224$

$\therefore \; -2 < \alpha < -1.75$

b) $f'(x) = 6x^2 + \frac{14}{x^3}$

$f(0.65) =$

$-0.0187973372 8$

$f'(0.65) = 53.51360719$

$x_0 - \left[\frac{f(x_0)}{f'(x_0)}\right]$

$= 0.65 - \left[\frac{-0.0187\ldots}{53.51\ldots}\right]$

$= \underline{\underline{0.6504}}$

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5. The rectangular hyperbola H has equation $xy = 16$

The point P, on H, has coordinates $\left(4p, \frac{4}{p}\right)$ where p is a non-zero constant.

(a) Show, using calculus, that the tangent to H at the point P has equation

$$x + p^2y = 8p$$

(4)

Given that the tangent to H at the point P passes through the point (7, 1)

(b) use algebra to find the coordinates of the two possible positions of P.

(4)

(a) $y = \frac{16}{x}$ $\quad \frac{dy}{dx} = \frac{-16}{x^2}$

at $x = 4p$.

$\frac{dy}{dx} = \frac{-16}{16p^2} = \frac{-1}{p^2}$

$y - y_0 = m(x - x_0)$

$y - \frac{4}{p} = -\frac{1}{p^2}(x - 4p)$

$p^2y - 4p = -x + 4p$.

$p^2y + x = 8p$ as req.

(b) $p^2(1) + 7 = 8p$.

$p^2 - 8p + 7 = 0$.

$\frac{8 \pm \sqrt{8^2 - 4(7)}}{2(1)}$

$p = 1$ or $p = 7$.

$x = 4$ $\quad$ $x = 7(4)$

$y = \frac{16}{4}$ $\quad$ $= 28$

$= 4$ $\quad$ $y = \frac{16}{28}$

$= \frac{4}{7}$

$\therefore$ P can be $(4, 4)$ or $(28, \frac{4}{7})$

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6. It is given that α and β are roots of the equation

$$12x^2 - 3x + 4 = 0$$

Without solving the equation,

(a) find the exact value of $\frac{2}{\alpha} + \frac{2}{\beta}$ **(3)**

(b) find a quadratic equation that has roots $\frac{2}{\alpha} - \beta$ and $\frac{2}{\beta} - \alpha$ giving your answer in the form $ax^2 + bx + c = 0$, where a, b and c are integers to be found. **(6)**

(a) $\alpha + \beta = -\frac{b}{a} = \frac{3}{12} = \frac{1}{4}$

$\alpha\beta = \frac{c}{a} = \frac{4}{12} = \frac{1}{3}$.

$\therefore \frac{2}{\alpha} + \frac{2}{\beta} = \frac{2(\alpha+\beta)}{\alpha\beta}$

$= \frac{2(0.25)}{1/3} = \frac{3}{2}$.

(b) Sum of roots:

$\frac{2}{\alpha} - \beta + \frac{2}{\beta} - \alpha$,

$\frac{2}{\alpha} + \frac{2}{\beta} - 1(\alpha+\beta)$

$\therefore = \frac{3}{2} - 1\left(\frac{1}{4}\right)$

$= \frac{5}{4}$.

Product of roots:

$\left(\frac{2}{\alpha} - \beta\right)\left(\frac{2}{\beta} - \alpha\right)$.

$\frac{4}{\alpha\beta} - 2 - 2 + \alpha\beta$.

$\frac{4}{1/3} + \frac{1}{3} - 4$

$= \frac{25}{3}$.

$\therefore x^2 - \frac{5}{4}x + \frac{25}{3} = 0$.

$12x^2 - 15x + 100 = 0$.

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7.

$$\mathbf{P} = \begin{pmatrix} -1 & -\sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix}$$

(a) Show that $\mathbf{P}^3 = 8\mathbf{I}$, where $\mathbf{I}$ is the 2×2 identity matrix. **(3)**

(b) Describe fully the transformation represented by the matrix $\mathbf{P}$ as a combination of two simple geometrical transformations. **(4)**

(c) Find the matrix $\mathbf{P}^{35}$, giving your answer in the form

$$\mathbf{P}^{35} = 2^k \begin{pmatrix} -1 & a \\ b & -1 \end{pmatrix}$$

where k is an integer and a and b are surds to be found. **(2)**

(a) $\underline{P^3}$

$$\begin{pmatrix} -1 & -\sqrt{3} \\ +\sqrt{3} & -1 \end{pmatrix}\begin{pmatrix} -1 & -\sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix}$$

$$\begin{pmatrix} 1-3 & \sqrt{3}+\sqrt{3} \\ -\sqrt{3}-\sqrt{3} & -3+1 \end{pmatrix}$$

$$= \begin{pmatrix} -2 & 2\sqrt{3} \\ -2\sqrt{3} & -2 \end{pmatrix}\begin{pmatrix} -1 & -\sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 8 & 0 \\ 0 & 8 \end{pmatrix}$$

$$8I = 8\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 8 & 0 \\ 0 & 8 \end{pmatrix} \quad \therefore \text{LHS} = \text{RHS}$$

$\therefore$ proved.

(b) → Enlargment centre (0, 0) S.F 2

→ Rotation 120° anticlockwise about (0, 0).

(c) $P^{35} = (P^3)^{11} \times P^2$

$P^3 = 8I$

$\therefore (P^3)^{11} = (8I)^{11} \times P^2$

$8 = 2^3 \quad \therefore (2I)^{33} \times \begin{pmatrix} -2 & 2\sqrt{3} \\ -2\sqrt{3} & -2 \end{pmatrix}$

$= 2^{34}\begin{pmatrix} -1 & \sqrt{3} \\ -\sqrt{3} & -1 \end{pmatrix}$ take 2 outside

$k = 34 \quad a = \sqrt{3} \quad b = -\sqrt{3}$.

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8. (i) Prove by induction that, for $n \in \mathbb{Z}^+$

$$\begin{pmatrix} 5 & -8 \\ 2 & -3 \end{pmatrix}^n = \begin{pmatrix} 1+4n & -8n \\ 2n & 1-4n \end{pmatrix}$$

(5)

(ii) A sequence of positive numbers is defined by

$$u_1 = 8, \quad u_2 = 40$$
$$u_{n+2} = 8u_{n+1} - 12u_n \qquad n \geqslant 1$$

Prove by induction that, for $n \in \mathbb{Z}^+$

$$u_n = 6^n + 2^n$$

(5)

(i) Basis:

Let $n=1$

LHS

$$\begin{pmatrix} 5 & -8 \\ 2 & -3 \end{pmatrix}^1 = \begin{pmatrix} 5 & -8 \\ 2 & -3 \end{pmatrix}$$

RHS

$$\begin{pmatrix} 1+4(1) & -8(1) \\ 2(1) & 1-4(1) \end{pmatrix}$$

$$= \begin{pmatrix} 5 & -8 \\ 2 & -3 \end{pmatrix}$$

$\therefore$ true for $n=1$.

Assumption:

Let $n=k$.

$$\begin{pmatrix} 5 & -8 \\ 2 & -3 \end{pmatrix}^k = \begin{pmatrix} 1+4k & -8k \\ 2k & 1-4k \end{pmatrix}$$

Induction

Let $n=k+1$

$$\begin{pmatrix} 5 & -8 \\ 2 & -3 \end{pmatrix}^{k+1} = \begin{pmatrix} 5 & -8 \\ 2 & -3 \end{pmatrix}^k \cdot \begin{pmatrix} 5 & -8 \\ 2 & -3 \end{pmatrix}$$

From previous step.

$$\begin{pmatrix} 5 & -8 \\ 2 & -3 \end{pmatrix}^k = \begin{pmatrix} 1+4k & -8k \\ 2k & 1-4k \end{pmatrix}$$

$$\begin{pmatrix} 1+4k & -8k \\ 2k & 1-4k \end{pmatrix} \begin{pmatrix} 5 & -8 \\ 2 & -3 \end{pmatrix}$$

$$= \begin{pmatrix} 5+20k-16k & -8-32k+24k \\ 10k+2-8k & -16k-3+12k \end{pmatrix}$$

$$= \begin{pmatrix} 1+4(k+1) & -8(k+1) \\ 2(k+1) & 1-4(k+1) \end{pmatrix}$$

$\therefore$ true for $n=k+1$

Conclusion $\Rightarrow$ If ~~proved~~ true for $n=k$ then proved true for $n=k+1$. Since true for $n=1$ $\therefore$ true for $n \in \mathbb{Z}^+$

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Question 8 continued

(ii) Basis

Let $n=1$

$u_1 = 6^1 + 2^1 = 8$ ($\therefore$ true)

$n=2$

$u_2 = 6^2 + 2^2 = 40$ ($\therefore$ true)

Assumption.

Let's assume true for $n=k$ and $n=k+1$.

$\cancel{u_{k+2} = 8u_{k+1} - 12u_k}$

$u_k = 6^k + 2^k$

$u_{k+1} = 6^{k+1} + 2^{k+1}$

Induction.

Let $n=k+2$.

$u_{k+2} = 8u_{k+1} - 12u_k$

$= 8(6^{k+1} + 2^{k+1}) - 12(6^k + 2^k)$

$= 48(6^k) + 16(2^k) - 12(6^k + 2^k)$

$= 36(6^k) + 4(2^k)$

$= 6^2(6^k) + 2^2(2^k)$

$\therefore = 6^{k+2} + 2^{k+2}$

$\therefore$ true for $n=k+2$.

Conclusion.

If true for $n=k$ and $n=k+1$ then proved true for $n=k+2$. Since true for $n=1, 2$ $\therefore$ true for all n

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9. The complex numbers z_1 and z_2 are given by

$$z_1 = -1 - \mathrm{i} \text{ and } z_2 = 3 - 4\mathrm{i}$$

(a) Find the argument of the complex number $z_1 - z_2$. Give your answer in radians to 3 decimal places.
(3)

(b) Find the complex number $\frac{z_1}{z_2}$ in the form $a + \mathrm{i}b$, where a and b are rational numbers.
(3)

(c) Find the modulus of $\frac{z_1}{z_2}$, giving your answer as a simplified surd.
(2)

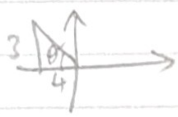
(d) Find the values of the real constants p and q such that

$$\frac{p + \mathrm{i}q - 8z_1}{p - \mathrm{i}q - 8z_2} = 3\mathrm{i}$$
(5)

(a) $z_1 - z_2 = (-1-i) - (3-4i)$

$= -4 + 3i.$

Arg



$\tan^{-1}\left(\frac{3}{4}\right) = \theta$

$= \cancel{36}\ 0.644.$

$\pi - 0.644 = \underline{2.498}$ (3dp)

(b) $\frac{z_1}{z_2} = \frac{-1-i\ (3+4i)}{(3-4i)(3+4i)}$

$= \frac{-3-4i-3i-4i^2}{9-16i^2}$

$= \frac{1-7i}{25}$

$= \frac{1}{25} - \frac{7}{25} i.$

(c) $\sqrt{(1/25)^2 + (-7/25)^2} = \frac{\sqrt{2}}{5}.$

(d) $p + iq - 8z_1 = 3i(p - iq - 8z_2)$

$p + iq + 8 + 8i = 3pi + 3q - 72i - 96.$

Equating real and imaginary parts.

$p + 8 = 3q - 96.$ (Real)

$q + 8 = 3p - 72$ (Im)

$p = 3q - 104.$

$q + 8 = 3(3q - 104) - 72$

$8q = 392$

$q = 49$

$p = 3(49) - 104$

$p = \underline{43}$