

Decision Mathematics D1 Mark scheme

Question	Scheme	Marks
1(a)	E.g. if use CD as shortcut get 807 or if use CF + AD get 793	M1 A1
		(2)
(b)	A F E D B C A	B1
	82 113 98 130 110 217 = 750	B1
		(2)
(c)	length of RMST = 439	B1
	439 + 82 + 113 = 634	M1 A1
		(3)
(d)	$634 < \text{optimal} \leq 750$	B1ft
		(1)
(8 marks)		
Notes:		
<p>(a) M1: Their plausible shortcut leading to a value < 810 and a length below 810 stated. A1: cao – shortcut and length must be consistent. (Examples shortcuts: $CD = 807$, $CF + AD = 793$, $CF + BD = 664$, $AD + EF + FC = 715$, $DF + FC = 785$ etc.)</p>		
<p>(b) B1: cao B1: cao</p>		
<p>(c) B1: cao M1: Adding two least weighted arcs to their RMST length A1: cao</p>		
<p>(d) B1: An interval that incorporates their lower bound from (c) and their best upper bound from either (a) or (b)</p>		

Question	Scheme	Marks
2(a)	e.g. accept (i) Every pair of nodes connected by a path (ii) Connected graph with no cycles (iii) All nodes connected	B1 B1 B1 (3)
(b)	$n - 1$	B1 (1)
(c)		M1 A1 (2)
(d)	Kruskal: AB, AD, BC, CG, reject BD, EG, reject CD, reject CE, reject AE, CF	M1 A1 A1 (3)
(e)	135 (km)	B1 (1)
(10 marks)		
Notes:		
<p>(a) In (a), all technical language used must be correct – for example, do not accept ‘point’ for node, etc (i)B1: every pair and path (or clear definition of path) – no bod - not describing complete graph (ii)B1: connected and no cycles (not ‘loops’, ‘circles’, etc. unless ‘cycle’ seen as well) (iii)B1: all nodes connected (accept definition of minimum spanning tree)</p>		
<p>(b) B1: cao</p>		
<p>(c) M1: Either all five arcs correct (ignore weights) or at least three arcs correct (including weights) A1: cso (arcs and weights) – no additional arcs</p>		

Question 2 notes *continued***(d)**

M1: Kruskal's – first three arcs (AB, AD, BC,... or weights 17, 19, 21, ...) chosen correctly **and at least one rejection seen at some point. For M1 only:** follow through from their diagram from (c)

A1: All six arcs (AB, AD, BC, CG, EG, CF or weights 17, 19, 21, 22, 25, 31) chosen correctly and no additional arcs (no follow through from an incorrect network in (c))

A1: cso All selections and rejections correct (in correct order and at the correct time) – do not accept weights or a contradiction between arcs and their weights (e.g. AB (16))

B1: cao (ignore lack of units)

Question	Scheme	Marks
3(a)	Bin 1: <u>12.1</u> <u>9.3</u> <u>10.9</u> Bin 2: <u>15.7</u> <u>6.4</u> <u>7.9</u> Bin 3: <u>17.4</u> 8.1 Bin 4: <u>20.1</u> Bin 5: 14.0	$\frac{M1}{A1}$ A1
		(3)
(b)	(i) 12.1 15.7 10.9 17.4 9.3 20.1 7.9 8.1 14.0 6.4 15.7 12.1 17.4 10.9 20.1 9.3 8.1 14.0 7.9 6.4	M1 A1
	(ii) Comparisons = $9 + 8 = 17$ Swaps = $7 + 5 = 12$	B1 B1
		(4)
(c)	e.g. middle right	M1 (quick)
	12.1 9.3 15.7 10.9 17.4 <u>6.4</u> 20.1 7.9 8.1 14.0 Pivot 6.4	A1 (1 st /2 nd passes/pivot for 3 rd)
	12.1 9.3 15.7 10.9 <u>17.4</u> 20.1 7.9 8.1 14.0 <u>6.4</u> Pivot 17.4	
	20.1 <u>17.4</u> 12.1 9.3 15.7 <u>10.9</u> 7.9 8.1 14.0 <u>6.4</u> Pivot (20.1) 10.9	A1ft (3 rd /4 th passes/pivot for 5 th)
	20.1 <u>17.4</u> 12.1 <u>15.7</u> 14.0 <u>10.9</u> 9.3 <u>7.9</u> 8.1 <u>6.4</u> Pivots 15.7 7.9	
	20.1 <u>17.4</u> <u>15.7</u> 12.1 <u>14.0</u> <u>10.9</u> 9.3 <u>8.1</u> <u>7.9</u> <u>6.4</u> Pivots 14.0 8.1	A1(cso + 'sort complete')
	(4)	
(d)	Bin 1: <u>20.1</u> <u>12.1</u> Bin 2: <u>17.4</u> <u>14.0</u> Bin 3: <u>15.7</u> <u>10.9</u> 6.4 Bin 4: <u>9.3</u> <u>8.1</u> 7.9	$\frac{M1}{A1}$ A1
		(3)
(e)	e.g. $\frac{121.9}{33} \approx 3.694$ so yes 4 bins is optimal	B1ft
		(1)
(15 marks)		

Question 3 *continued*

Notes:

(a)

M1: First four numbers placed correctly (therefore Bin 1 correct and 15.7 in Bin 2) and at least seven numbers put in bins – condone cumulative totals here only

A1: First eight numbers placed correctly (therefore Bins 1 and 2 correct and 17.4 in Bin 3 and 20.1 in Bin 4)

A1: cso All correct

(b)

(i)M1: Bubble sort – first pass correct

(i)A1: cao both passes correct (ignore additional passes)

(ii)B1: cao on total number of comparisons

(ii)B1: cao on total number of swaps

SC in b(ii): If B0B0, award B1B0 if correct numbers referred to but not summed

(c)

M1: Quick sort, pivot, p , chosen (must be choosing middle left or right – **choosing first/last item as pivot is M0**) and first pass gives $>p$, p , $<p$. So after the first pass the list should read (values greater than the pivot), pivot, (values less than the pivot). **If only choosing one pivot per iteration M1 only**

A1: First and second passes correct **and** next pivot(s) chosen correctly for third pass (but third pass does not need to be correct)

A1ft: Third and fourth passes correct (follow through from their second pass and choice of pivots) – **and** next pivot(s) chosen correctly for the fifth pass

A1: cso (correct solution only – all previous marks in this part **must** have been awarded) including ‘sort complete’ – this could be shown by the final list being re-written or ‘sorted’ statement or each item being used (**not** just stated) as a pivot

(d)

M1: **Must be using ‘sorted’ list** in decreasing order (independent of (c)). First four numbers placed correctly and at least seven numbers put in bins – condone cumulative totals here only. First-fit increasing is M0

A1: First eight numbers placed correctly

A1: cso – all correct

SC for (d): if the ‘sorted’ list they use in (d) has one ‘error’ from (c) (e.g. a missing number, an extra number or one number incorrectly placed) then M1 only can be awarded in (d) (for the first four numbers). If there is more than one ‘error’ then M0. Allow full marks in (d) if a correct list is used in (d) even if the list is incorrect at the end of (c).

(e)

B1ft: $\frac{121.9}{33}$ **or** awrt 3.7 (**or** 3.6 with correct calculation seen) **and** 4 together with a correct conclusion

based on their answer to (d) (a correct calculation etc. with an answer of 4 with no conclusion (as a minimum accept ‘yes’) scores B0)

middle left

12. 1 9.3 15.7 10.9 17.4 6.4 20.1 7.9 8.1 14.0

Pivot 17.4

20.1 17.4 12.1 9.3 15.7 10.9 6.4 7.9 8.1 14.0

Pivot (20.1) 10.9

20.1 17.4 12.1 15.7 14.0 10.9 9.3 6.4 7.9 8.1

Pivots 15.7 6.4

20.1 17.4 15.7 12.1 14.0 10.9 9.3 7.9 8.1 6.4

Pivots 12.1 7.9

20.1 17.4 15.7 14.0 12.1 10.9 9.3 8.1 7.9 6.4

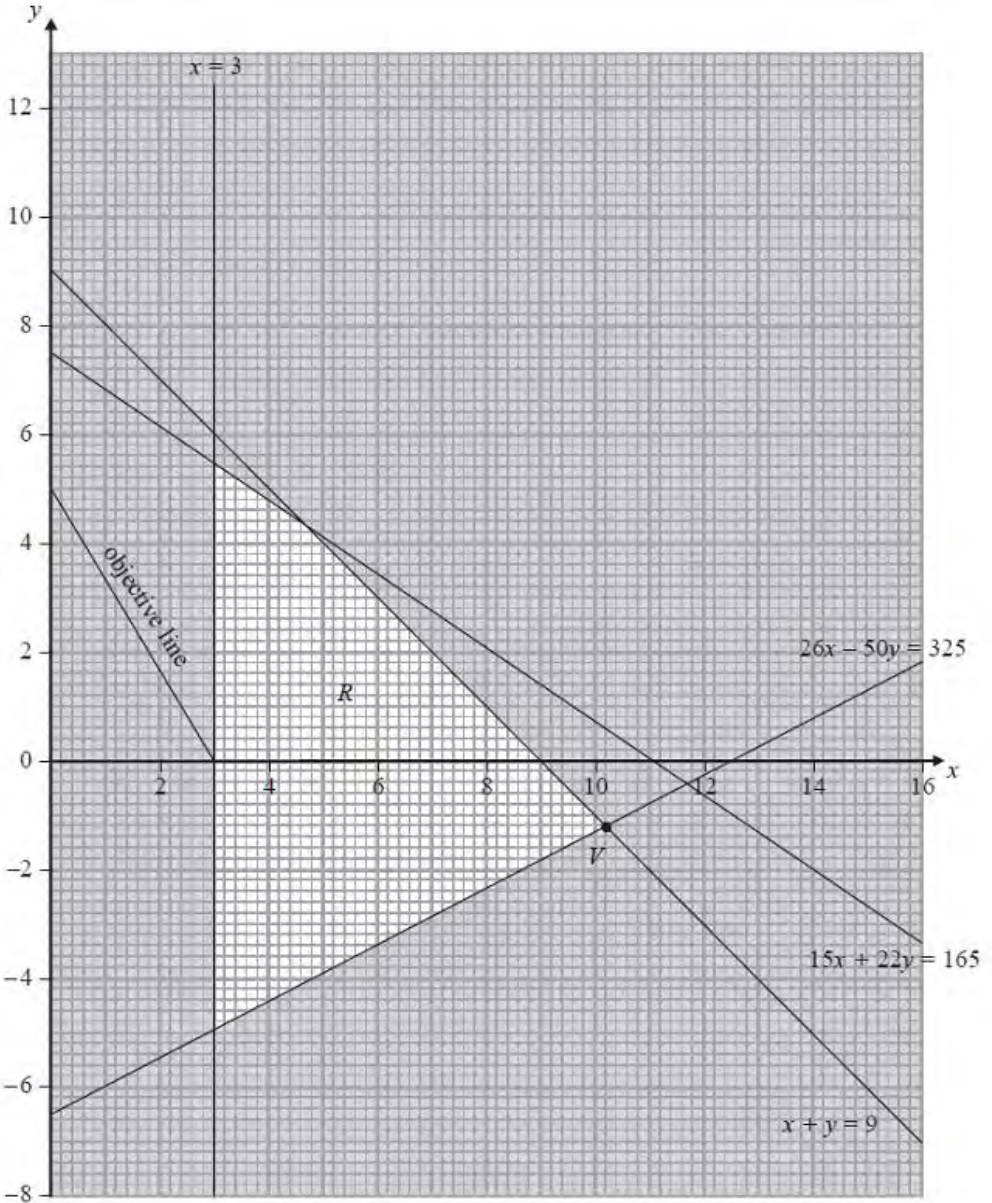
Pivot (14.0) 9.3

20.1 17.4 15.7 14.0 12.1 10.9 9.3 8.1 7.9 6.4

(sort complete (8.1))

Question	Scheme	Marks
<p>4(a)</p>	<p>Quickest route: A – G – H – K</p> <p>Shortest time: 32 (mins)</p>	<p>M1 A1 (JEFD) A1 (BG) A1ft (HK)</p>
	<p>Quickest route: A – G – H – K</p>	<p>A1</p>
	<p>Shortest time: 32 (mins)</p>	<p>A1ft</p>
		<p>(6)</p>
<p>(b)</p>	<p>Route from B to K via A: B – D – E – A – G – H – K Length: 51 (mins)</p>	<p>B1 B1ft</p>
		<p>(2)</p>
<p>(c)</p>	<p>$A(ED)B + F(G)H = 19 + 15 = 34$ $AF + B(K)H = 16 + 18 = 34$ $A(G)H + B(DE)F = 29 + 11 = 40$</p> <p>Arcs AF, BK, KH or AE, ED, DB, FG, GH will be traversed twice Route length = $196 + 34 = 230$ (mins)</p>	<p>M1 A1ft A1ft A1ft</p> <p>A1A1 A1</p>
		<p>(7)</p>
<p>Notes:</p>		
<p>(a)</p> <p>M1: A larger value replaced by a smaller value at least once in the working values at either B or H or K</p> <p>A1: All values in J, E, F and D correct and the working values in the correct order. Penalise order of labelling only once per question. Condone an additional working value at F of 22</p> <p>A1: All values in B and G correct and the working values in the correct order. Penalise order of labelling only once per question (B and G must be labelled in that order and B must be labelled after J, E, F, D). Condone an additional working value of 20 at B and an additional working value of 26 at G</p> <p>A1ft: All values in H and K correct on the follow through and the working values in the correct order. Penalise order of labelling only once per question (H and K must be labelled in that order and H labelled after all other nodes (excluding K))</p> <p>A1: CAO (AGHK)</p> <p>A1ft: Follow through on their final value at K – if their answer is not 32 follow through their final value at K (condone lack of units)</p>		

Question 4 notes *continued***(b)****B1:** CAO (BDEAGHK)**B1ft:** 51 or their final value at B + their final value at K (condone lack of units)**(c)****M1:** Three distinct pairings of the correct four odd nodes**A1ft:** One row correct including pairing **and** total (the ft on the first three A marks in (c) is for using their final values at B, F and H from (a) for the lengths of AB, AF and AH only)**A1ft:** Two rows correct including pairing **and** totals**A1ft:** All three rows correct including pairing **and** totals**A1:** CAO one combination of arcs that need traversing twice (arcs must be explicitly stated and not implied by working)**A1:** CAO both combination of arcs that need traversing twice (arcs must be explicitly stated and not implied by working)**A1:** CAO (230)

Question	Scheme	Marks
<p>5(a)(b)</p>		<p>B1 B1 B1 B1 (R) (4) B1 B1 (2)</p>
<p>(c)</p>	<p>$V\left(\frac{775}{76}, -\frac{91}{76}\right)$</p> <p>$P = \frac{1801}{38}$</p>	<p>M1 A1 A1 (3)</p>
<p>(d)</p>	<p>$x = 3, y = -4$ minimum value is 3</p>	<p>B1 B1 (2)</p>
<p>(11 marks)</p>		

Question 5 *continued*

Notes:

(a)

In (a), lines must be long enough to define the correct feasible region **and** pass through one small square of the points stated:

$x + y = 9$ passes through (5, 4) and (9,0) but in most cases check (0, 9) and (9,0)

$26x - 50y = 325$ passes through (5, -3.9) and (10, -1.3) but in most cases check (0, -6.5) and (12.5, 0)

$15x + 22y = 165$ passes through $\left(3, \frac{60}{11}\right)$ and $\left(4, \frac{105}{22}\right)$ but in most cases check (0, 7.5) and (11, 0)

B1: Any two lines correctly drawn

B1: Any three lines correctly drawn

B1: All four lines correctly drawn

B1: Region, R, correctly labelled – not just implied by shading – dependent on scoring the first three marks in (a)

(b)

B1: Drawing the correct objective line on the graph, use line drawing tool to check if necessary. Line must not pass outside of a small square if extended from axis to axis

B1: V labelled clearly on their graph. **This mark is dependent on both the correct feasible region (but maybe not labelled) and the correct objective line**

(c)

M1: Candidates **must** have drawn either the correct objective line **or** its reciprocal. If they have drawn the correct objective line they must be solving $x + y = 9$ and $26x - 50y = 325$. If they have drawn the reciprocal objective line they must be solving $x = 3$ and $15x + 22y = 165$. Must get to either $x = \dots$ or $y = \dots$ (condone one error in the solving of the simultaneous equations).

The correct exact answer $\left(\frac{775}{76}, -\frac{91}{76}\right)$, or for the reciprocal $\left(3, \frac{60}{11}\right)$, can imply this mark

A1: cao $\left(\frac{775}{76}, -\frac{91}{76}\right)$ or $\left(10\frac{15}{76}, -1\frac{15}{76}\right)$ (coordinates must be exact) – **if correct answer stated**

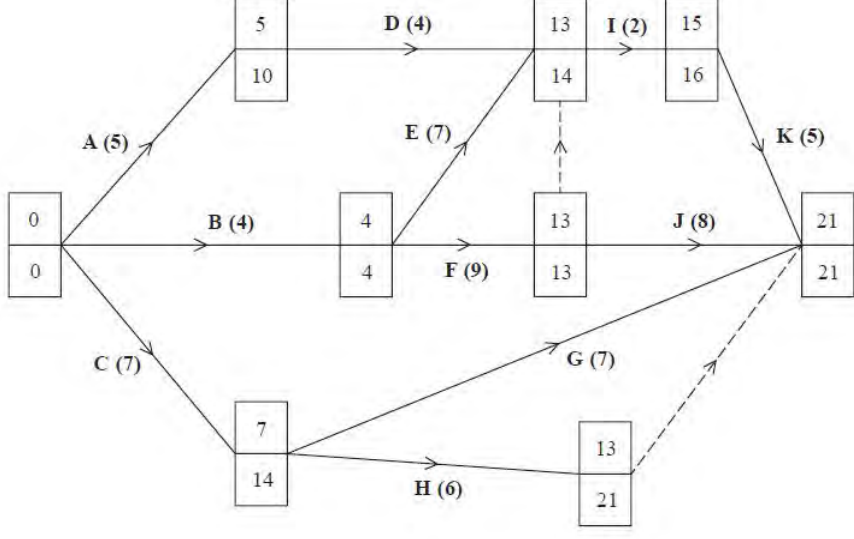
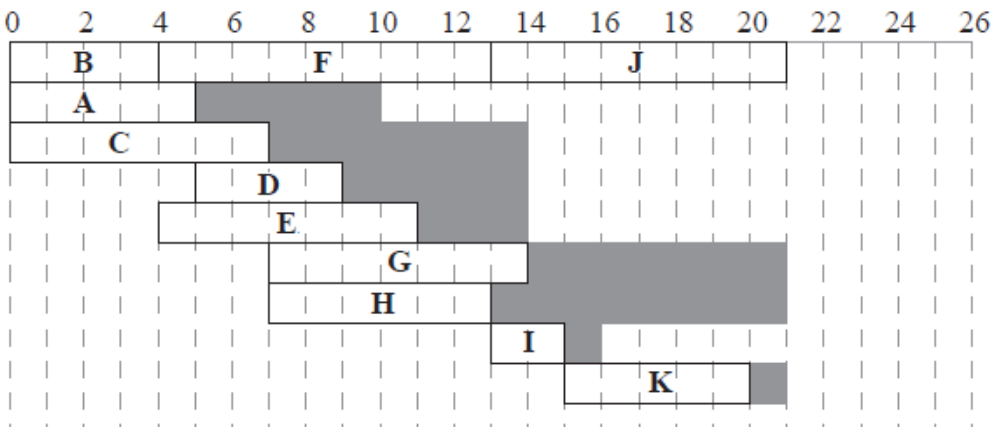
with no working seen then award M1A0 only (however, they can still earn the next A mark for the corresponding value of P at V). **This mark is dependent on the correct feasible region (but maybe not labelled)**

A1: cao $\frac{1801}{38}$ or $47\frac{15}{38}$ (must be exact). **This mark is dependent on the correct feasible region (but maybe not labelled)**

(d)

B1: cao $x = 3, y = -4$ or $(3, -4)$

B1: cao of 3

Question	Scheme	Marks
6(a)	(i) The dummy from event 5 to event 6 is needed to show that J depends on F but I depends on D, E and F	B1
	(ii) The dummy from event 7 to event 9 is because activities G and H must be able to be described uniquely in terms of the events at each end	B1
		(2)
(b)	 <p>The network diagram shows activities A through K. Each activity is represented by an arrow with its duration in parentheses. Each event is represented by a box with four numbers: top-left (early start), top-right (early finish), bottom-left (late start), and bottom-right (late finish). Activity A (5) starts at event 0 (0,0) and ends at event 5 (5,10). Activity B (4) starts at event 0 (0,0) and ends at event 4 (4,4). Activity C (7) starts at event 0 (0,0) and ends at event 7 (7,14). Activity D (4) starts at event 5 (5,10) and ends at event 13 (13,14). Activity E (7) starts at event 4 (4,4) and ends at event 13 (13,14). Activity F (9) starts at event 4 (4,4) and ends at event 13 (13,13). Activity G (7) starts at event 7 (7,14) and ends at event 13 (13,21). Activity H (6) starts at event 7 (7,14) and ends at event 13 (13,21). Activity I (2) starts at event 13 (13,14) and ends at event 15 (15,16). Activity J (8) starts at event 13 (13,13) and ends at event 21 (21,21). Activity K (5) starts at event 15 (15,16) and ends at event 21 (21,21). There are dummy activities: a dashed arrow from event 5 (5,10) to event 13 (13,13), and a dashed arrow from event 13 (13,21) to event 15 (15,16).</p>	M1 A1 M1 A1
		(4)
(c)	21 (hours)	B1
		(1)
(d)	$\frac{64}{21} \approx 3.048$ so at least 4 workers required	M1 A1
		(2)
(e)	 <p>The Gantt chart shows the timing of activities A through K on a scale from 0 to 26. Activity A starts at 0 and ends at 5. Activity B starts at 0 and ends at 4. Activity C starts at 0 and ends at 7. Activity D starts at 5 and ends at 9. Activity E starts at 4 and ends at 11. Activity F starts at 5 and ends at 14. Activity G starts at 7 and ends at 14. Activity H starts at 7 and ends at 13. Activity I starts at 13 and ends at 15. Activity J starts at 13 and ends at 21. Activity K starts at 15 and ends at 21.</p>	M1 A1 M1 A1
		(4)

Question	Scheme	Marks
6(f)	e.g. 	M1 A1 A1
		(3)
(16 marks)		
Notes:		
(a)	<p>In (a) any use of the terms ‘activity’ and ‘event’ must be correct</p> <p>B1: cao dependency - all relevant activities must be referred to - activities I, J, F and either D or E must be mentioned.</p> <p>B1: cao uniqueness – please note that, for example, ‘so that activities can be defined uniquely’ is not sufficient to earn this mark. There must be some mention of describing activities in terms of the event at each end. However, give bod on statements that imply that an activity begins and ends at the same event</p>	
(b)	<p>M1: All top boxes complete, values generally increasing in the direction of the arrows (‘left to right’), condone one rogue</p> <p>A1: cao (top boxes)</p> <p>M1: All bottom boxes complete, values generally decreasing in the opposite direction of the arrows (‘right to left’), condone one rogue</p> <p>A1: cao (bottom boxes)</p>	
(c)	<p>B1: cao (21)</p>	
(d)	<p>M1: Attempt to find lower bound: (a value in the interval $[55 - 73] / \text{their finish time}$) or (sum of the activities / their finish time) or (as a minimum) an awrt 3.05 or 3.04 (truncated)</p> <p>A1: cso – either a correct calculation seen or awrt 3.05 (or 3.04) then 4. An answer of 4 with no working scores M0A0</p>	
(e)	<p>M1: At least 8 activities added including 5 floats. Scheduling diagram scores M0</p> <p>A1: Critical activities dealt with correctly and 4 non-critical activities dealt with correctly</p> <p>M1: All 11 activities including all 8 floats (on the correct non-critical activities)</p> <p>A1: cao (all activities correct and present only once)</p>	

Question 4 notes *continued*

(f)

M1: Not a cascade chart. 3 workers used and at least 9 activities placed. The completion time must be no greater than one hour more than the minimum completion time stated in (c) or seen in (b)

A1: 3 workers, All 11 activities present (just once). Condone one error either precedence or activity length. The completion time must be one hour greater than the minimum completion time stated in (c) or seen in (b)

A1: 3 workers. All 11 activities present (just once). No errors. The completion time must be 22

Activity	Duration	IPA
A	5	-
B	4	-
C	7	-
D	4	A
E	7	B
F	9	B
G	7	C
H	6	C
I	2	D, E, F
J	8	F
K	5	I

