

Pearson Edexcel International Advanced Level

Sample Assessment Material for first teaching September 2018

Time: 1 hour 30 minutes

Paper Reference **WDM11/01**

Mathematics

International Advanced Subsidiary/Advanced Level

Decision Mathematics D1

You must have:

Decision Mathematics Answer Book (enclosed), calculator

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Write your answers for this paper in the Decision Mathematics answer book provided.
- **Fill in the boxes** at the top of the answer book with your name, centre number and candidate number.
- Do not return the question paper with the answer book.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
– *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- There are 6 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
– *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ►

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Write your answers in the D1 answer book for this paper.

1. The table shows the least distances, in km, between six towns, A, B, C, D, E and F.

	A	B	C	D	E	F
A	–	122	217	137	109	82
B	122	–	110	130	128	204
C	217	110	–	204	238	135
D	137	130	204	–	98	211
E	109	128	238	98	–	113
F	82	204	135	211	113	–

Liz must visit each town at least once. She will start and finish at A and wishes to minimise the total distance she will travel.

- (a) Starting with the minimum spanning tree given in your answer book, use the shortcut method to find an upper bound below 810km for Liz's route. You must state the shortcut(s) you use and the length of your upper bound. (2)
- (b) Use the nearest neighbour algorithm, starting at A, to find another upper bound for the length of Liz's route. (2)
- (c) Starting by deleting F, and all of its arcs, find a lower bound for the length of Liz's route. (3)
- (d) Use your results to write down the smallest interval which you are confident contains the optimal length of the route. (1)

(Total for Question 1 is 8 marks)

Question 1 continued

(b)

	(1) A	(5) B	(6) C	(4) D	(3) E	(2) F
A	-	122	217	137	109	82
B	122	-	110	(30)	128	204
C	217	(110)	-	204	238	135
D	137	130	204	-	(98)	211
E	109	128	238	98	-	(113)
F	(82)	204	135	211	113	-

b) AFEDBCA

$$82 + 113 + 98 + 130 + 110 + 217 = 750 \text{ km}$$

(c)

	A	B	C	D	E	F
A	-	122	217	137	109	82
B	122	-	110	130	128	204
C	217	110	-	204	238	135
D	137	130	204	-	98	211
E	109	128	238	98	-	113
F	82	204	135	211	113	-

$$\text{RMST} : 98 + 109 + 122 + 110 = 439 \text{ km}$$

$$439 + 82 + 113 = 634 \text{ km}$$

d) $634 < \text{optimal length} \leq 750$

Q1

(Total for Question 1 is 8 marks)

2. Kruskal’s algorithm finds a minimum spanning tree for a connected graph with n vertices.

(a) Explain the terms

- (i) connected graph,
- (ii) tree,
- (iii) spanning tree.

(3)

(b) Write down, in terms of n , the number of arcs in the minimum spanning tree.

(1)

The table below shows the lengths, in km, of a network of roads between seven villages, A, B, C, D, E, F and G.

	A	B	C	D	E	F	G
A	–	17	–	19	30	–	–
B	17	–	21	23	–	–	–
C	–	21	–	27	29	31	22
D	19	23	27	–	–	40	–
E	30	–	29	–	–	33	25
F	–	–	31	40	33	–	39
G	–	–	22	–	25	39	–

(c) Complete the drawing of the network on Diagram 1 in the answer book by adding the necessary arcs from vertex C together with their weights.

(2)

(d) Use Kruskal’s algorithm to find a minimum spanning tree for the network. You should list the arcs in the order that you consider them. In each case, state whether you are adding the arc to your minimum spanning tree.

(3)

(e) State the weight of the minimum spanning tree.

(1)

(Total for Question 2 is 10 marks)

2.

a) Every pair of nodes are connected by a path

ii) A connected graph with no cycles

iii) A tree which connects all nodes

b) $n-1$ = number of arcs in the minimum spanning tree

(c)

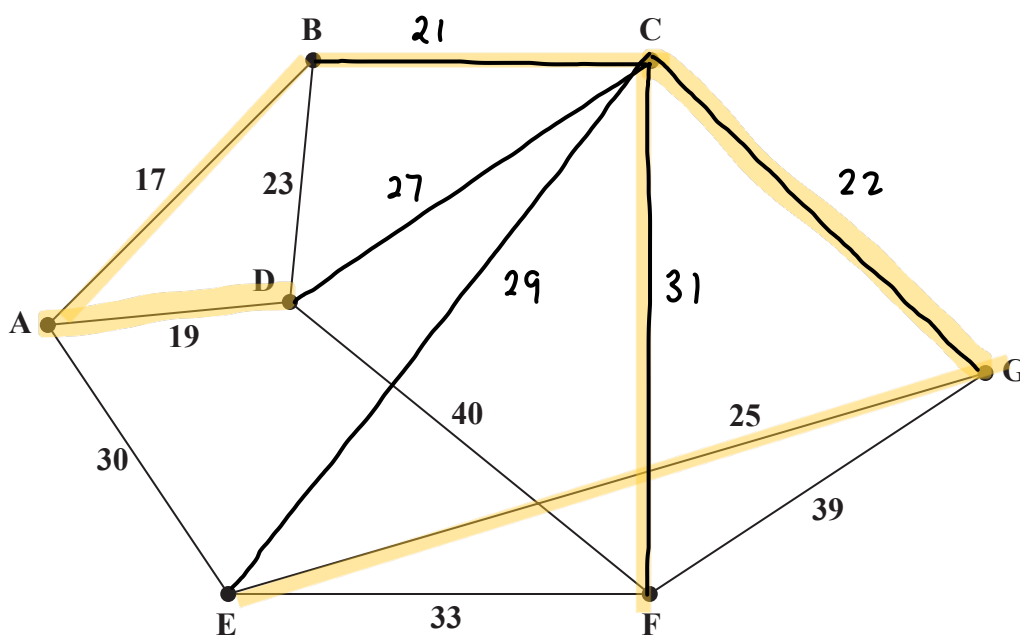


Diagram 1

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Question 2 continued

- d) 7 - AB
- 19 - AD
- 21 - BC
- 22 - CG
- 23 - BD reject (completes a cycle)
- 25 - EG
- 27 - CD reject (completes a cycle)
- 29 - CE reject (completes a cycle)
- 30 - AE reject (completes a cycle)
- 31 - CF
- 33 - EF reject (completes a cycle)
- 39 - FG reject (completes a cycle)
- 40 - DF reject (completes a cycle)

e) $17 + 19 + 21 + 22 + 25 + 31 = 135 \text{ km}$

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Q2

(Total for Question 2 is 10 marks)

3. 12.1 9.3 15.7 10.9 17.4 6.4 20.1 7.9 8.1 14.0

- (a) Use the first-fit bin packing algorithm to determine how the numbers listed above can be packed into bins of size 33
(3)

The list is to be sorted into **descending** order.

- (b) (i) Starting at the left-hand end of the list, perform two passes through the list using a bubble sort. Write down the state of the list that results at the end of each pass.

- (ii) Write down the total number of comparisons and the total number of swaps performed during your two passes.
(4)

- (c) Use a quick sort on the **original** list to obtain a fully sorted list in **descending** order. You must make your pivots clear.
(4)

- (d) Use the first-fit decreasing bin packing algorithm to determine how the numbers listed can be packed into bins of size 33
(3)

- (e) Determine whether your answer to (d) uses the minimum number of bins. You must justify your answer.
(1)

(Total for Question 3 is 15 marks)

3.

12.1 9.3 15.7 10.9 17.4 6.4 20.1 7.9 8.1 14.0

a) Bin 1 : 12.1 9.3 10.9

Bin 2 : 15.7 6.4 7.9

Bin 3 : 17.4 8.1

Bin 4 : 20.1

Bin 5 : 14.0

b i) 12.1 15.7 10.9 17.4 9.3 20.1 7.9 8.1 14.0 6.4

15.7 12.1 17.4 10.9 20.1 9.3 8.1 14.0 7.9 6.4

ii) No. of comparisons : $9+8=17$

No. of swaps : $7+5=12$

c) 12.1 9.3 15.7 10.9 17.4 (6.4) 20.1 7.9 8.1 14.0

12.1 9.3 15.7 10.9 (17.4) 20.1 7.9 8.1 14.0 (6.4)

20.1 (17.4) 12.1 9.3 15.7 (10.9) 7.9 8.1 14.0 (6.4)

20.1 (17.4) 12.1 (15.7) 14.0 (10.9) 9.3 (7.9) 8.1 (6.4)

20.1 (17.4) (15.7) 12.1 (14.0) (10.9) 9.3 (8.1) (7.9) (6.4)

20.1 (17.4) (15.7) (14.0) 12.1 (10.9) 9.3 (8.1) (7.9) (6.4)

sort complete

d) Bin 1 : 20.1 12.1

Bin 2 : 17.4 14.0

Bin 3 : 15.7 10.9 6.4

Bin 4 : 9.3 8.1 7.9

e) $(20.1 + 17.4 + 15.7 + 14.0 + 12.1 + 10.9 + 9.3 + 8.1 + 7.9 + 6.4) \div 33$

$$= 121.9 \div 33$$

$$= 3.69$$

$$\approx 4$$

\therefore 4 bins is the minimum

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4.

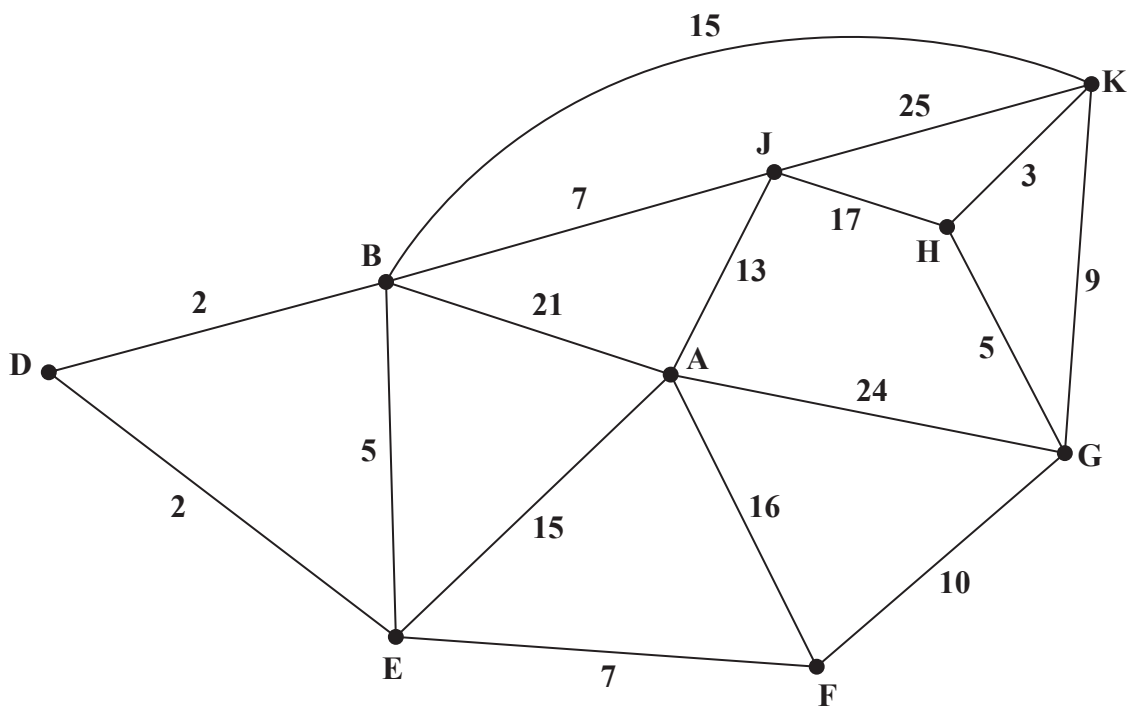


Figure 1

[The total weight of the network is 196]

Figure 1 models a network of roads. The number on each edge gives the time, in minutes, taken to travel along that road. Oliver wishes to travel by road from A to K as quickly as possible.

- (a) Use Dijkstra’s algorithm to find the shortest time needed to travel from A to K. State the quickest route. (6)

On a particular day Oliver must travel from B to K via A.

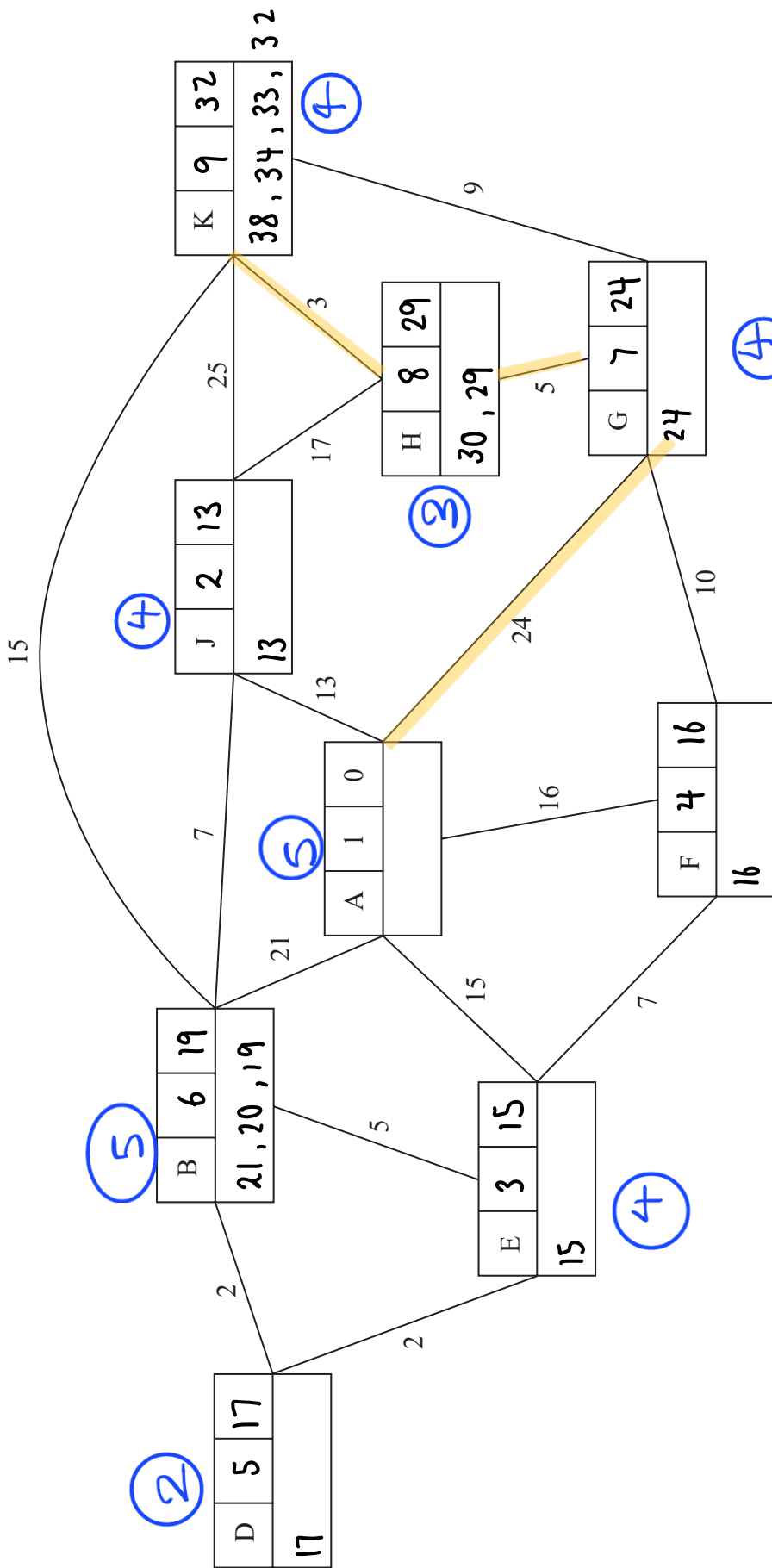
- (b) Find a route of minimal time from B to K that includes A, and state its length. (2)

Oliver needs to travel along each road to check that it is in good repair. He wishes to minimise the total time required to traverse the network.

- (c) Use the route inspection algorithm to find the shortest time needed. You must state all combinations of edges that Oliver could repeat, making your method and working clear. (7)

(Total for Question 4 is 15 marks)

Leave blank



Key:

Vertex	Order of labelling	Final value
	Working values	

Quickest route: **AGHK**
 Shortest time: **32 mins**

4.

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Question 4 continued

b) B D E A G H K

$$19 + 32 = 51 \text{ mins}$$

c) Odd nodes: A, B, F, H

$$AB + FH : 19 + (10+5) = 34 \quad \text{shortest route}$$

$$AF + BH : 16 + (15+3) = 34 \quad \text{shortest route}$$

$$AH + BF : 29 + (2+2+7) = 40$$

ATCS AE, ED, DB, FG & GH or

AF, BF, KH can be traversed twice

$$34 + 196 = 230 \text{ mins}$$

Q4

(Total for Question 4 is 15 marks)

5. A linear programming problem in x and y is described as follows.

$$\text{Maximise } P = 5x + 3y$$

$$\text{subject to: } x \geq 3$$

$$x + y \leq 9$$

$$15x + 22y \leq 165$$

$$26x - 50y \leq 325$$

- (a) Add lines and shading to Diagram 2 in the answer book to represent these constraints. Hence determine the feasible region and label it R. (4)
- (b) Use the objective line method to find the optimal vertex, V, of the feasible region. You must draw and label your objective line and label vertex V clearly. (2)
- (c) Calculate the exact coordinates of vertex V and hence calculate the corresponding value of P at V. (3)

The objective is now to **minimise** $5x + 3y$, where x and y are **integers**.

- (d) Write down the minimum value of $5x + 3y$ and the corresponding value of x and corresponding value of y . (2)

(Total for Question 5 is 11 marks)

5.

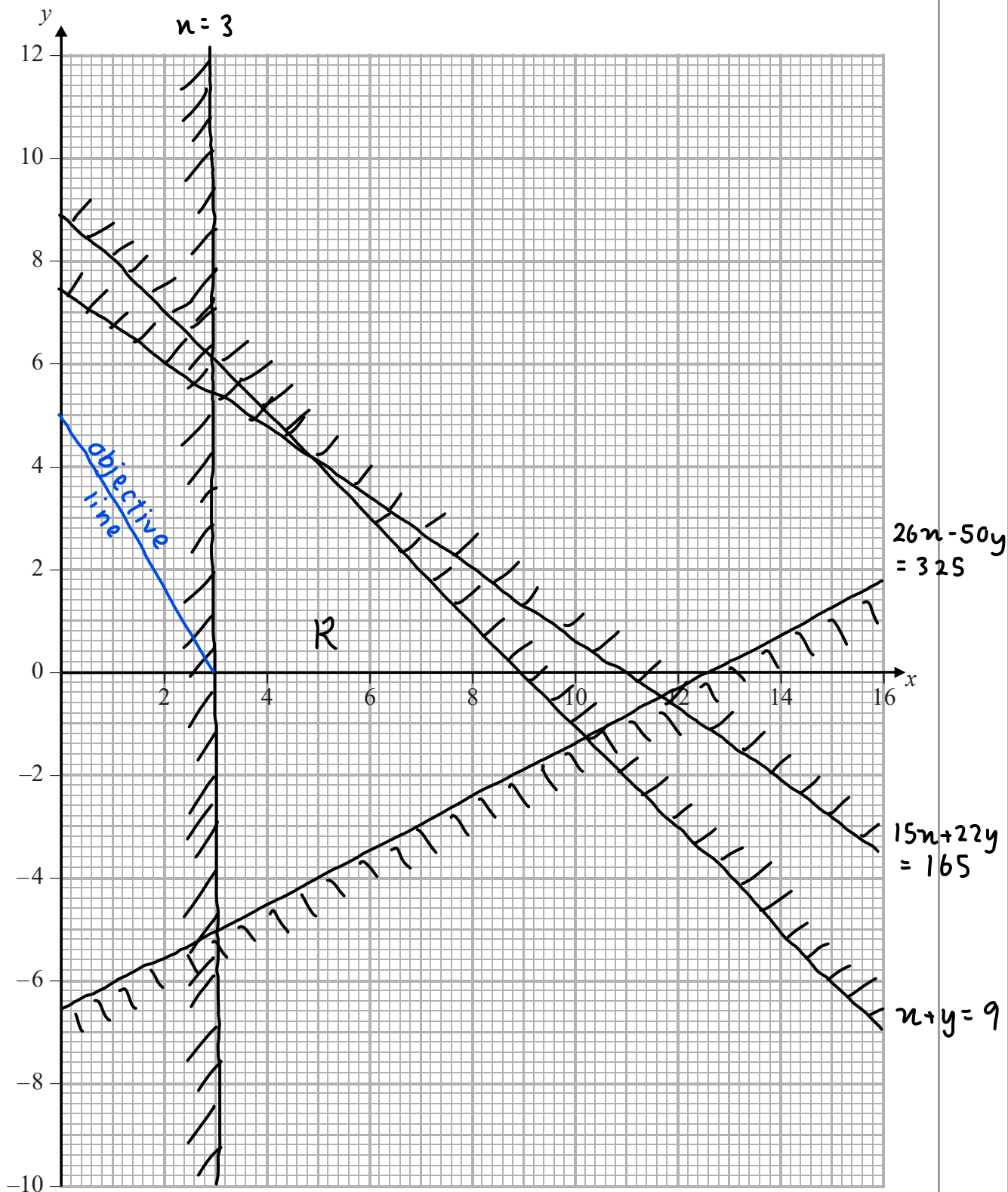


Diagram 2

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Question 5 continued

c) $x + y = 9$

$$y = 9 - x \quad \text{--- (1)}$$

$$26x - 50y = 325 \quad \text{--- (2)}$$

$$26x - (9-x)50 = 325$$

$$76x = 775$$

$$x = \frac{775}{76}$$

$$y = 9 - \frac{775}{76}$$

$$= -\frac{91}{76}$$

$$P = 5\left(\frac{775}{76}\right) + 3\left(-\frac{91}{76}\right)$$

$$= \frac{1801}{38}$$

$$\approx 47.4$$

d) $x = 3$

$$26x - 50y = 325$$

$$26(3) - 50y = 325$$

$$y = -4.94 \quad y \text{ is an integer so } y = -4 \text{ or } y = -5$$

$$\text{when } y = -4, \quad 26x - 50y = 278 < 325$$

$$y = -5, \quad 26x - 50y = 328 > 325 \quad n/a$$

$$\therefore y = -4, \quad x = 3$$

$$\text{min value: } 5(3) + 3(-4) = 3$$

Q5

(Total for Question 5 is 11 marks)

6.

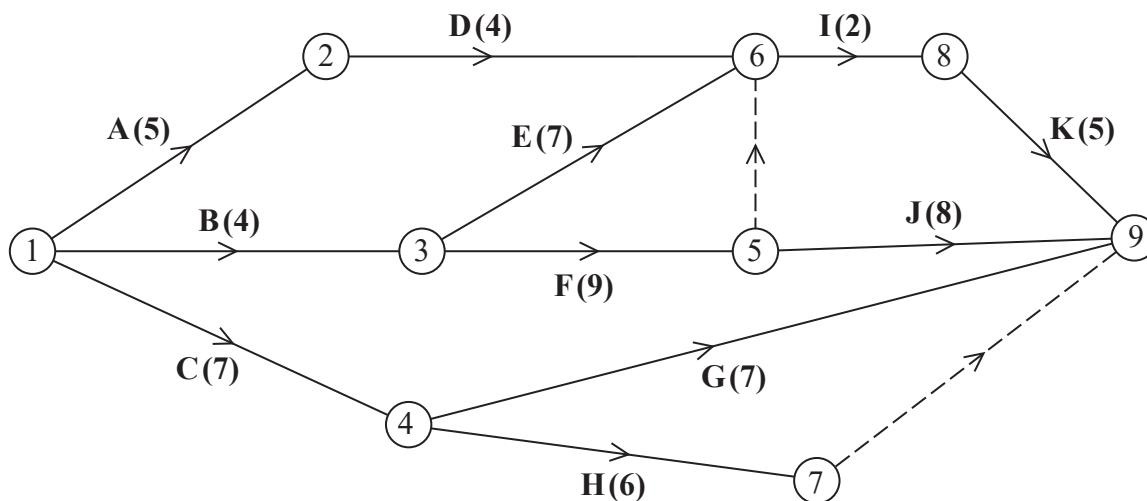


Figure 2

A project is modelled by the activity network shown in Figure 2. The activities are represented by the arcs. The number in brackets on each arc gives the time required, in hours, to complete the activity. The numbers in circles are the event numbers. Each activity requires one worker.

- (a) Explain the significance of the dummy activity
 - (i) from event 5 to event 6
 - (ii) from event 7 to event 9. (2)
- (b) Complete Diagram 3 in the answer book to show the early event times and the late event times. (4)
- (c) State the minimum project completion time. (1)
- (d) Calculate a lower bound for the minimum number of workers required to complete the project in the minimum time. You must show your working. (2)
- (e) On Grid 1 in your answer book, draw a cascade (Gantt) chart for this project. (4)
- (f) On Grid 2 in your answer book, construct a scheduling diagram to show that this project can be completed with three workers in just one more hour than the minimum project completion time. (3)

(Total for Question 6 is 16 marks)

TOTAL FOR PAPER IS 75 MARKS

6. (a)

i) To show that J only depends on F but I depends on D, E and F.

ii) To describe G and H uniquely in terms of events at each end.

(b)

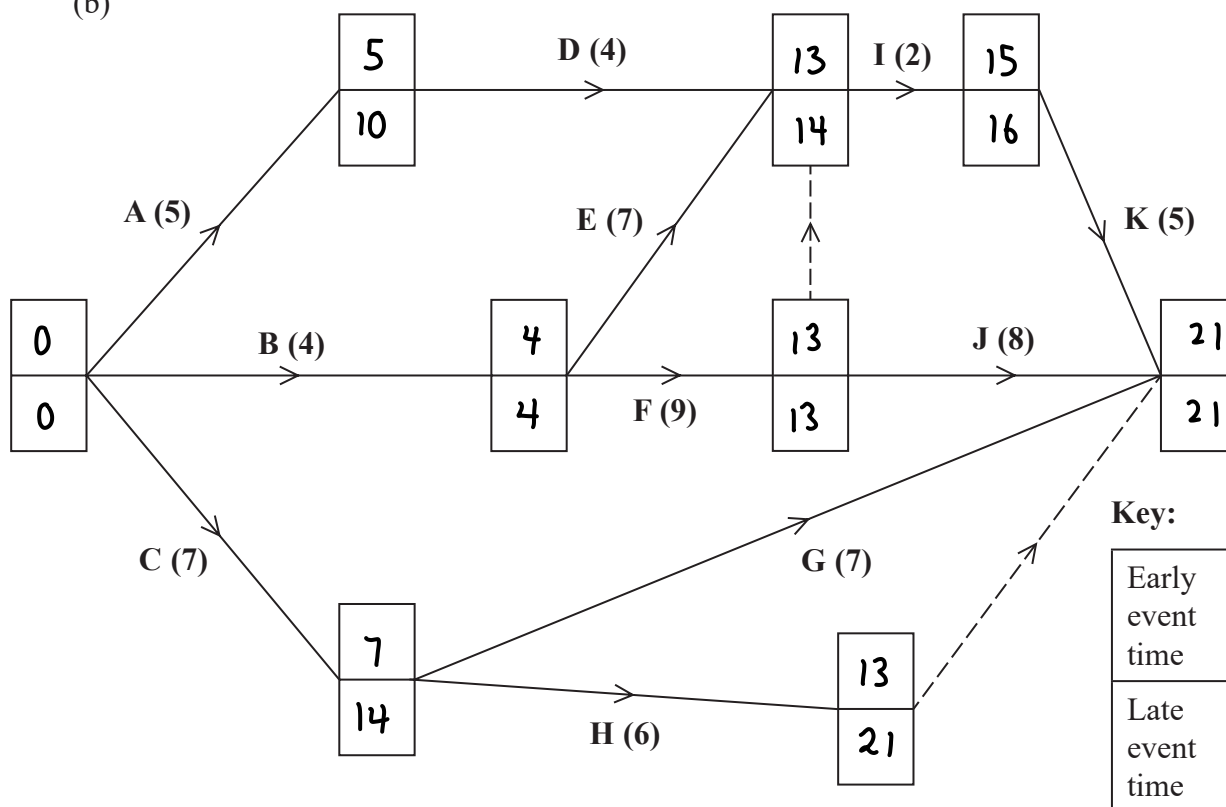


Diagram 3

c) 21 hours

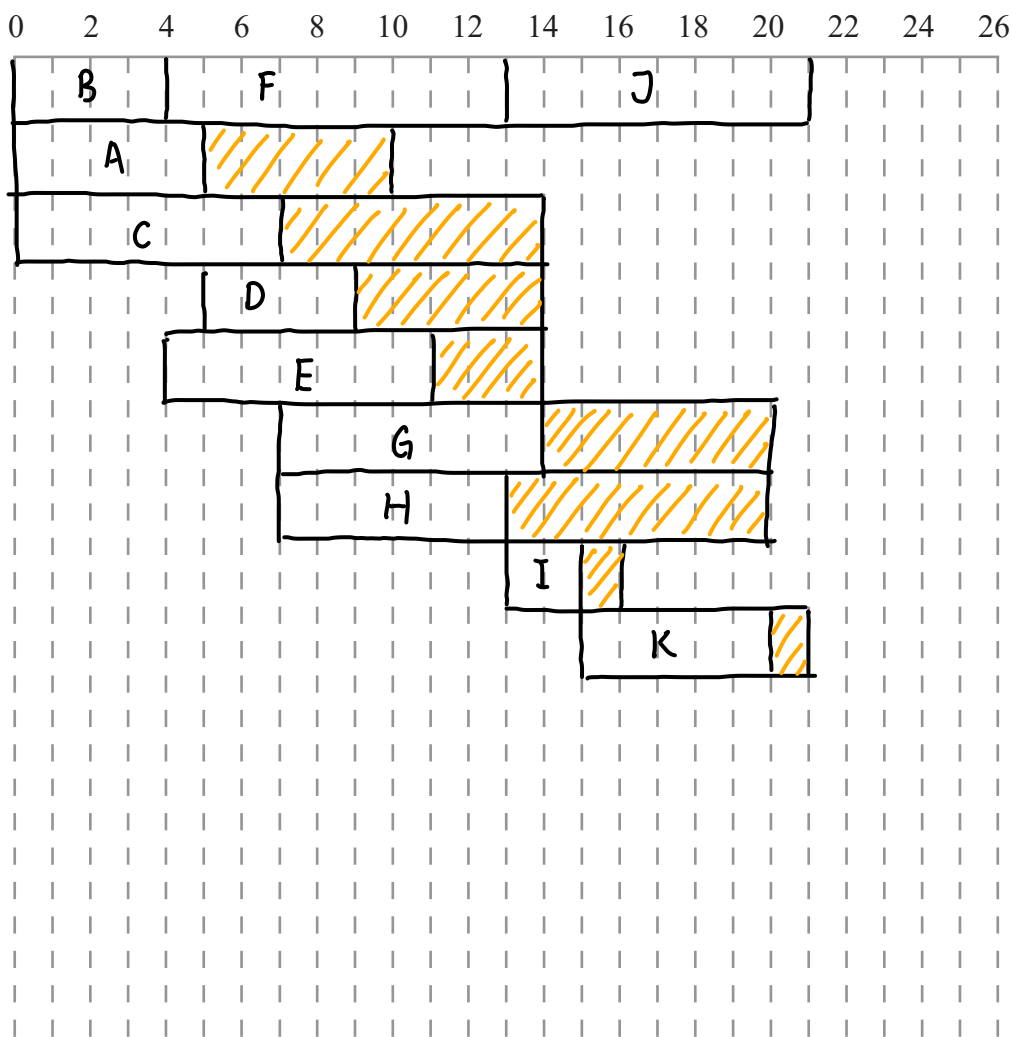
$$d) (5 + 4 + 7 + 4 + 7 + 9 + 7 + 6 + 2 + 8 + 5) \div 21 = 64 \div 21$$

$$= 3.0476$$

\therefore lower bound = 4 workers

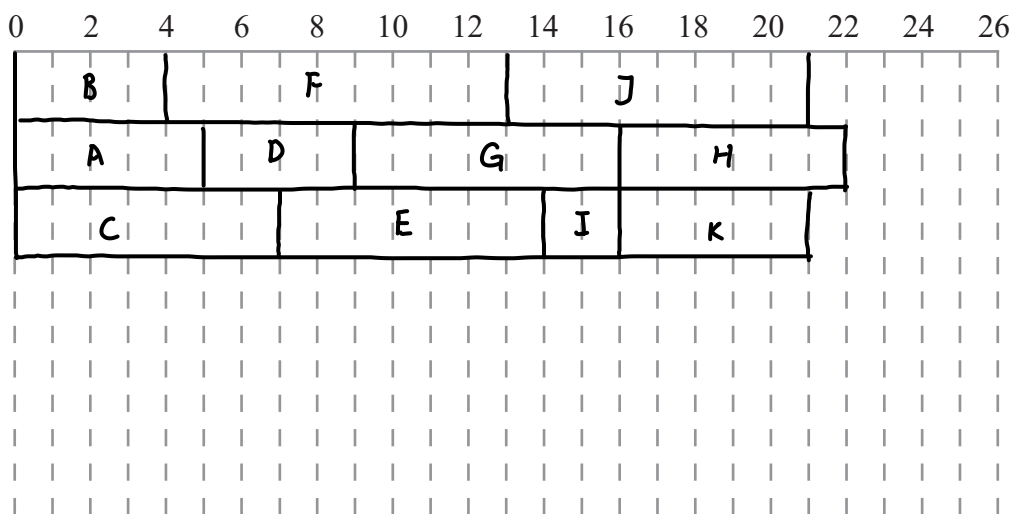
Question 6 continued

(e)



Grid 1

(f)



Grid 2

(Total for Question 6 is 16 marks)

TOTAL FOR PAPER IS 75 MARKS

Q6

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