

Pearson Edexcel International Advanced Level

Decision Mathematics D1

Advanced/Advanced Subsidiary

Friday 22 June 2018 – Morning

Time: 1 hour 30 minutes

Paper Reference

WDM01/01

You must have:

D1 Answer Book

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** on the top of the answer book with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the D1 answer book provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.
- Do not return the question paper with the answer book.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ►

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Write your answers in the D1 answer book for this paper.

1.

Kerry (K)
Nikki (N)
Violet (V)
Dev (D)
Henri (H)
Leslie (L)
Enlai (E)
Sylvester (S)
Joan (J)

A binary search is to be performed on the names in the list above to locate the name Leslie.

- (a) Explain why a binary search cannot be performed with the list in its present form. **(1)**
- (b) Using an appropriate algorithm, alter the list so that a binary search can be performed. You should state the name of the algorithm you use and show the list after each complete iteration. **(4)**
- (c) Use the binary search algorithm to locate the name Leslie in the altered list you obtained in (b). You must make your method clear. **(3)**

The binary search algorithm is to be used to search for a name in an alphabetical list of 727 names.

- (d) Find the maximum number of iterations needed. You should justify your answer. **(2)**

(Total 10 marks)

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1.

a) The names are not in alphabetical order.

b) Using quick sort :

K N V D (H) L E S J

D (E) (H) K N V (L) S J

(D) (E) (H) K (J) (L) N (V) S

D E H J K L N S V

D E H J K L N S V

Sort complete

c) 1 D

2 E

3 H

4 J

5 K

6 L

7 N

8 S

9 V

$$\text{Pivot 1} = \frac{9+1}{2} = 5 \text{ Kerry}$$

reject 1-5

$$\text{Pivot 2} = \frac{6+9}{2} = 7.5 \approx 8 \text{ Sylvester}$$

reject 8-9

$$\text{Pivot 3} = \frac{6+7}{2} = 6.5 \approx 7 \text{ Nikki}$$

reject 7

$$\text{Pivot 4} = 6 \text{ Leslie}$$

Search complete. Leslie is found at 6.

$$\text{d) } \log_2 727 = 9.5058$$

≈ 10 iterations.

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2. (a) Define the terms

(i) alternating path,

(ii) complete matching.

(4)

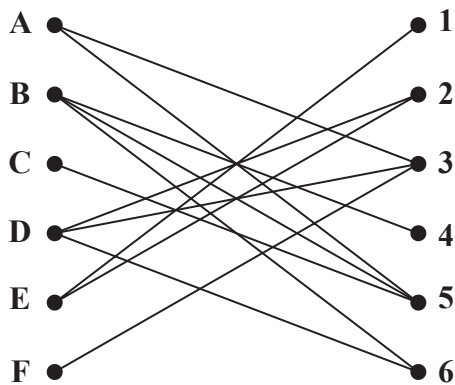


Figure 1

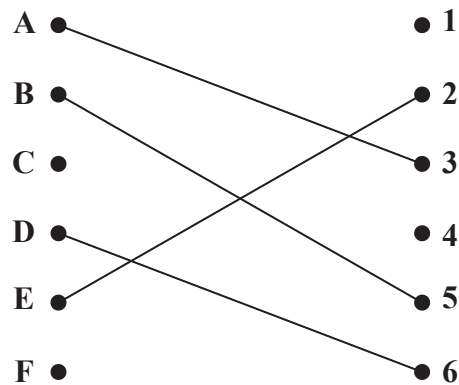


Figure 2

Figure 1 shows the possible allocations of six workers, A, B, C, D, E and F, to six tasks, 1, 2, 3, 4, 5 and 6. Each task must be assigned to only one worker and each worker must be assigned to exactly one task.

Figure 2 shows an initial matching.

(b) Starting from the given initial matching, use the maximum matching algorithm to find an alternating path from F to 1. Hence find an improved matching. You must write down the alternating path used and state your improved matching.

(3)

(c) Explain why it is not possible to find a complete matching.

(1)

Worker C has task 1 added to his possible allocations.

(d) Starting from the improved matching found in (b), use the maximum matching algorithm to find a complete matching. You must write down the alternating path used and state your complete matching.

(3)

(Total 11 marks)

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2. (a)

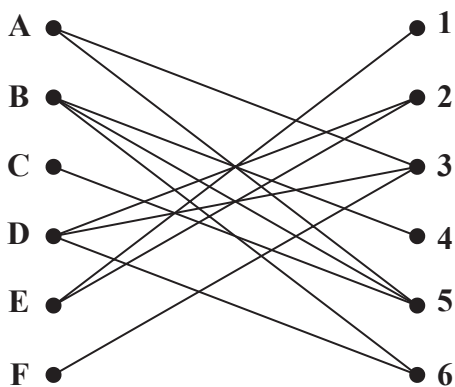


Figure 1

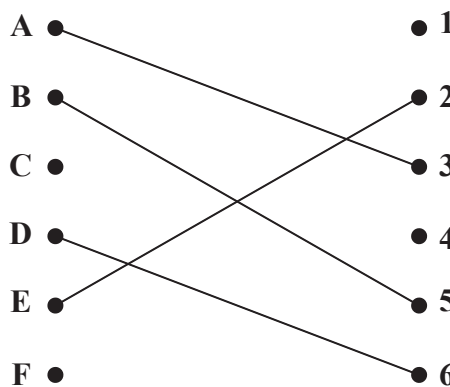


Figure 2

- ai) Alternating path starts at an unmatched node on one side of a bipartite graph and finishes at an unmatched node on the other side. It uses arcs that are alternately not in and in the initial matching.
- ii) Each element of set X is paired with a single element of set Y and vice versa.

b) Alternating path:

$$F - 3 = A - 5 = B - 6 = D - 2 = E - 1$$

Change status:

$$F = 3 - A = 5 - B = 6 - D = 2 - E = 1$$

Improved matching:

$$A = 5$$

$$B = 6$$

$$C =$$

$$D = 2$$

$$E = 1$$

$$F = 3$$

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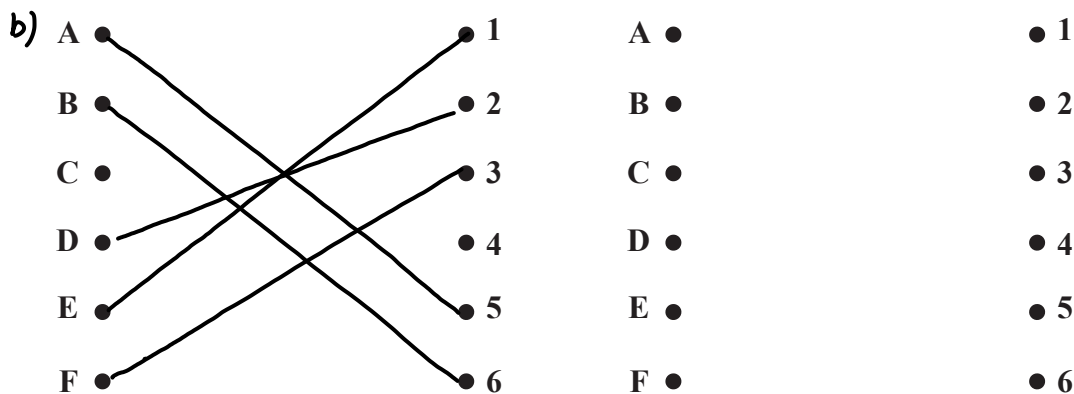
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Question 2 continued



c) F can only do task 3 so A has to do task 5 as A can only do task 3 and 5. C can only do task 5 so C will have no task.

d) Alternating path:
 $C - 1 = E - 2 = D - 6 = B - 4$

Change status:
 $C = 1 - E = 2 - D = 6 - B = 4$

Complete matching:

$A = 5$

$B = 4$

$C = 1$

$D = 6$

$E = 2$

$F = 3$

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Q2

(Total 11 marks)



3.

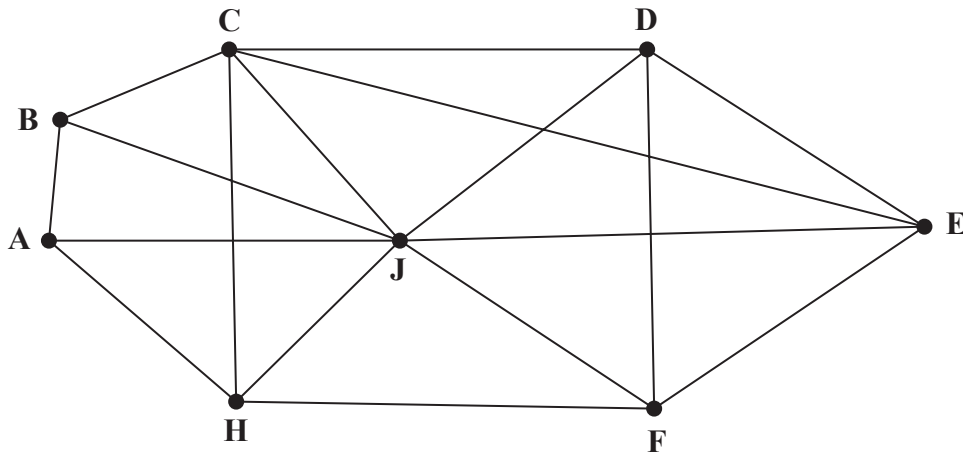


Figure 3

Figure 3 shows a graph G that contains 17 arcs and 8 vertices.

- (a) State how many arcs there are in a spanning tree for G . (1)
- (b) Explain why a path on G cannot contain 10 vertices. (2)
- (c) Determine the number of arcs that would need to be added to G to make G a complete graph with 8 vertices. (1)

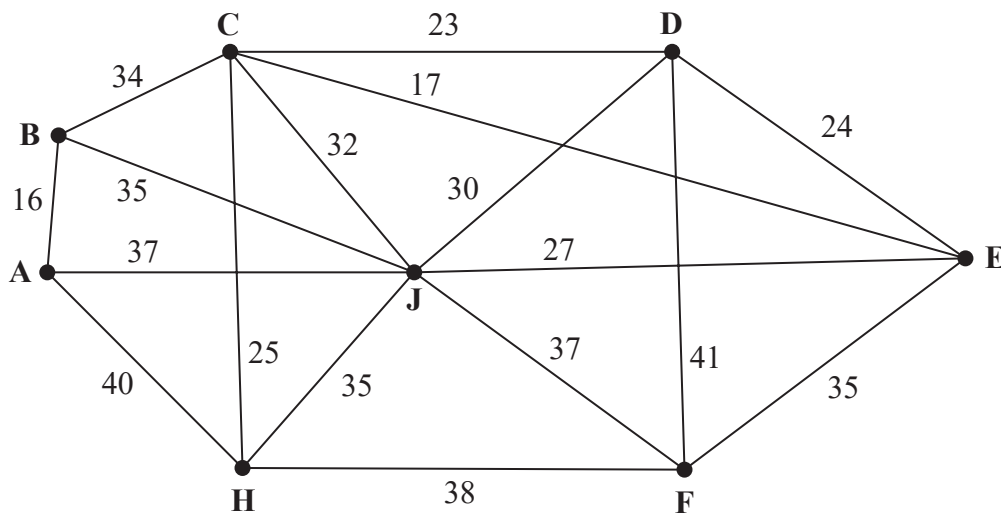


Figure 4

Figure 4 shows a weighted graph.

- (d) Use Prim's algorithm, starting at C , to find the minimum spanning tree for the weighted graph. You must clearly state the order in which you select the arcs of the tree. (3)
- (e) State the weight of the minimum spanning tree. (1)

(Total 8 marks)

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3.

a) $7 \quad (n-1 = 8-1)$

b) A vertex cannot appear in a path more than once. G has 8 vertices so maximum number of vertices is 8

c) $7+6+5+4+3+2+1-17=11$

d) CE 17

CD 23

CH 25

EJ 27

CB 34

BA 16

EF 35

e) $17+23+25+27+34+16+35=177$

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4.

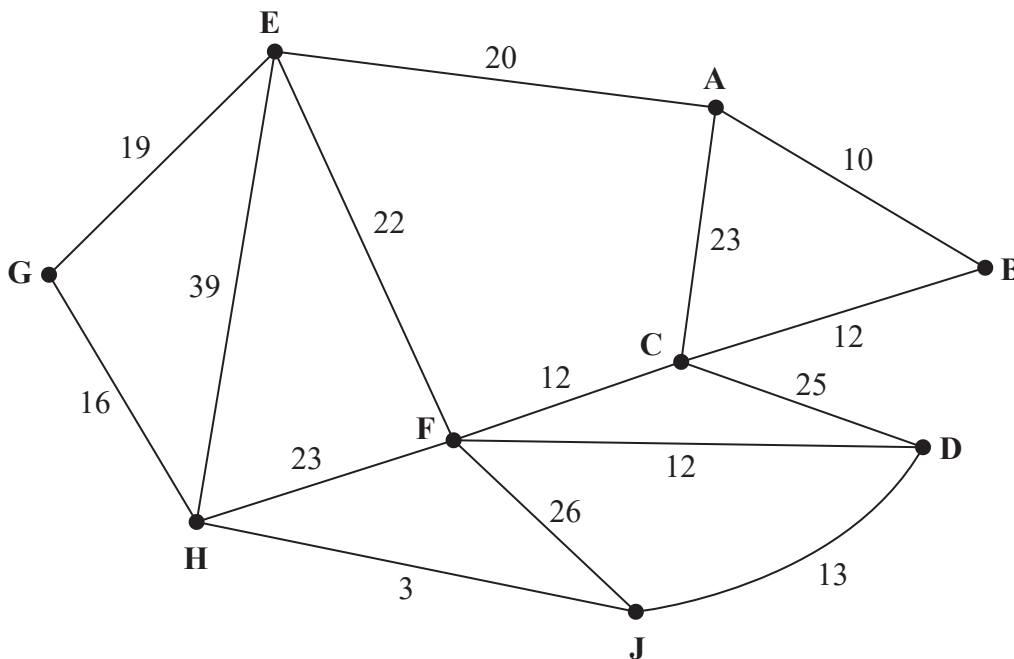


Figure 5

[The total weight of the network is 275]

Figure 5 models a network of roads between nine villages, A, B, C, D, E, F, G, H and J. The number on each edge gives the time, in minutes, to travel along the corresponding road. Mandeep wishes to travel from A to J as quickly as possible.

- (a) Use Dijkstra’s algorithm to find the shortest time needed to travel from A to J. State the quickest route. (6)

On Monday, Mandeep must travel from D to H via A.

- (b) Find the shortest time needed to travel from D to H via A. State the quickest route. (2)

On Wednesday, Mandeep needs to travel along each road to check that it is in good repair. She wishes to minimise the total time required to traverse the network. Mandeep plans to start and finish her inspection route at G.

- (c) Use an appropriate algorithm to find the roads that need to be traversed twice. You must make your method and working clear. (5)
- (d) Write down a possible route, giving its duration. (2)

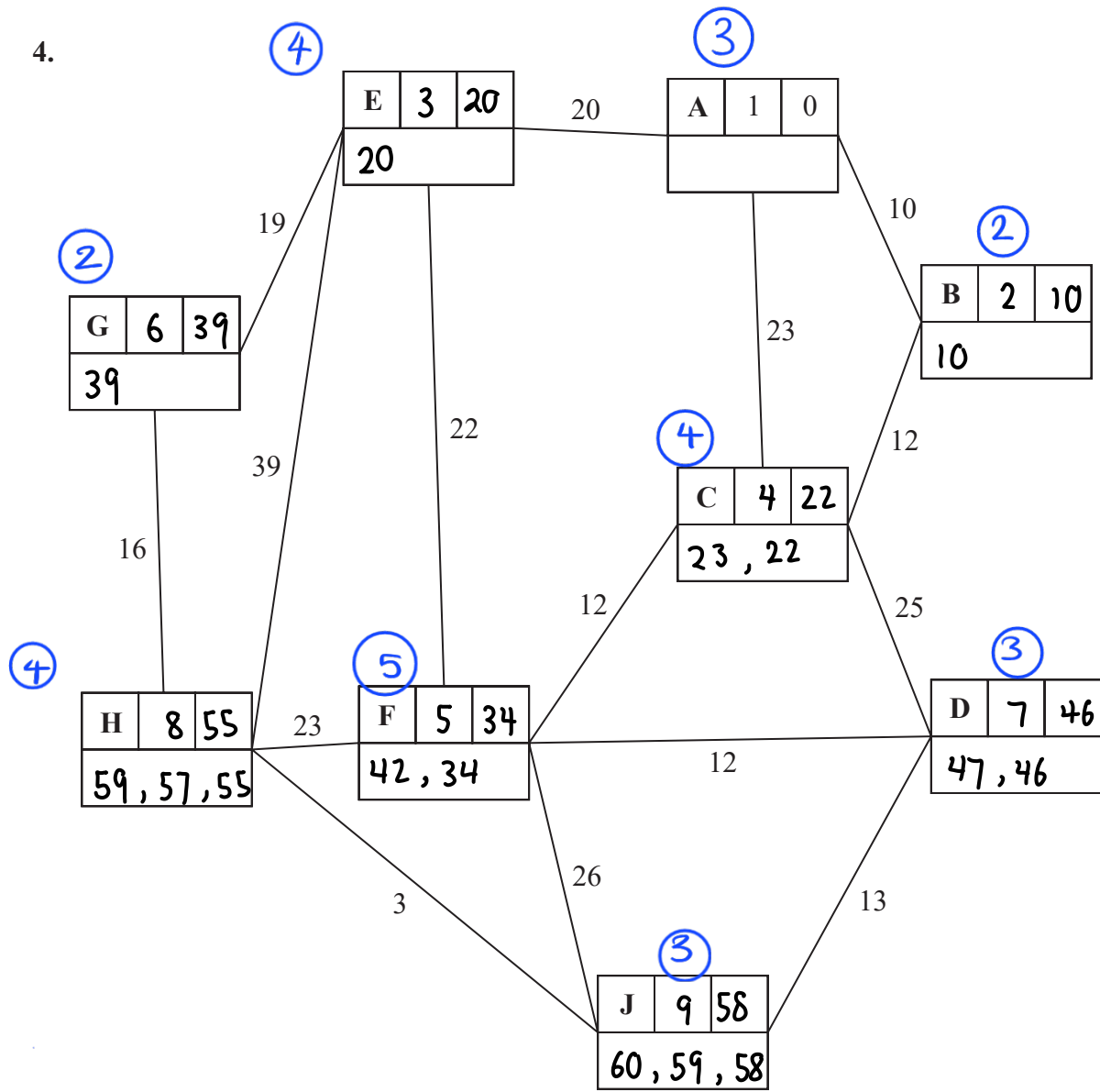
On Friday, all the roads leading directly to B are closed. Mandeep needs to check all the remaining roads and may start at any village and finish at any village. A route is required that excludes all those roads leading directly to B.

- (e) State all possible combinations of starting and finishing points so that the duration of Mandeep’s route is minimised. Calculate the duration of Mandeep’s minimum route. (3)

(Total 18 marks)

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4.



Key:

Vertex	Order of labelling	Final value
Working values		

Shortest time: 58 mins

Quickest route: A-E-G-H-J

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Question 4 continued

b) $D \rightarrow A \rightarrow H: D - F - C - B - A - E - G - H$

$$46 + 55 = 101 \text{ mins}$$

c) Odd valencies: A, D, F, J

$$\begin{aligned} \overset{(BCF)}{AD} + \overset{(D)}{FJ} &= 46 + 25 \\ &= 71 \end{aligned}$$

$$\overset{(BC)}{AF} + DJ = 34 + 13$$

$$= 47 \quad \times \text{Lowest weight} \times$$

$$\begin{aligned} \overset{(EGH)}{AJ} + DF &= 58 + 12 \\ &= 70 \end{aligned}$$

\therefore Arcs AB, BC, CF and DJ should be traversed twice

d) GEHFEABCDFCBACFJDJHG

$$275 + 47 = 322 \text{ mins}$$

e) Odd valencies: C, D, F, J

$$CD \quad 24$$

$$CF \quad 12$$

$$CJ \quad 37$$

$$DF \quad 12 \quad \text{least weight: CF \& DF}$$

$$DJ \quad 13$$

$$FJ \quad 26$$

Possible combinations: start at C, end at J or vice versa, repeat DF
or start at D, end at J or vice versa, repeat CF

$$\text{length: } 275 - 12 - 10 + 12 = 265 \text{ mins}$$

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5.

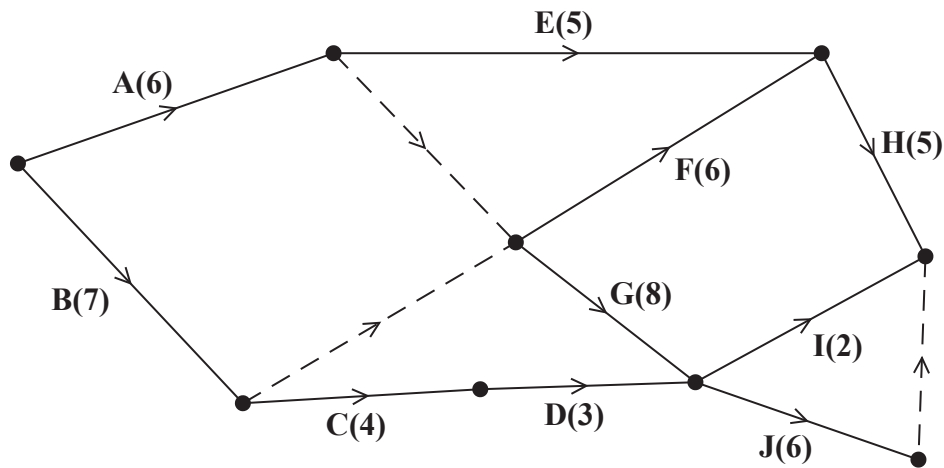


Figure 6

A project is modelled by the activity network shown in Figure 6. The activities are represented by the arcs. The number in brackets on each arc gives the time, in hours, to complete the corresponding activity. Each activity requires one worker. The project is to be completed in the shortest possible time.

- (a) Complete the precedence table in the answer book. (2)
- (b) Complete Diagram 1 in the answer book to show the early event times and late event times. (3)
- (c) State the minimum project completion time and list the critical activities. (2)
- (d) Calculate the maximum number of hours by which activity E could be delayed without affecting the shortest possible completion time of the project. You must make the numbers used in your calculation clear. (1)
- (e) Calculate a lower bound for the number of workers needed to complete the project in the minimum time. You must show your working. (1)

The project is to be completed in the minimum time using as few workers as possible.

- (f) Schedule the activities using Grid 1 in the answer book. (3)

Before the project begins it becomes apparent that activity E will require an additional 6 hours to complete. The project is still to be completed in the shortest possible time and the time to complete all other activities is unchanged.

- (g) State the new minimum project completion time and list the new critical activities. (2)

(Total 14 marks)

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5.

Activity	Immediately preceding activities
A	-
B	-
C	B
D	C
E	A

Activity	Immediately preceding activities
F	A, B
G	A, B
H	E, F
I	D, G
J	D, G

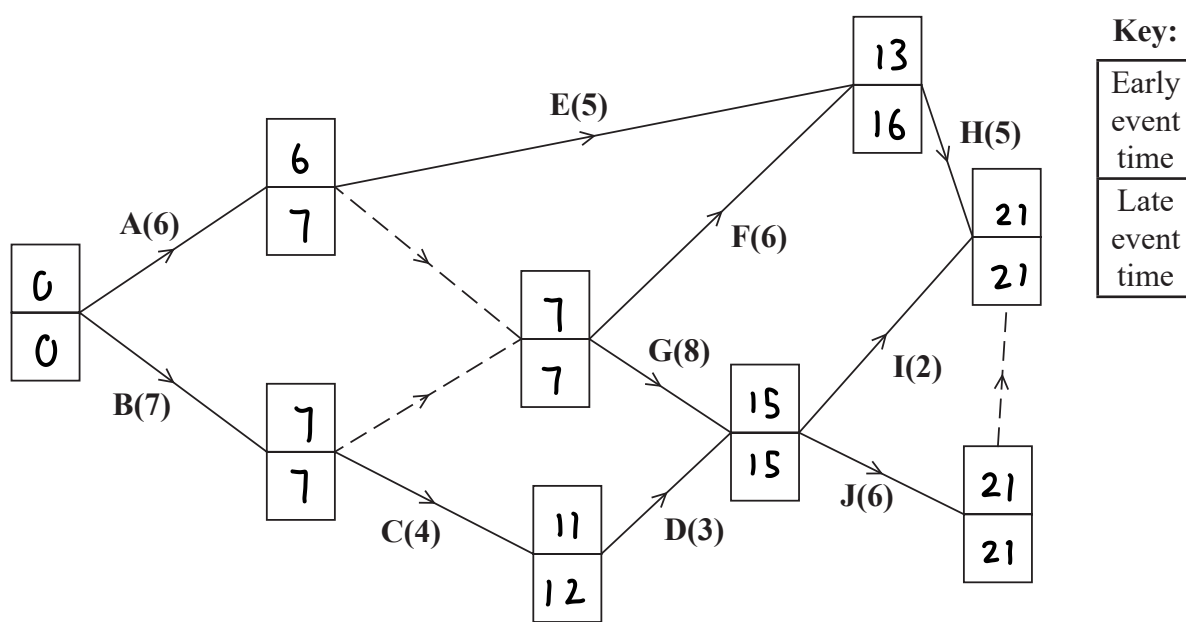


Diagram 1

c) 21 hours

Critical activity: B, G, J

d) Float on E: $16 - 5 - 6 = 5$ hours

e) $(6 + 7 + 4 + 3 + 5 + 6 + 8 + 5 + 2 + 6) \div 21 = 52 \div 21$

$= 2.476$

≈ 3

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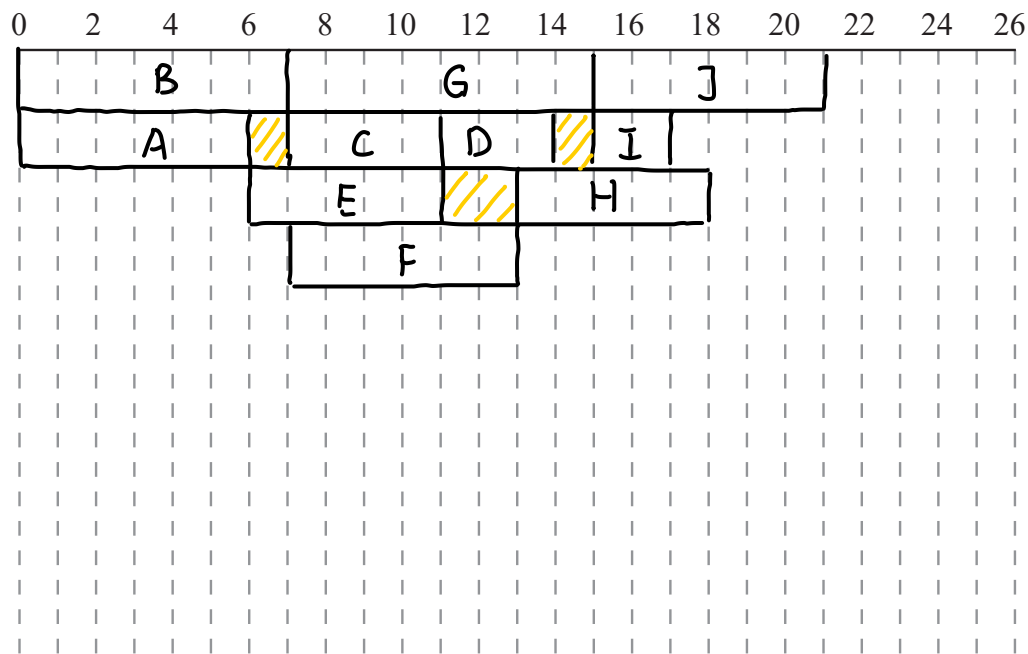
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Question 5 continued

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Grid 1

g) $21 - 5 + 6 = 22$ hours
critical activities : A, E & H

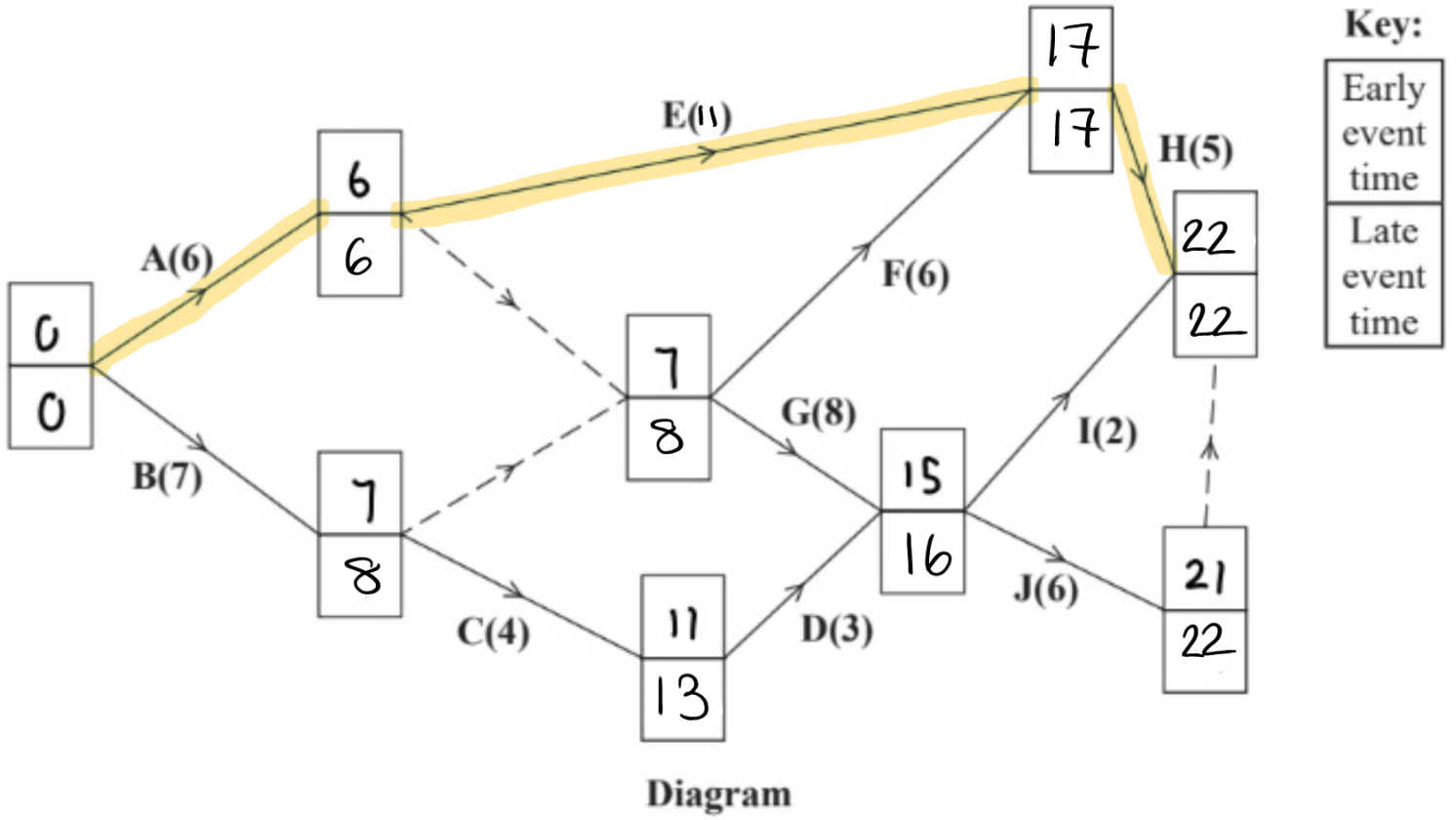
(Total 14 marks)

Q5

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g) * Working out * PhysicsAndMathsTutor.com



6. A linear programming problem in x , y and z is described as follows.

$$\begin{array}{ll} \text{Maximise} & P = 2x + 7y + 2z \\ \text{subject to} & 4x + 3y \leq 30 \\ & -7x + 4z \leq 36 \\ & x \geq 1 \\ & z \geq 10 \end{array}$$

Given that $x + 15 = y + z$,

(a) (i) show that the constraints can be written as

$$\begin{array}{l} 4x + 3y \leq 30 \\ 3x + 4y \geq 24 \\ x \geq 1 \\ -x + y \leq 5 \end{array}$$

(ii) express P in terms of x and y only.

(3)

(b) Add lines and shading to Diagram 1 in the answer book to represent the constraints from (a)(i). Hence determine the feasible region and label it R .

(4)

(c) Use the objective line method to find the optimal vertex V of the feasible region. You must draw and label your objective line and label V clearly.

(2)

(d) Use algebra to calculate the exact coordinates of V and hence calculate the corresponding value of P at V .

(3)

Given that an integer solution is required,

(e) determine the optimal solution for the original problem in x , y and z and state the corresponding value of P .

(2)

(Total 14 marks)

TOTAL FOR PAPER: 75 MARKS

END

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6.

$$\text{ai) } x + 15 = y + z$$

$$z = x + 15 - y \quad \text{--- (1)}$$

$$\text{sub (1) into } -7x + 4z \leq 36$$

$$-7x + 4(x + 15 - y) \leq 36$$

$$-7x + 4x + 60 - 4y \leq 36$$

$$-3x - 4y + 24 \leq 0$$

$$3x + 4y \geq 24$$

$$\text{sub (1) into } z \geq 10$$

$$x + 15 - y \geq 10$$

$$x - y \geq -5$$

$$-x + y \leq 5$$

$$\text{ii) } P = 2x + 7y + 2(x + 15 - y)$$

$$= 4x + 5y + 30$$

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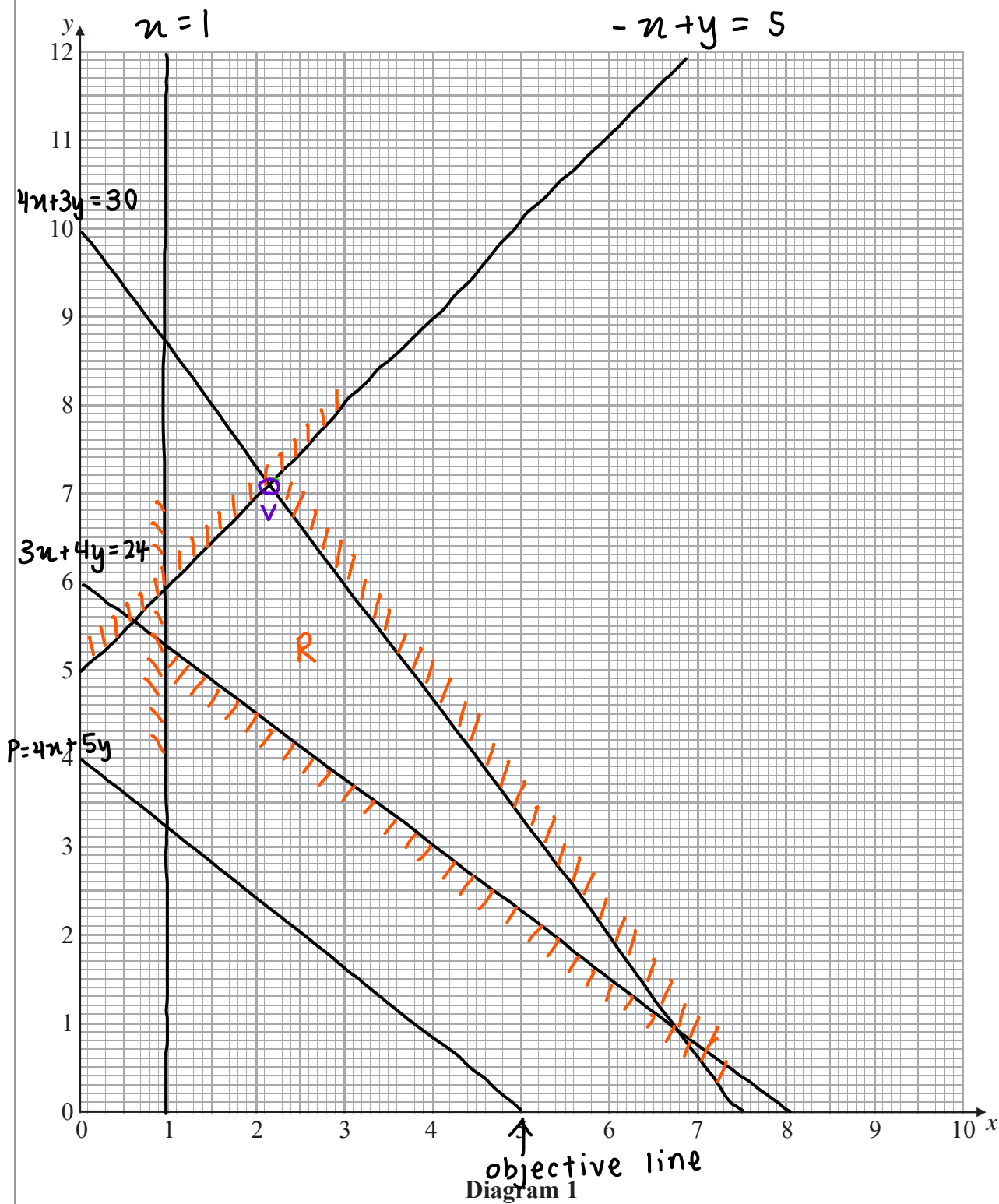
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Question 6 continued



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Question 6 continued

$$d) \quad -x + y \leq 5$$

$$4x + 3y \leq 30$$

$$y = 5 + x$$

$$4x + 3(5 + x) = 30$$

$$4x + 15 + 3x = 30$$

$$7x = 15$$

$$x = \frac{15}{7}$$

$$y = 5 + \frac{15}{7}$$

$$= \frac{50}{7}$$

$$v \left(\frac{15}{7}, \frac{50}{7} \right)$$

$$P = 4 \left(\frac{15}{7} \right) + 5 \left(\frac{50}{7} \right) + 30$$

$$= \frac{520}{7}$$

$$e) \quad x = 2, \quad y = 7$$

$$z = 15 + 2 - 7$$

$$= 10$$

$$P = 2(2) + 7(7) + 2(10)$$

$$= 73$$

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