MODEL ANSWERS

Pearson Edexcel International Advanced Level

Decision Mathematics D1

Advanced/Advanced Subsidiary

Friday 16 June 2017 – Afternoon
Time: 1 hour 30 minutes

Paper Reference
WDM01/01

You must have:
D1 Answer Book

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** on the top of the answer book with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the D1 answer book provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.
- Do not return the question paper with the answer book.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

P46680A

1/2/1/1/

P 4 6 6 8 0 A

Pearson

Write your answers in the D1 answer book for this paper.

1.

2.5 0.9 3.1 1.4 1.5 2.0 1.9 1.2 0.3 0.4 3.9

The numbers in the list are the lengths, in metres, of eleven pieces of wood. They are to be cut from planks of wood of length 5 metres. You should ignore wastage due to cutting.

(a) Calculate a lower bound for the number of planks needed. You must make your method clear.

(2)

(b) Use the first-fit bin packing algorithm to determine how these pieces could be cut from 5 metre planks.

(3)

(c) Carry out a quick sort to produce a list of the lengths in descending order. You should show the result of each pass and identify your pivots clearly.

(4)

(d) Use the first-fit decreasing bin packing algorithm to determine how these pieces could be cut from 5 metre planks.

(2)

(Total 11 marks)

Leave blank

1.

2.5 0.9 3.1 1.4 1.5 2.0 1.9 1.2 0.3 0.4 3.9

a) $(2.5 + 0.9 + 3.1 + 1.4 + 1.5 + 2.0 + 1.9 + 1.2 + 0.3 + 0.4 + 3.9) \div 5 = 19.1 \div 5$

$= 3.82$

$\simeq$ 4 bins

b) Bin 1 : 2.5 0.9 1.4

Bin 2 : 3.1 1.5 0.3

Bin 3 : 2.0 1.9 0.4

Bin 4 : 1.2

Bin 5 : 3.9

c) 2.5 0.9 3.1 1.4 1.5 (2.0) 1.9 1.2 0.3 0.4 3.9

2.5 (3.1) 3.9 [2.0] 0.9 1.4 1.5 (1.9) 1.2 0.3 0.4

3.9 [3.1] 2.5 [2.0] [1.9] 0.9 1.4 1.5 (1.2) 0.3 0.4

3.9 [3.1] 2.5 [2.0] [1.9] 1.4 (1.5) [1.2] 0.9 (0.3) 0.4

3.9 [3.1] 2.5 [2.0] [1.9] [1.5] 1.4 [1.2] 0.9 (0.4) [0.3]

3.9 [3.1] 2.5 [2.0] [1.9] [1.5] 1.4 [1.2] 0.9 [0.4] [0.3]

Sort complete

d) Bin 1 : 3.9 0.9

Bin 2 : 3.1 1.9

Bin 3 : 2.5 2.0 0.4

Bin 4 : 1.5 1.4 1.2 0.3

2.

	S	A	B	C	D	E	F	G
S	–	150	225	275	135	200	280	255
A	150	–	265	300	185	170	385	315
B	225	265	–	245	190	155	215	300
C	275	300	245	–	250	310	280	275
D	135	185	190	250	–	145	205	270
E	200	170	155	310	145	–	220	380
F	280	385	215	280	205	220	–	250
G	255	315	300	275	270	380	250	–

The table shows the costs, in pounds, of connecting seven computer terminals, A, B, C, D, E, F and G, to a server, S.

(a) Use Prim's algorithm, starting at S, to find the minimum spanning tree for this table of costs. You must clearly state the order in which you select the edges of your tree.

(3)

(b) Draw the minimum spanning tree using the vertices given in Diagram 1 in the answer book. State the minimum cost, in pounds, of connecting the seven computer terminals to the server.

(2)

(c) Explain why it is not necessary to check for cycles when using Prim's algorithm.

(1)

(Total 6 marks)

Leave blank

2.

1 4 5 7 2 3 6

	S	A	B	C	D	E	F	G
S		~~150~~	~~225~~	~~275~~	~~135~~	~~200~~	~~280~~	~~255~~
A	150	–	~~265~~	~~300~~	~~185~~	~~170~~	~~385~~	~~315~~
B	~~225~~	~~265~~		~~245~~	~~190~~	155	~~215~~	~~300~~
C	~~275~~	~~300~~	245		~~250~~	~~310~~	~~280~~	~~275~~
D	135	~~185~~	~~190~~	~~250~~		~~145~~	~~205~~	~~270~~
E	~~200~~	~~170~~	~~155~~	~~310~~	145		~~220~~	~~380~~
F	~~280~~	~~385~~	~~215~~	~~280~~	205	~~220~~		~~250~~
G	255	315	300	275	270	380	250	–

SD , DE , SA , EB , DF , BC , FG

DO NOT WRITE IN THIS AREA

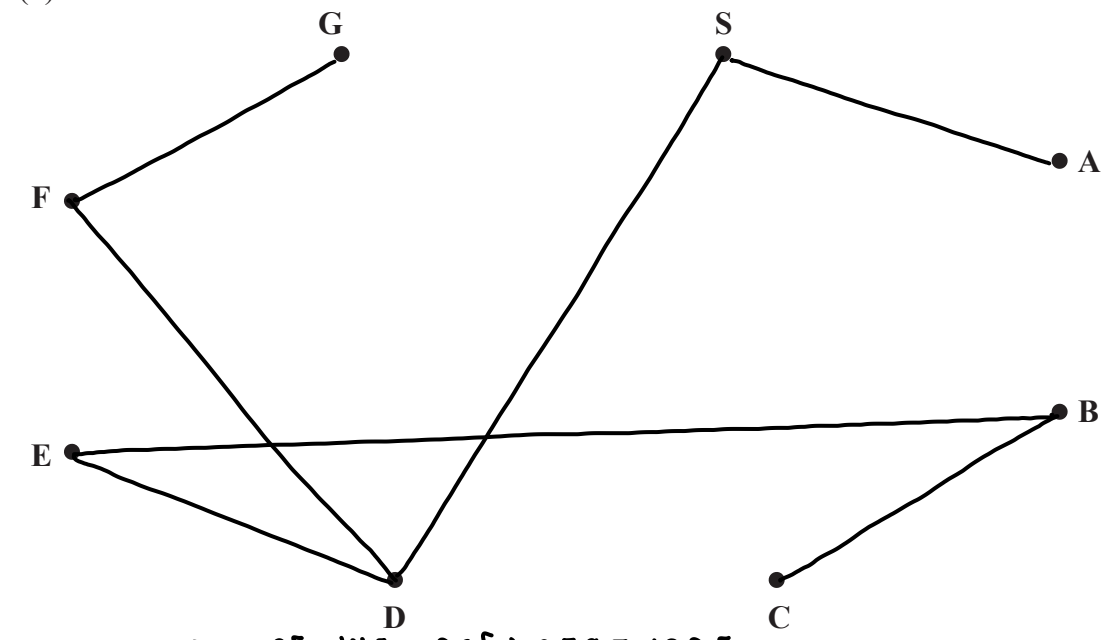
DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Leave blank

Question 2 continued

(b)



150+155+245+135+145+205+250 = 1285

Diagram 1

Minimum cost £ 1285

(c)

In Prim's, a new vertex is added to the tree so no cycles are formed

Q2

(Total 6 marks)

3.

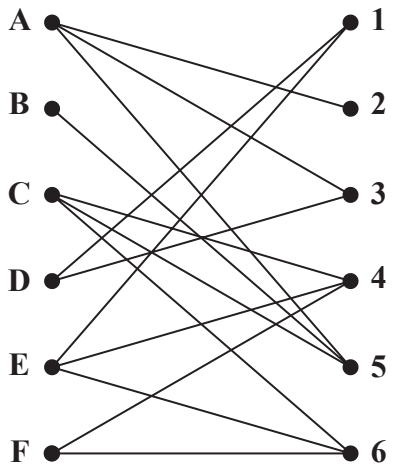


Figure 1

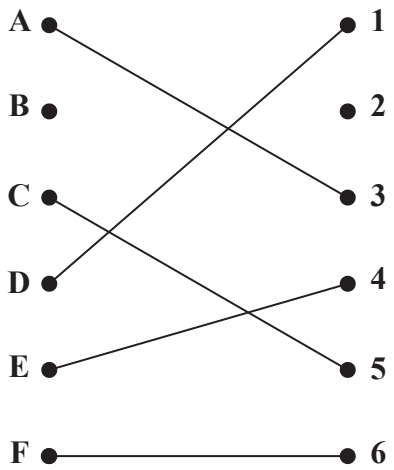


Figure 2

Figure 1 shows the possible allocations of six workers, Andrea (A), Baasim (B), Charlie (C), Deirdre (D), Ean (E) and Fen-Fang (F), to six tasks, 1, 2, 3, 4, 5 and 6.

(a) Write down the technical name given to the type of graph shown in Figure 1.

(1)

Figure 2 shows an initial matching.

(b) Starting from the initial matching, use the maximum matching algorithm to find a complete matching. You must list the alternating path you used and state your complete matching.

(3)

(c) State a different complete matching from the one found in (b).

(1)

(d) By considering the workers who must be allocated to particular tasks, explain why there are exactly two different complete matchings.

(2)

(Total 7 marks)

Leave blank

3.

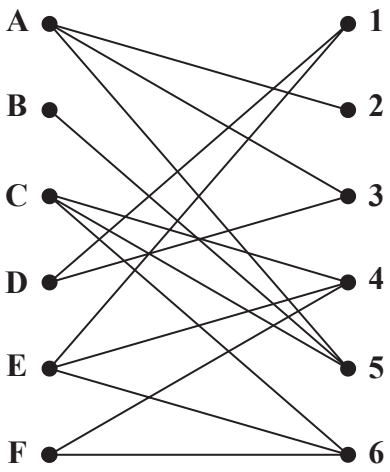


Figure 1

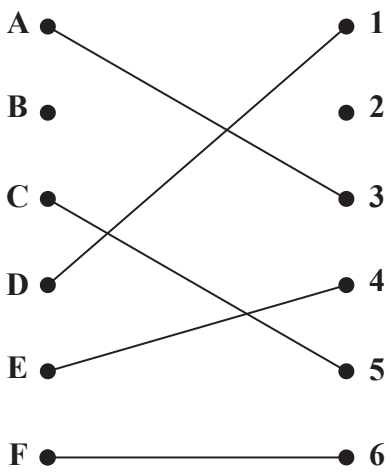


Figure 2

a) Bipartite graph

b) Alternating path:

B - 5 = C - 4 = E - 1 = D - 3 = A - 2

Change status:

B = 5 - C = 4 - E = 1 - D = 3 - A = 2

Complete matching:

A = 2

B = 5

C = 4

D = 3

E = 1

F = 6

c) A = 2

B = 5

C = 6

F = 3

E = 1

F = 4

d) Only worker A can do task 2 and B can only do task 5

∴ A = 2 , B = 5

C and F can both do either task 4 or 6 so there are exactly 2 different complete matching

4.

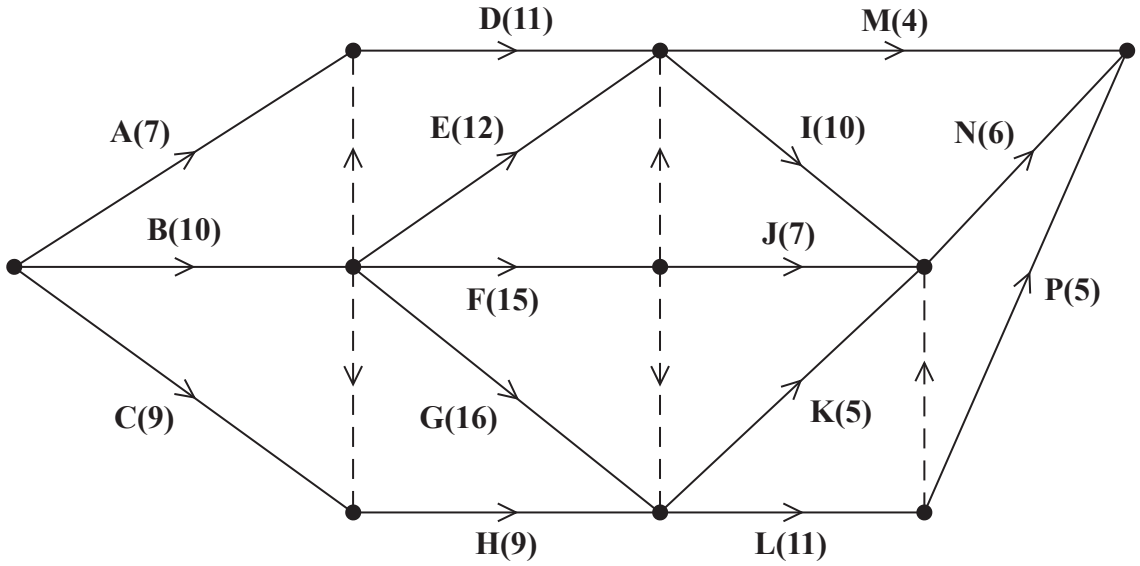


Figure 3

A project is modelled by the activity network shown in Figure 3. The activities are represented by the arcs. The number in brackets on each arc gives the time, in days, to complete the corresponding activity. Each activity requires exactly one worker. The project is to be completed in the shortest possible time.

(a) Complete Diagram 1 in the answer book to show the early event times and late event times.

(4)

(b) Determine the critical activities and the length of the critical path.

(2)

(c) Calculate the total float for activity D. You must make the numbers you use in your calculation clear.

(2)

(d) Draw a cascade (Gantt) chart for this project on Grid 1 in the answer book.

(4)

(e) Use your cascade chart to determine the minimum number of workers needed to complete the project in the shortest possible time. You must make specific reference to times and activities.

(2)

(Total 14 marks)

Leave blank

4. (a)

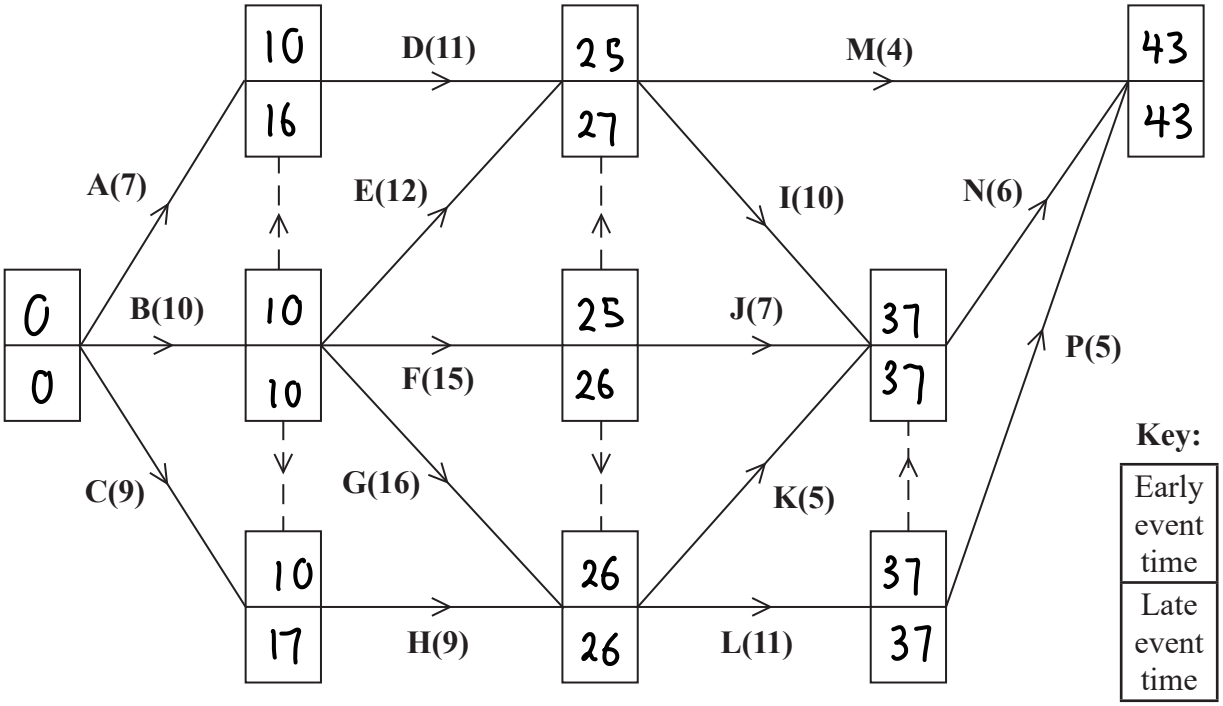


Diagram 1

b) B, G, L & N

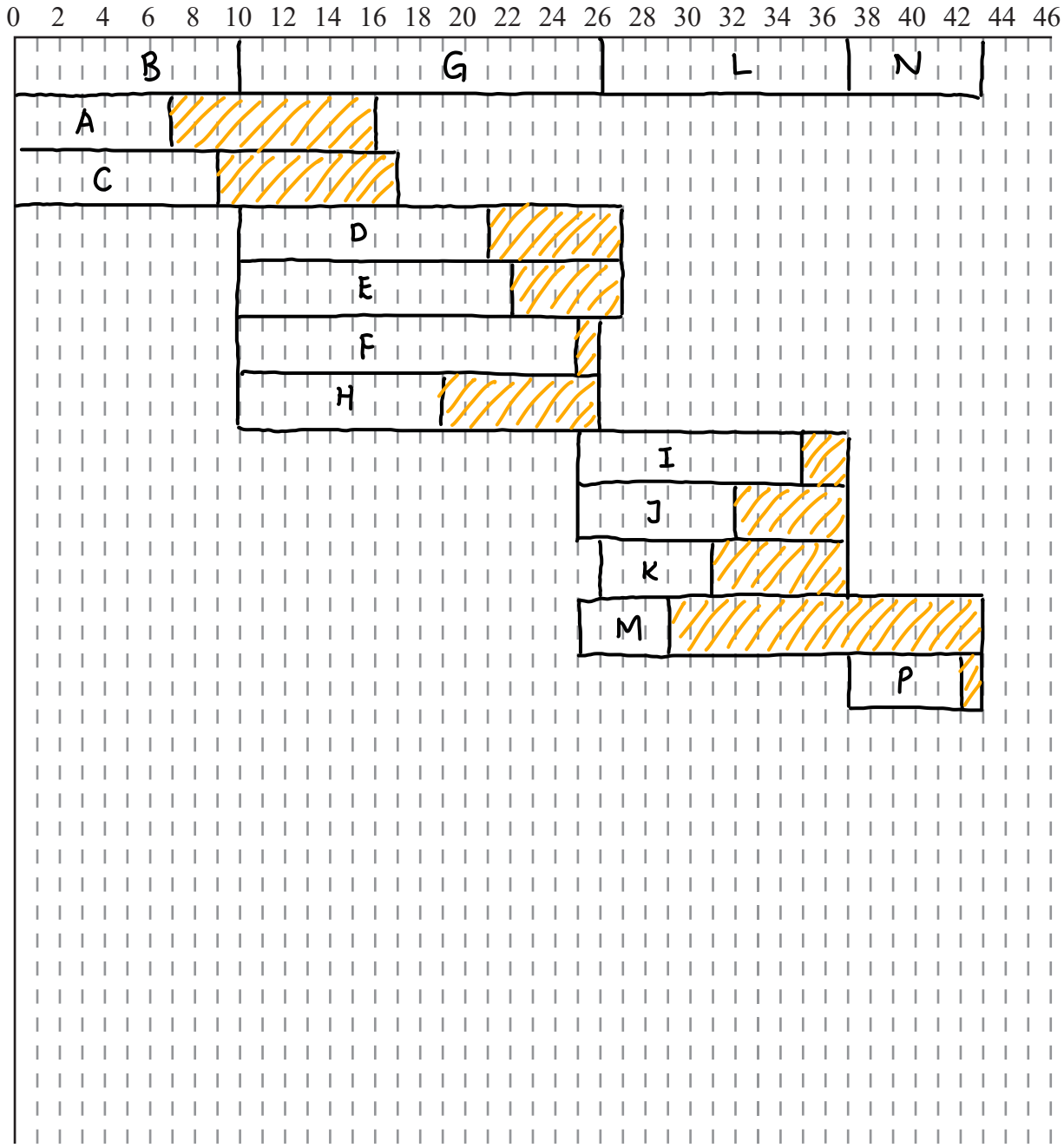
length : 43 days

c) 27 - 11 - 10 = 6 days

Leave blank

Question 4 continued

(d)



Grid 1

e) 5 workers, working on G, D, E, F and H from time 16 to 19

Q4

(Total 14 marks)

5. A school awards two types of prize, junior and senior.

The school decides that it will award at least 25 junior prizes and at most 60 senior prizes.

Let x be the number of junior prizes that the school awards and let y be the number of senior prizes that the school awards.

(a) Write down two inequalities to model these constraints.

(2)

Two further constraints are

$$2x + 5y \geqslant 250$$

$$5x - 3y \leqslant 150$$

(b) Add lines and shading to Diagram 1 in the answer book to represent all four of these constraints. Hence determine the feasible region and label it R.

(4)

The cost of a senior prize is three times the cost of a junior prize. The school wishes to minimise the cost of the prizes.

(c) State the objective function, giving your answer in terms of x and y.

(1)

(d) Determine the exact coordinates of the vertices of the feasible region. Hence use the vertex method to find the number of junior prizes and the number of senior prizes that the school should award. You should make your working clear.

(8)

(Total 15 marks)

5.

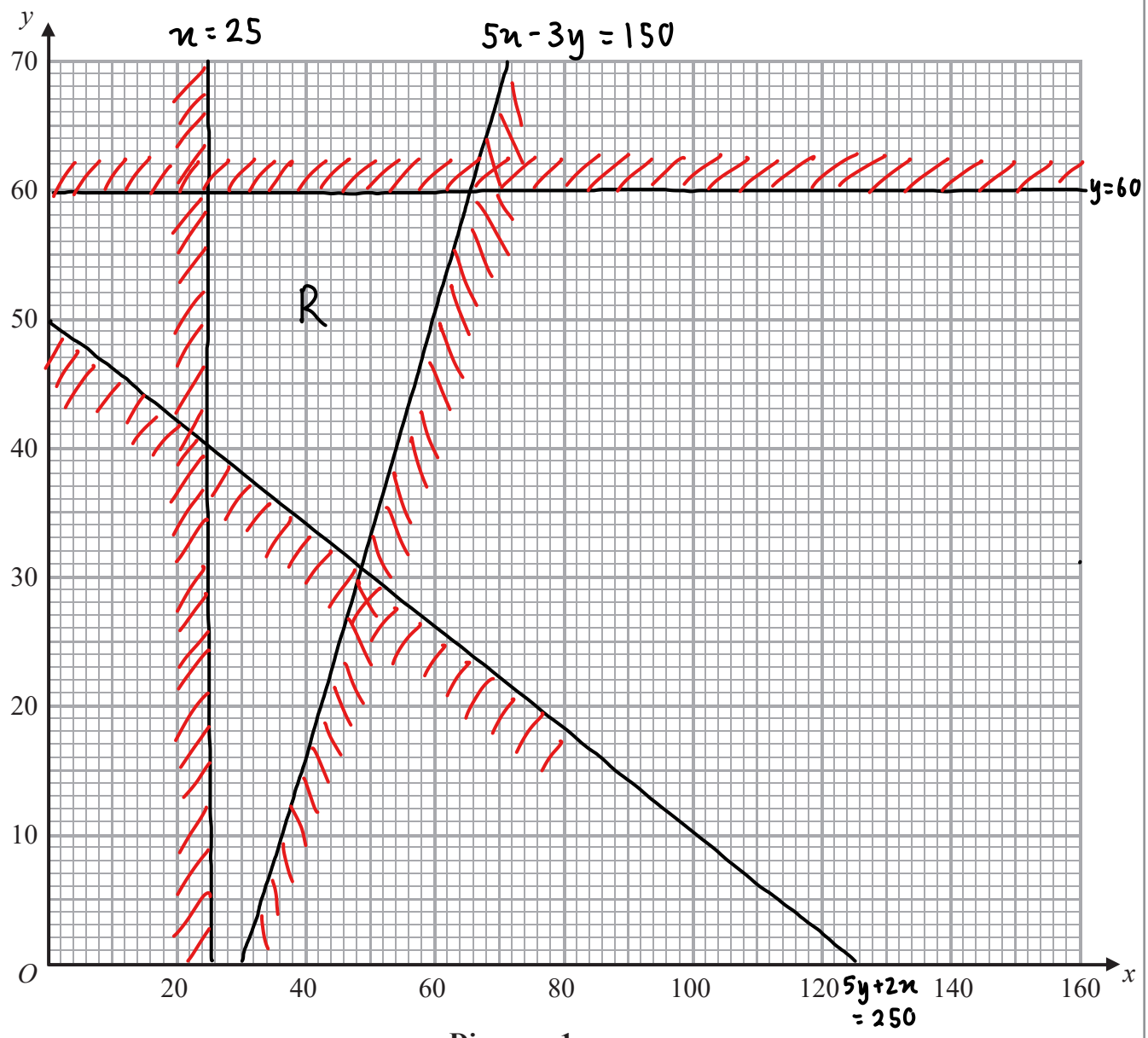


Diagram 1

a) $n \geq 25$, $y \leq 60$

c) $C = n + 3y$

d) $n = 25$, $y = 60$ $(25, 60)$ $C = 25 + 3(60)$
$= 205$

$n = 25$, $5y + 2n = 250$
$y = [250 - 2(25)]\frac{1}{5}$
$= 40$
$(25, 40)$ $C = 25 + 3(40)$
$= 145$

Leave blank

Question 5 continued

$y=60 \quad 5x-3y=150$

$5x = 150+3(60)$

$x = 66$

$(66, 60)$ $\qquad C = 66+3(60)$

$= 246$

$5x-3y=150$ —①

$5y+2x=250$ —②

① × 5 $\quad 25x-15y=750$ —③

② × 3 $\quad 6x+15y=750$ —④

③ + ④ $\quad 31x=1500$

$x = \frac{1500}{31}$

$5y+2\left(\frac{1500}{31}\right)=250$

$y = \frac{950}{31}$

$\left(\frac{1500}{31}, \frac{950}{31}\right)$ $\qquad C = \frac{1500}{31}+3\left(\frac{950}{31}\right)$

$= 140.32$

$\therefore$ optimal vertex : $\left(\frac{1500}{31}, \frac{950}{31}\right)$

x	y	$5y+2x$	$5x-3y$	
48	30	$246 < 250$		n/a
48	31	251	147	
49	30	$248 < 250$		n/a
49	31	253	$150 > 150$	n/a

$\therefore$ 48 junior prizes and 31 senior prizes should be awarded

6.

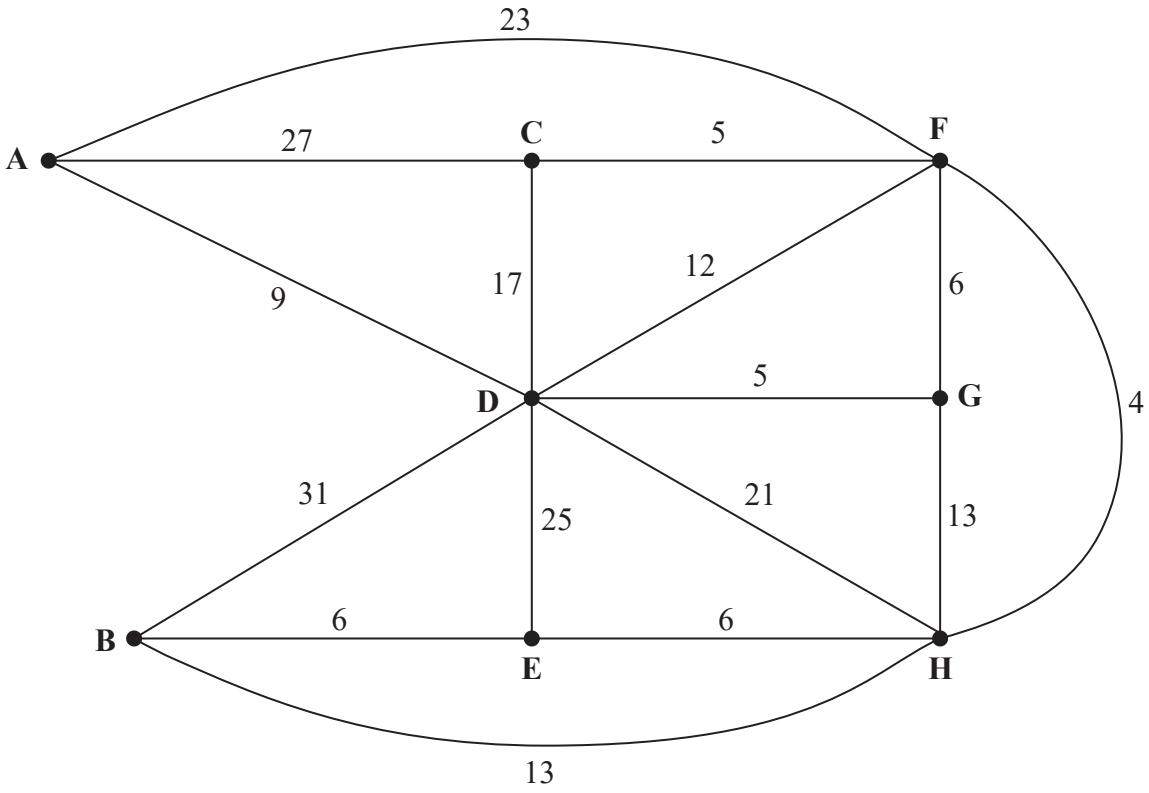


Figure 4

[*The total weight of the network is 223*]

Figure 4 models a network of roads. The number on each arc represents the length, in km, of the corresponding road. Pamela wishes to travel from A to B.

(a) Use Dijkstra's algorithm to find the shortest path from A to B. State your path and its length.

(6)

On a particular day, Pamela must include C in her route.

(b) Find the shortest route from A to B that includes C, and state its length.

(2)

Due to damage, the three roads in and out of C are closed and cannot be used. Faith needs to travel along all the remaining roads to check that there is no damage to any of them. She must travel along each of the remaining roads at least once and the length of her inspection route must be minimised. Faith will start and finish at A.

(c) Use an appropriate algorithm to find the arcs that will need to be traversed twice. You must make your method and working clear.

(4)

(d) Write down a possible route, and calculate its length. You must make your calculation clear.

(3)

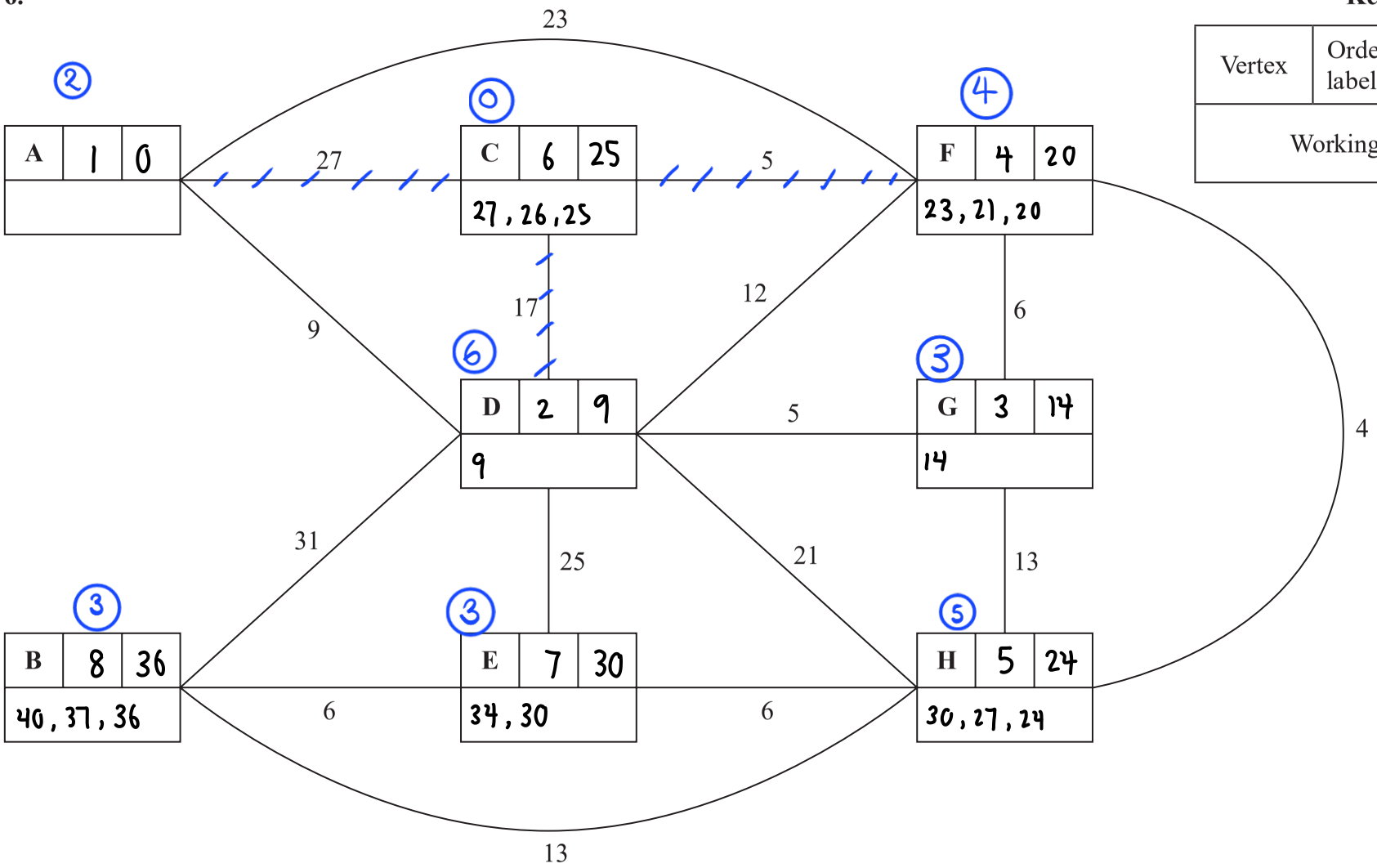
Faith now decides to start at vertex B and finish her inspection route at a different vertex. A route of minimum length that includes each road, excluding those directly connected to C, needs to be found.

(e) State the finishing vertex of Faith's route. Calculate the difference between the length of this new route and the route found in (d).

(2)

(Total 17 marks)

6.



Shortest path from A to B: A D G F H E B

Length of shortest path from A to B: 36 km

Leave blank

Leave blank

Question 6 continued

b) ADGFCFHEB

36 + 5 + 5 = 46 km

c) Odd nodes: B, E, G, H

BE + GH (F): 6 + (6+4) = 16 *Shortest route*

BG (EHF) + EH: (6+4+6+6) + 6 = 28

BH (E) + EG (HF): (6+6) + (6+4+6) = 28

∴ BE, FG and FH are traversed twice

d) ADBEHFGHBEDHFGDFA

length = 223 − 27 − 5 − 17 + 16

= 190 km

e) GH = 10

EH = 6

EG = 16

∴ Repeat EH, finish at G

Difference = 16 − 6

= 10 km

7. Draw the activity network described in this precedence table, using activity on arc and dummies only where necessary.

Activity	Immediately preceding activities
A	–
B	–
C	A
D	A
E	C, D
F	C, D
G	C, D
H	B, E
I	B, E, F, G
J	G
K	G

(Total 5 marks)

TOTAL FOR PAPER: 75 MARKS

END

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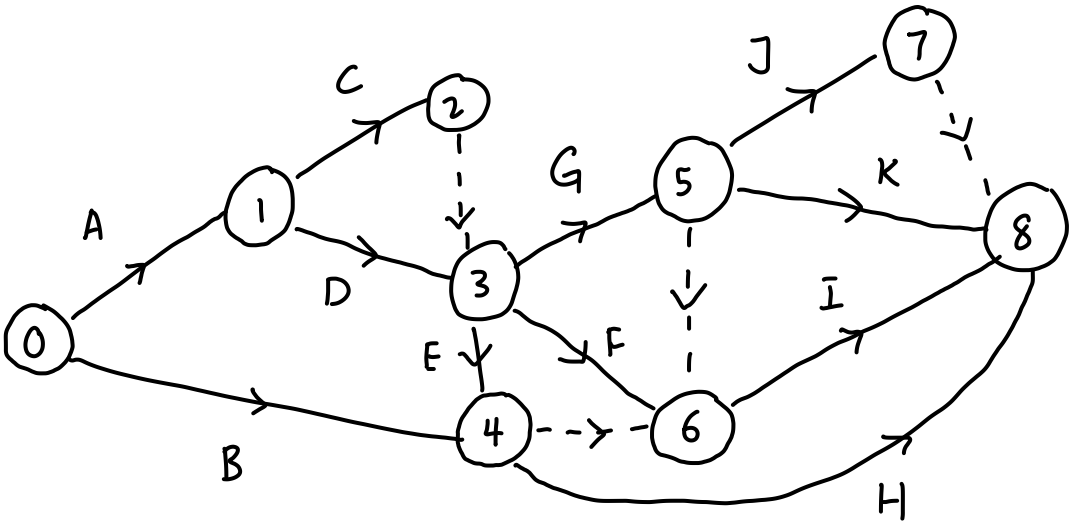
7.



Q7

(Total 5 marks)

TOTAL FOR PAPER: 75 MARKS

END