MODEL ANSWERS

Pearson Edexcel International Advanced Level

Decision Mathematics D1

Advanced/Advanced Subsidiary

Friday 17 June 2016 – Afternoon
Time: 1 hour 30 minutes

Paper Reference
WDM01/01

You must have:
D1 Answer Book

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** on the top of the answer book with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the D1 answer book provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.
- Do not return the question paper with the answer book.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

P46943A

1/1/1/1/e2

P 4 6 9 4 3 A

Turn over

PEARSON

Write your answers in the D1 answer book for this paper.

1.

4.2 1.8 3.1 1.3 4.0 4.1 3.7 2.3 2.7

(a) Use the first-fit bin packing algorithm to determine how the numbers listed above can be packed into bins of size 7.8

(2)

(b) Determine whether the number of bins used in (a) is optimal. Give a reason for your answer.

(2)

(c) The list of numbers is to be sorted into ascending order. Use a quick sort to obtain the sorted list. You should show the result of each pass and identify your pivots clearly.

(4)

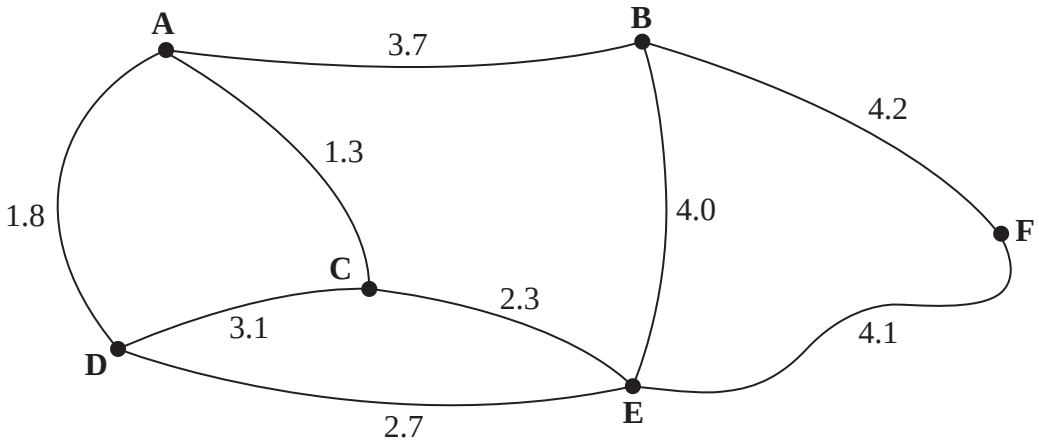

Figure 1

(d) Use Kruskal's algorithm to find a minimum spanning tree for the network in Figure 1. You must show clearly the order in which you consider the arcs. For each arc, state whether or not you are including it in your minimum spanning tree.

(2)

A new spanning tree is required which includes the arcs AB and DE, and which has the lowest possible total weight.

(e) Explain why Prim's algorithm could not be used to complete the tree.

(1)

(Total 11 marks)

Leave blank

1.

4.2 1.8 3.1 1.3 4.0 4.1 3.7 2.3 2.7

a) Bin 1 : 4.2 , 1.8 , 1.3

Bin 2 : 3.1 , 4.0

Bin 3 : 4.1 , 3.7

Bin 4 : 2.3 , 2.7

b) $(4.2+1.8+3.1+1.3+4.0+4.1+3.7+2.3+2.7) \div 7.8 = 27.2 \div 7.8$

$= 3.487$

∴ Yes, 4 bins is optimal.

c) 4.2 1.8 3.1 1.3 (4.0) 4.1 3.7 2.3 2.7

1.8 3.1 1.3 (3.7) 2.3 2.7 [4.0] 4.2 (4.1)

1.8 3.1 (1.3) 2.3 2.7 [3.7] [4.0] [4.1] (4.2)

[1.3] 1.8 3.1 (2.3) 2.7 [3.7] [4.0] [4.1] [4.2]

[1.3] (1.8) [2.3] 3.1 (2.7) [3.7] [4.0] [4.1] [4.2]

[1.3] [1.8] [2.3] [2.7] 3.1 [3.7] [4.0] [4.1] [4.2]

Sort complete

Leave blank

Question 1 continued

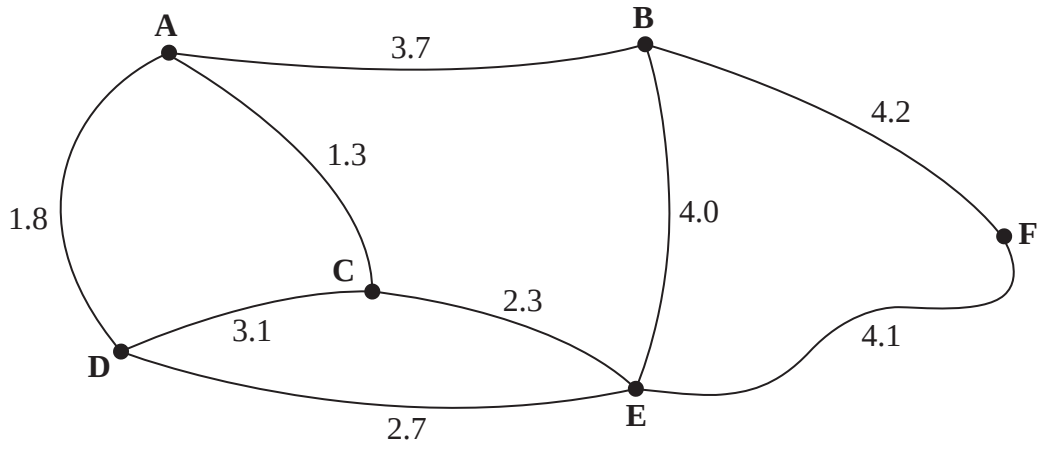

Figure 1

AC 1.3
AD 1.8
CE 2.3
DE 2.7
CD 3.1
AB 3.7
BE 4.0
EF 4.1
BF 4.2

* Some arcs are rejected as they form cycles. (minimum spanning trees requires no cycles) *

AC, AD, CE, reject DE, reject CD, AB, reject BE, EF, reject BF

e) The spanning tree in Prim grows in a connected fashion. AB and DE will not be connected using Prim.

Q1

(Total 11 marks)

2. Six film critics, Bronwen (B), Greg (G), Jean (J), Mick (M), Renee (R) and Susan (S), must see six films, 1, 2, 3, 4, 5 and 6. Each critic must attend a different film and each critic needs to be allocated to exactly one film. The critics are asked which films they would prefer and their preferences are given in the table below.

Critic	Preference
B	2, 3, 6
G	1
J	2, 5, 6
M	1, 5
R	2, 4, 6
S	3, 5

(a) Using Diagram 1 in the answer book, draw a bipartite graph to show the possible allocations of critics to their preferred films.

(1)

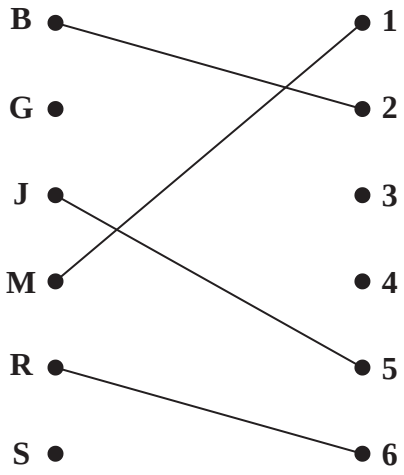

Figure 2

Figure 2 shows an initial matching.

(b) Starting from the given initial matching, apply the maximum matching algorithm to find an alternating path from G to 3. Hence find an improved matching. You should list the alternating path that you use, and state your improved matching.

(3)

(c) Starting with the improved matching found in (b), apply the maximum matching algorithm to obtain a complete matching. You should list the alternating path that you use, and state your complete matching.

(3)

(Total 7 marks)

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2. (a)

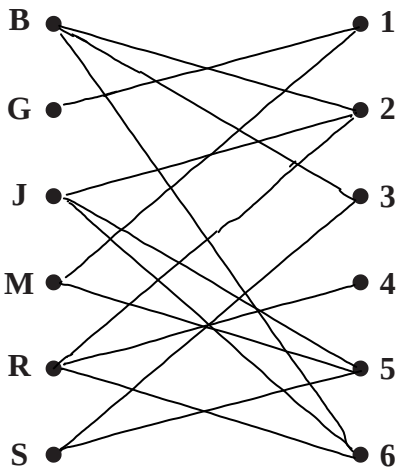


Diagram 1

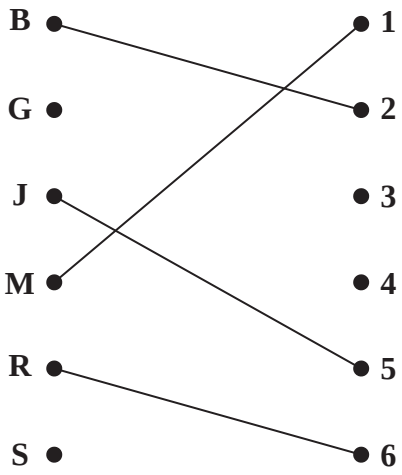


Figure 2

b) Alternating path:

G - 1 = M - 5 = J $\begin{cases} 2 = B - 3 \\ 6 = R - 2 = B - 3 \end{cases}$

Change status:

G = 1 - M = 5 - J = 6 - R = 2 - B = 3

Improved matching:

B = 3

G = 1

J = 6

M = 5

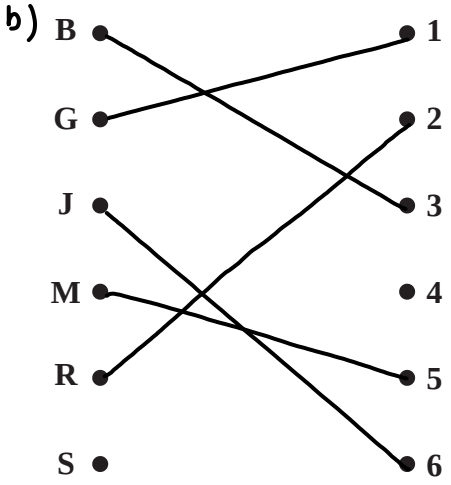
R = 2

S

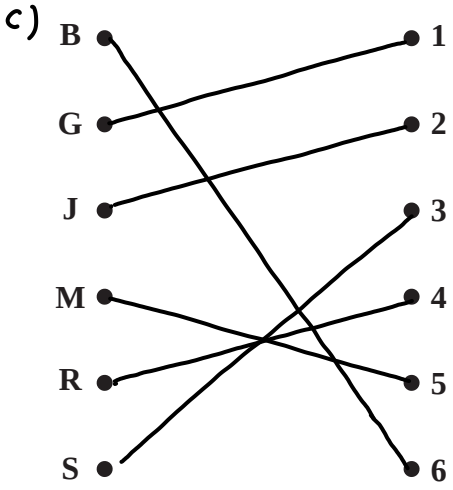
Leave blank

Question 2 continued

b)



c)



c) Alternating path:

S
- 3 = B
  - 2 = R
    - 6 = J
  - 6 = J
    - 2 = R - 4
    - 5 = M - 1
- 5 = M - 1 = G

S - 3 = B - 6 = J - 2 = R - 4

Change status:

S = 3 - B = 6 - J = 2 - R = 4

Complete matching

B = 6

G = 1

J = 2

M = 5

R = 4

S = 3

Q2

(Total 7 marks)

3.

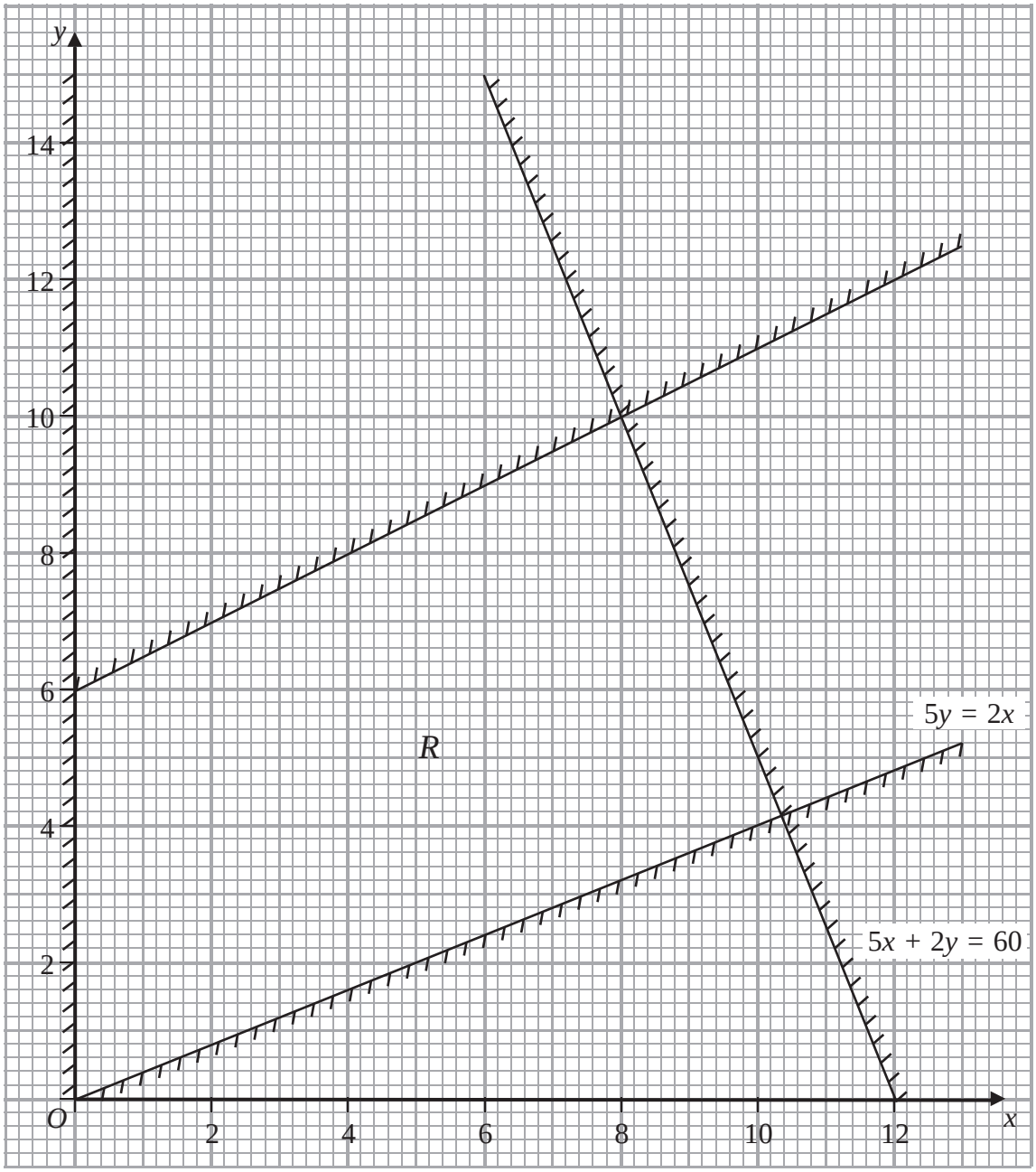


Figure 3

Figure 3 shows the constraints of a linear programming problem in x and y, where R is the feasible region. The equations of two of the lines have been given.

(a) Determine the inequalities that define the feasible region.

(4)

(b) Find the exact coordinates of the vertices of the feasible region.

(3)

The objective is to maximise P, where $P = kx + y$.

(c) For the case $k = 2$, use point testing to find the optimal vertex of the feasible region.

(3)

(d) For the case $k = 2.5$, find the set of points for which P takes its maximum value.

(3)

(Total 13 marks)

Leave blank

3.

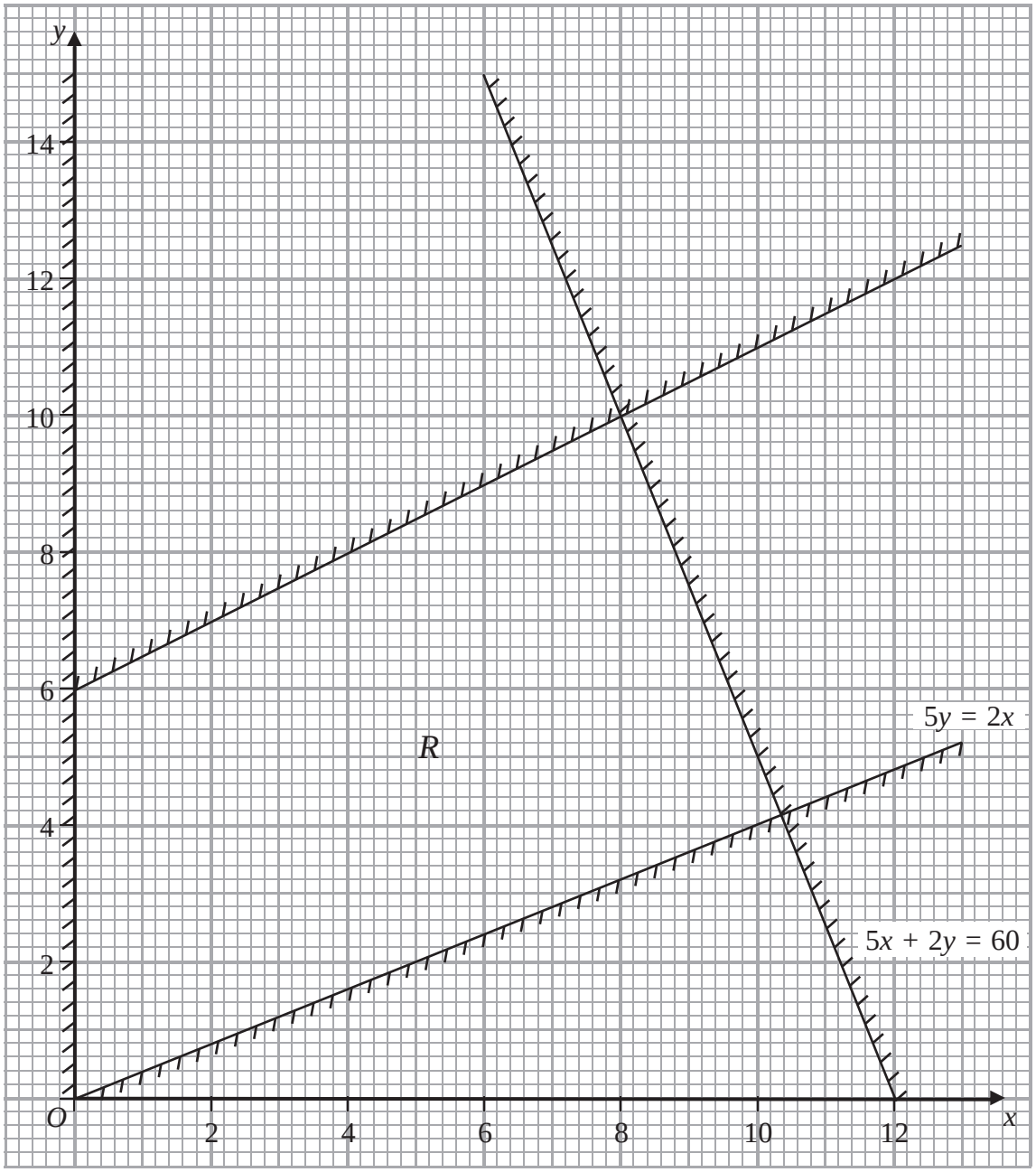


Figure 3

a) $m = \frac{12-6}{12}$, $c = 6$

$= \frac{1}{2}$

$l : y = \frac{1}{2}x + 6$

$2y = x + 12$

$2y \leq x + 12$

$5y \geq 2x$

$5x + 2y \leq 60$

$x \geq 0$

Leave blank

Question 3 continued

b) $5x + 2y = 60$ —①

$5y = 2x$ —②

$x = \frac{5}{2}y$ —③

Sub ③ into ①

$5(\frac{5}{2})y + 2y = 60$

$y = \frac{120}{29}$

$x = \frac{5}{2}(\frac{120}{29})$

$= \frac{300}{29}$

$(\frac{300}{29}, \frac{120}{29})$

$5x + 2y = 60$ —①

$2y = x + 12$ —②

Sub ② into ①

$5x + x + 12 = 60$

$x = 8$

$2y = 8 + 12$

$y = 10$

$(8, 10)$

Ans: $(0,0)$, $(0,6)$, $(8,10)$, $(\frac{300}{29}, \frac{120}{29})$

c) $P = 2x + y$

At $(0,0)$, $P = 0$

$(0,6)$, $P = 6$

$(8,10)$, $P = 2(8) + 10$

$= 26$

$(\frac{300}{29}, \frac{120}{29})$, $P = 2(\frac{300}{29}) + \frac{120}{29}$

$= \frac{720}{29}$

∴ Optimal vertex: $(8,10)$

d) $P = 2.5x + y$

At $(0,0)$, $P = 0$

$(0,6)$, $P = 6$

$(8,10)$, $P = 2.5(8) + 10$

$= 30$

$(\frac{300}{29}, \frac{120}{29})$, $P = 2.5(\frac{300}{29}) + (\frac{120}{29})$

$= 30$

∴ value of P is maximum when $5x + 2y = 60$ for $8 \leq x \leq \frac{300}{29}$

4. (a) Draw the activity network described in the precedence table below, using activity on arc and the minimum number of dummies.

Activity	Immediately preceding activities
A	-
B	-
C	-
D	A
E	A
F	A, B, C
G	C
H	E, F, G
I	E, F, G
J	H, I
K	H, I
L	D, J

(5)

A project is modelled by the activity network drawn in (a). Each activity requires one worker. The project is to be completed in the shortest possible time. The table below gives the time, in days, to complete some of the activities.

Activity	Duration (in days)
B	7
F	4
J	4
L	6

The critical activities for the project are B, F, I, J and L and the length of the critical path is 30 days.

(b) Calculate the duration of activity I.

(1)

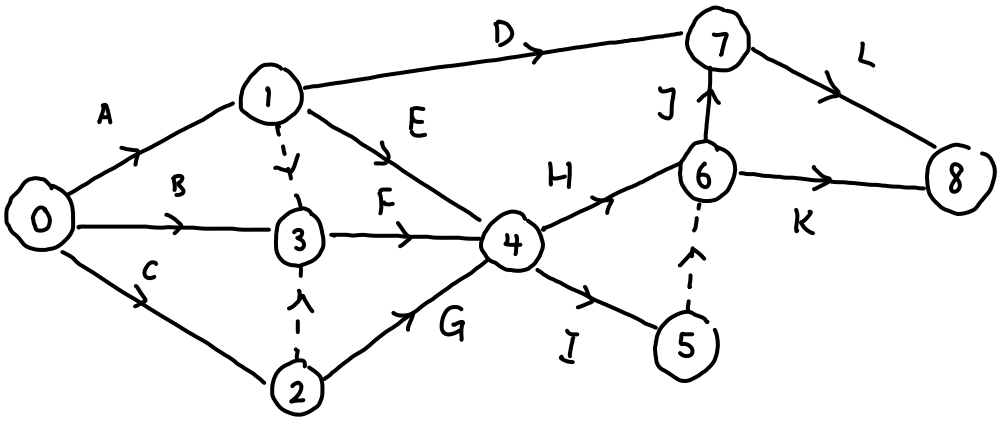
(c) Find the range of possible values for the duration of activity K.

(2)

(Total 8 marks)

Leave blank

4. (a)



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Question 4 continued

b) $30 - 7 - 4 - 4 - 6 = 9$

$I = 9$

c) $K < J + L$

$< 4 + 6$

< 10

$0 < K < 10$

Q4

(Total 8 marks)

5.

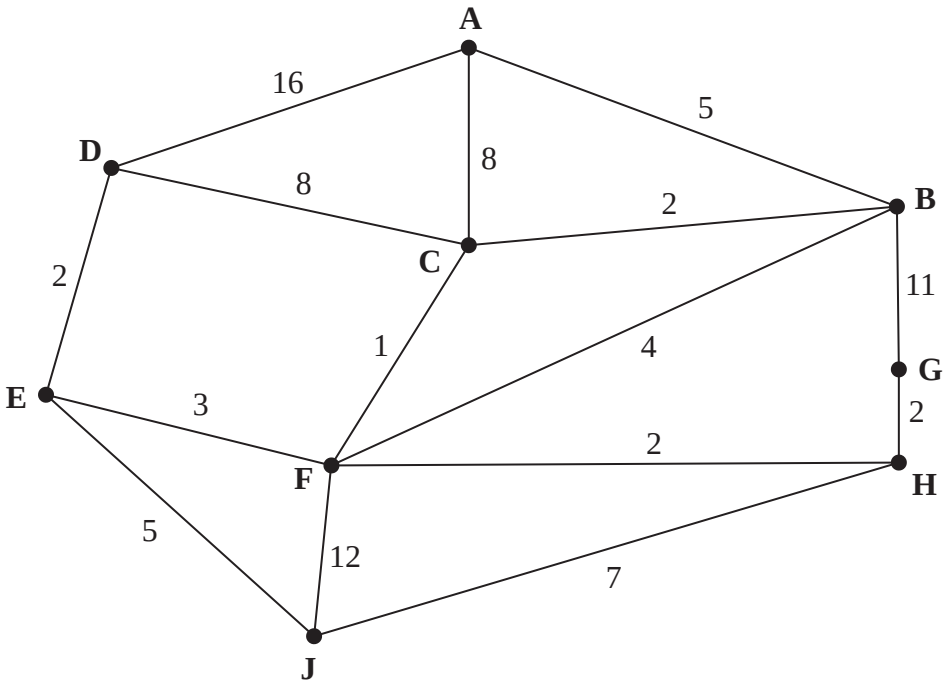


Figure 4

[*The total weight of the network is 88*]

(a) Explain what is meant by the term ‘path’.

(2)

Figure 4 represents a network of roads. The number on each arc represents the length, in km, of the corresponding road. Tomek wishes to travel from A to J.

(b) Use Dijkstra’s algorithm to find the shortest path from A to J. State your path and its length.

(6)

On a particular day, Tomek needs to travel from G to J via A.

(c) Find the shortest route from G to J via A, and find its length.

(3)

The road HJ becomes damaged and cannot be used. Tomek needs to travel along all the remaining roads to check that there is no damage to any of them. The inspection route he uses must start and finish at B.

(d) Use an appropriate algorithm to find the length of a shortest inspection route. State the arcs that should be repeated. You should make your method and working clear.

(5)

(e) Write down a possible shortest inspection route.

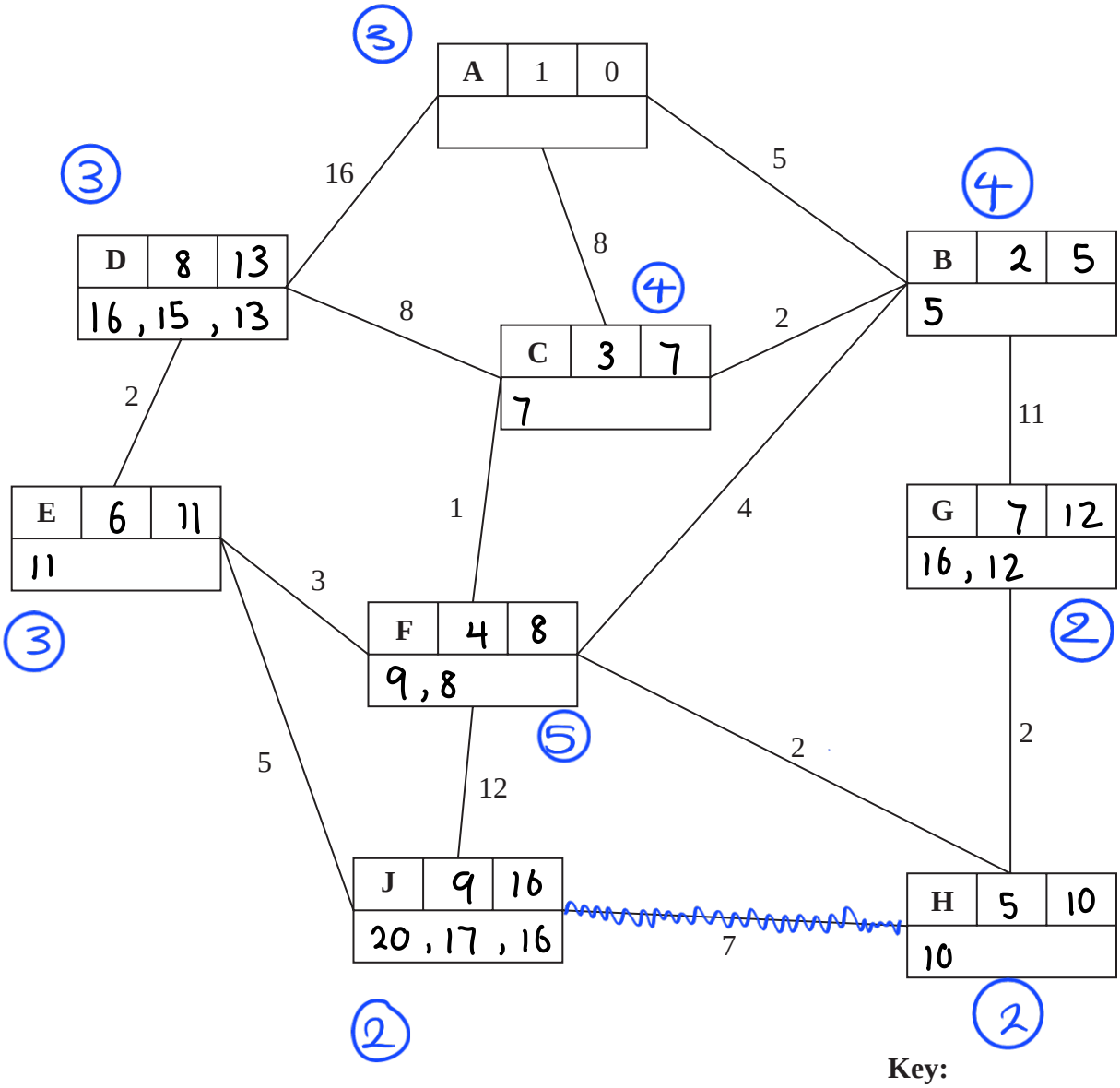
(1)

(Total 17 marks)

Leave blank

5. (a)

A finite sequence of edges such that the end vertex of one edge is the start vertex of the next, and in which no vertex appears more than once.



Key:

Vertex	Order of labelling	Final value
Working values		

Shortest path: A - B - C - F - F - J

Length of shortest path: 16

Leave blank

Question 5 continued

c) From A to G : A - B - C - F - H - G

A to J : A - B - C - F - E - J

G → A → J : G - H - F - C - B - A - B - C - F - E - J

length : 16 + 12 = 28 km

d) Odd valencies : A , D , E , F

AD + EF = 13 + 3

= 16

(CF) (E)
AE + DF = 11 + 5

= 16

(BC)
AF + DE = 8 + 2

= 10 * Shortest path *

Arcs AB , BC , CF and DE are repeated

Length : 88 - 7 + 10 = 91 km

e) BADEJFEDCFHGBACFBCB

Q5

(Total 17 marks)

6.

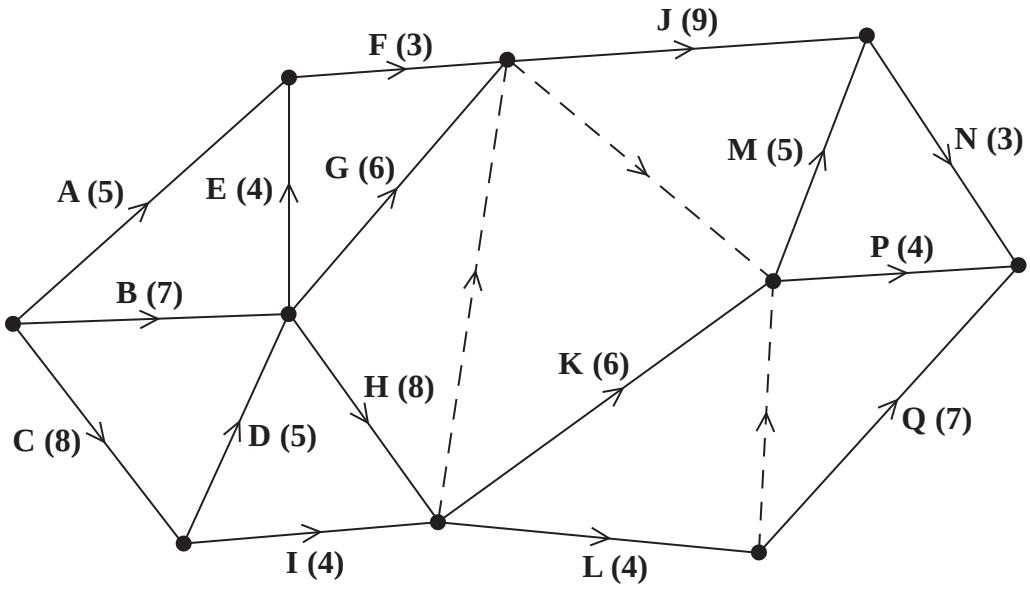


Figure 5

A project is modelled by the activity network shown in Figure 5. The activities are represented by the arcs. The number in brackets on each arc gives the time, in days, to complete the activity. Each activity requires exactly one worker. The project is to be completed in the shortest possible time.

(a) Complete Diagram 1 in the answer book to show the early event times and late event times.

(4)

(b) State the critical activities.

(1)

(c) Calculate the maximum number of days by which activity E could be delayed without lengthening the completion time of the project. You must make the numbers used in your calculation clear.

(2)

(d) Calculate a lower bound for the number of workers needed to complete the project in the minimum time. You must show your working.

(2)

(e) Draw a cascade (Gantt) chart for this project on the grid provided in the answer book.

(4)

(Total 13 marks)

Leave blank

6. (a)

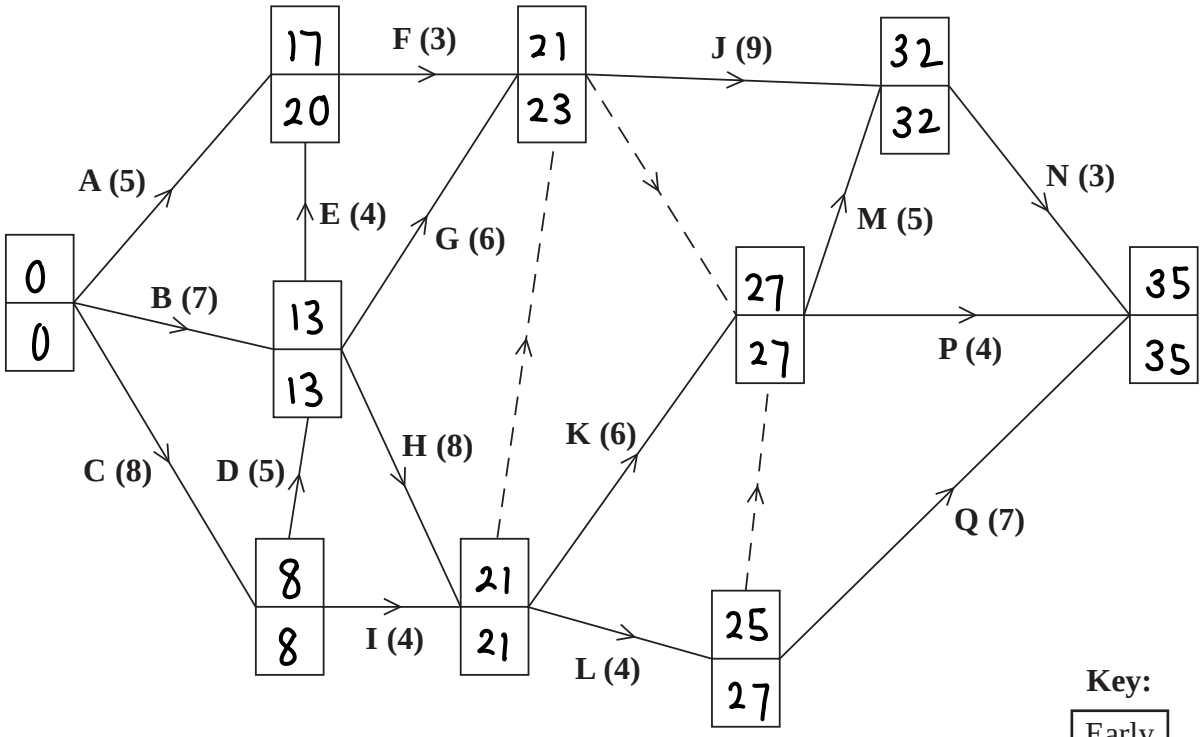


Diagram 1

b) C, D, H, K, M, N

c) 20 - 4 - 13 = 3 days

d) 5+7+8+5+4+3+6+8+4+9+6+4+5+3+4+7 = 88

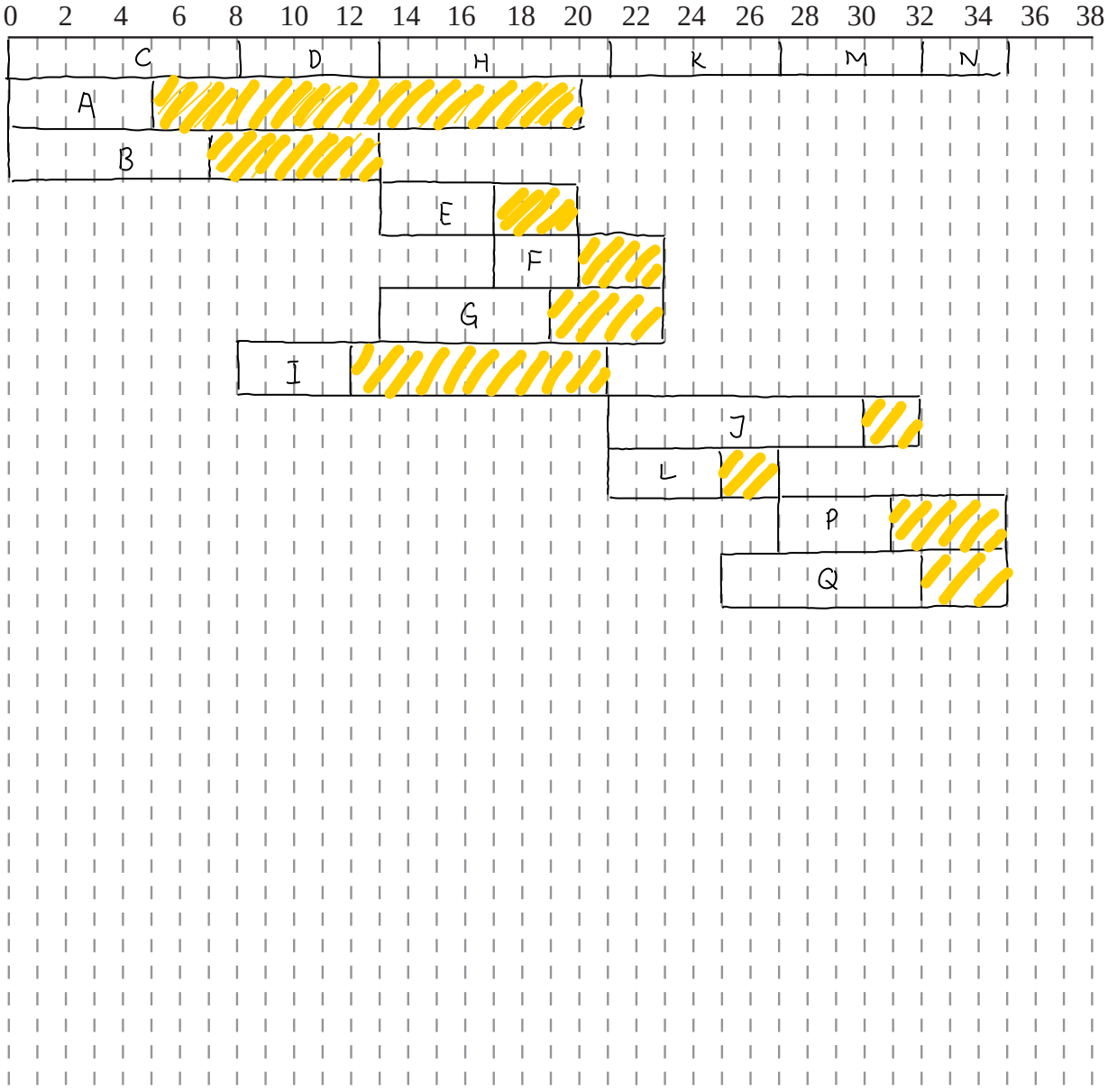
$\frac{88}{35} = 2.514$

≈ 3

Leave blank

Question 6 continued

(e)



Q6

(Total 13 marks)

7. A theatre company is planning to sell two types of ticket, standard and premier. The theatre company has completed some market research and has used this to form the following constraints.

- They will sell at most 450 tickets.
- They will sell at least three times as many standard tickets as premier tickets.
- At most 85% of all the tickets sold will be standard.

The theatre wants to maximise its profit.

The profit on each standard ticket sold is £5 and the profit on each premier ticket sold is £8

Let x represent the number of standard tickets sold and y represent the number of premier tickets sold.

Formulate this as a linear programming problem, stating the objective and listing the constraints as simplified inequalities with integer coefficients.

You should **not** attempt to solve the problem.

(Total 6 marks)

TOTAL FOR PAPER: 75 MARKS

END

Leave blank

7. Maximise $P = 5x + 8y$

$x + y \leqslant 4.50$

$x \geqslant 3y$

$0.85(x+y) \geqslant x \Rightarrow 3x \leqslant 17y$

$x \geqslant 0, y \geqslant 0$

Q7

(Total 6 marks)

TOTAL FOR PAPER: 75 MARKS

END