MODEL ANSWERS

Pearson Edexcel International Advanced Level

Decision Mathematics D1

Advanced/Advanced Subsidiary

Thursday 19 January 2017 – Afternoon
Time: 1 hour 30 minutes

Paper Reference
WDM01/01

You must have:
D1 Answer Book

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** on the top of the answer book with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the D1 answer book provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.
- Do not return the question paper with the answer book.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

P48346A

1/1/1/1/

P 4 8 3 4 6 A

Pearson

Write your answers in the D1 answer book for this paper.

1. Use the binary search algorithm to try to locate the name Hilbert in the following alphabetical list. Clearly indicate how you chose your pivots and which part of the list is being rejected at each stage.

Descartes
Euler
Gauss
Hamilton
Lagrange
Noether
Poisson
Ramanujan
Stokes
Turing

(Total 4 marks)

Leave blank

1.

$\frac{1+10}{2} = 5.5$

$\therefore 6$ 6th name : Noether

$\therefore$ reject 6-10

$\frac{1+5}{2} = 3$ 3rd name : Gauss

$\therefore$ reject 1-3

$\frac{4+5}{2} = 4.5$

$\therefore 5$ 5th name : Langrange

$\therefore$ reject 5

4th name : Hamilton

$\therefore$ reject 4

Hilbert is not on the list

2.

	A	B	C	D	E	F	G	H
A	–	27	51	32	29	23	47	40
B	27	–	24	35	20	42	33	28
C	51	24	–	37	43	31	26	34
D	32	35	37	–	39	45	44	30
E	29	20	43	39	–	38	45	55
F	23	42	31	45	38	–	53	45
G	47	33	26	44	45	53	–	39
H	40	28	34	30	55	45	39	–

The table represents a network that shows the average journey time, in minutes, between eight towns, A, B, C, D, E, F, G and H.

(a) Use Prim's algorithm, starting at A, to find the minimum spanning tree for this network. You must clearly state the order in which you select the edges of your tree.

(3)

(b) Draw the minimum spanning tree using the vertices given in Diagram 1 in the answer book.

(1)

(c) State the weight of the minimum spanning tree.

(1)

(Total 5 marks)

Leave blank

2.

1 3 5 8 4 2 6 7

	A	B	C	D	E	F	G	H
A	–	~~27~~	~~51~~	~~32~~	~~29~~	~~23~~	~~47~~	~~40~~
B	27	–	~~24~~	~~35~~	~~20~~	~~42~~	~~33~~	~~28~~
C	~~51~~	24	–	~~37~~	~~43~~	~~31~~	~~26~~	~~34~~
D	32	35	37	–	39	45	44	30
E	~~29~~	20	~~43~~	~~39~~	–	~~38~~	~~45~~	~~55~~
F	23	~~42~~	~~31~~	~~45~~	~~38~~	–	~~53~~	~~45~~
G	~~47~~	~~33~~	26	~~44~~	~~45~~	~~53~~	–	~~39~~
H	~~40~~	28	~~34~~	~~30~~	~~55~~	~~45~~	~~39~~	–

AF , AB , BE , BC , CG , BH , DH

Leave blank

Question 2 continued

(b)

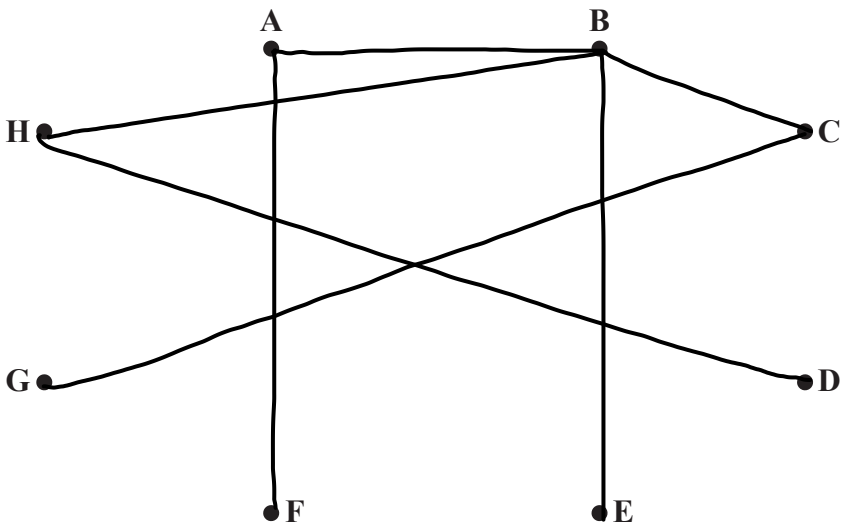

Diagram 1

c) 27 + 24 + 30 + 20 + 23 + 26 + 28 = 178 mins

Q2

(Total 5 marks)

3.

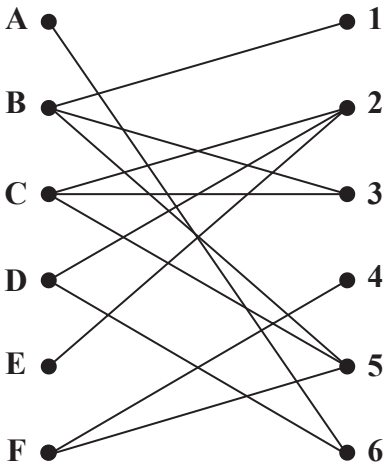

Figure 1

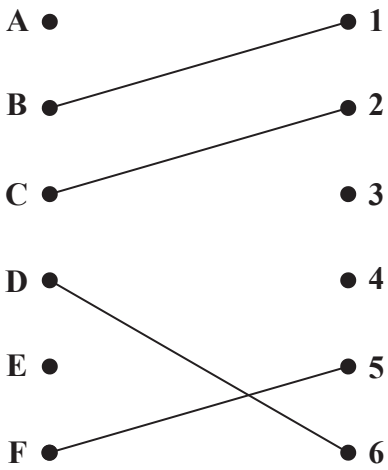

Figure 2

Figure 1 shows the possible allocations of six workers, A, B, C, D, E and F, to six tasks, 1, 2, 3, 4, 5 and 6. Each task must be assigned to only one worker and each worker must be assigned to exactly one task.

Figure 2 shows an initial matching.

(a) Starting from the given initial matching, use the maximum matching algorithm to find an alternating path from A to 4. Hence find an improved matching. You should list the alternating path you use, and state your improved matching.

(3)

(b) Explain why it is not possible to find a complete matching.

(1)

After training, task 1 is added to worker A's possible allocations.

(c) Starting from the improved matching found in (a), use the maximum matching algorithm to find a complete matching. You should list the alternating path you use, and state your complete matching.

(3)

(Total 7 marks)

Leave blank

3.

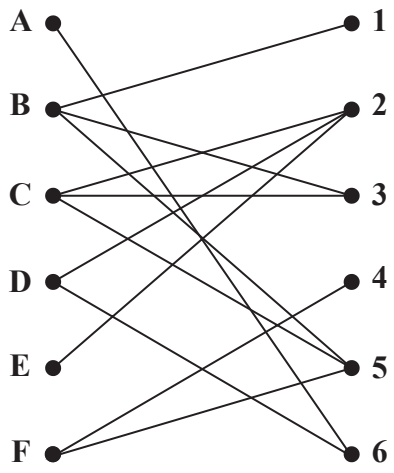

Figure 1

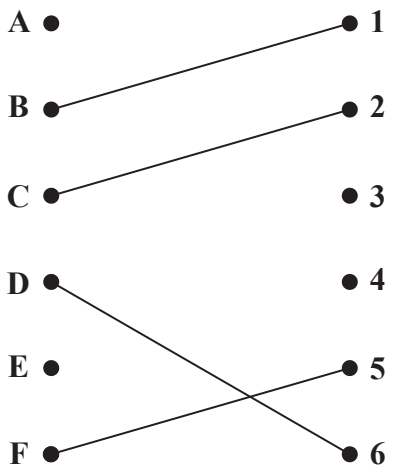

Figure 2

a) Alternative path :

A - 6 = D - 2 = C - 5 = F - 4

Change status:

A = 6 - D = 2 - C = 5 - F = 4

Improved matching :

A = 6

B = 1

C = 5

D = 2

E =

F = 4

b) A can only do task 6 so D must do task 2. E can only do task 2 so E will have no task

c) Alternative path :

E - 2 = D - 6 = A - 1 = B - 5 = C - 3

Change status:

E = 2 - D = 6 - A = 1 - B = 5 - C = 3

Complete matching :

A = 1 F = 4

B = 5

C = 3

D = 6

E = 2

4. 23 18 27 9 25 10 12 30 24

The numbers in the list represent the weights, in kilograms, of nine suitcases. The suitcases are to be transported in containers that will each hold a maximum weight of 45 kilograms.

(a) Calculate a lower bound for the number of containers that will be needed to transport the suitcases.

(2)

(b) Use the first-fit bin packing algorithm to allocate the suitcases to the containers.

(2)

(c) Using the list provided, carry out a bubble sort to produce a list of the weights in descending order. You need only give the state of the list after each complete pass.

(4)

(d) Use the first-fit decreasing bin packing algorithm to allocate the suitcases to the containers.

(2)

(e) Explain why it is not possible to transport the suitcases using fewer containers than the number used in (d).

(1)

(Total 11 marks)

Leave blank

4. 23 18 27 9 25 10 12 30 24

a) $(23+18+27+9+25+10+12+30+24) \div 45 = 178 \div 45$

$= 3.955$

$\simeq 4$ bins

b) Bin 1 : 23 18

Bin 2 : 27 9

Bin 3 : 25 10

Bin 4 : 12 30

Bin 5 : 24

c) 23 18 27 9 25 10 12 30 24

23 27 18 25 10 12 30 24 9

27 23 25 18 12 30 24 10 9

27 25 23 18 30 24 12 10 9

27 25 23 30 24 18 12 10 9

27 25 30 24 23 18 12 10 9

27 30 25 24 23 18 12 10 9

30 27 25 24 23 18 12 10 9

Sort complete

d) Bin 1 : 30 12

Bin 2 : 27 18

Bin 3 : 25 10 9

Bin 4 : 24

Bin 5 : 23

e) The weight is each suitcase is more than half of the maximum weight so no 2 suitcases can be combined.

5.

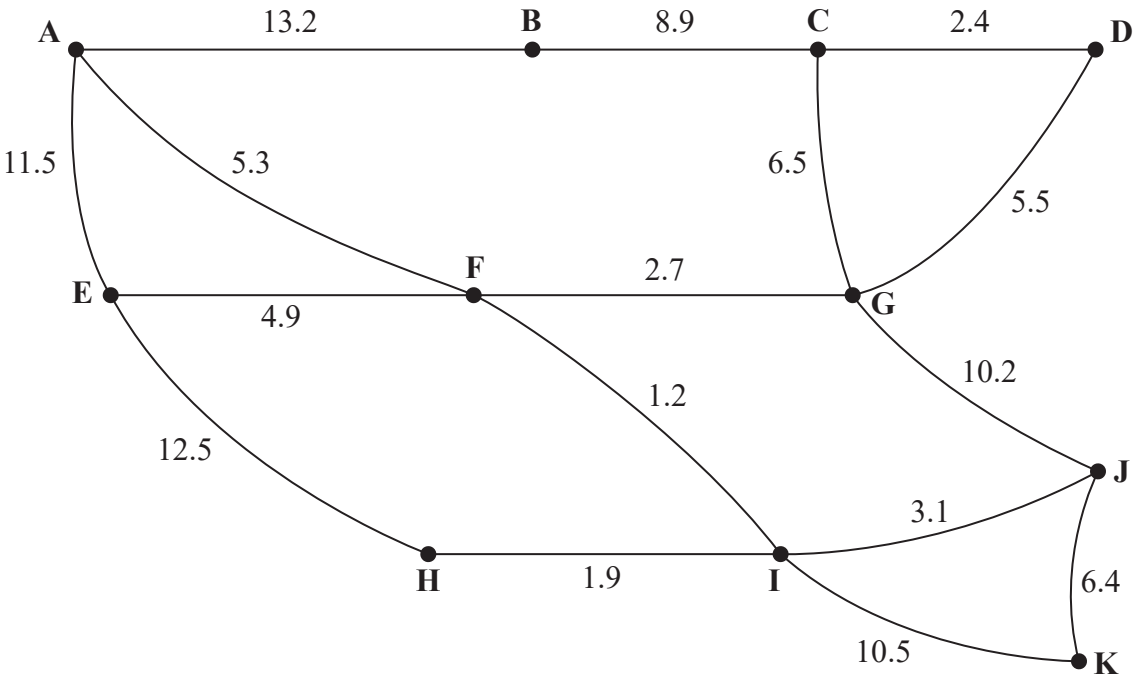

Figure 3

[*The total weight of the network is 106.7*]

Figure 3 models a network of cycle tracks that have to be inspected. The number on each arc represents the length, in km, of the corresponding track. Angela needs to travel along each cycle track at least once and wishes to minimise the length of her inspection route. She must start and finish at A.

(a) Use an appropriate algorithm to find the tracks that will need to be traversed twice. You should make your method and working clear.

(5)

(b) Find a route of minimum length, starting and finishing at A. State the length of your route.

(2)

A new cycle track, AC, is under construction. It will be 15 km long. Angela will have to include this new track in her inspection route.

(c) State the effect this new track will have on the total length of her route. Justify your answer.

(2)

(Total 9 marks)

Leave blank

5.

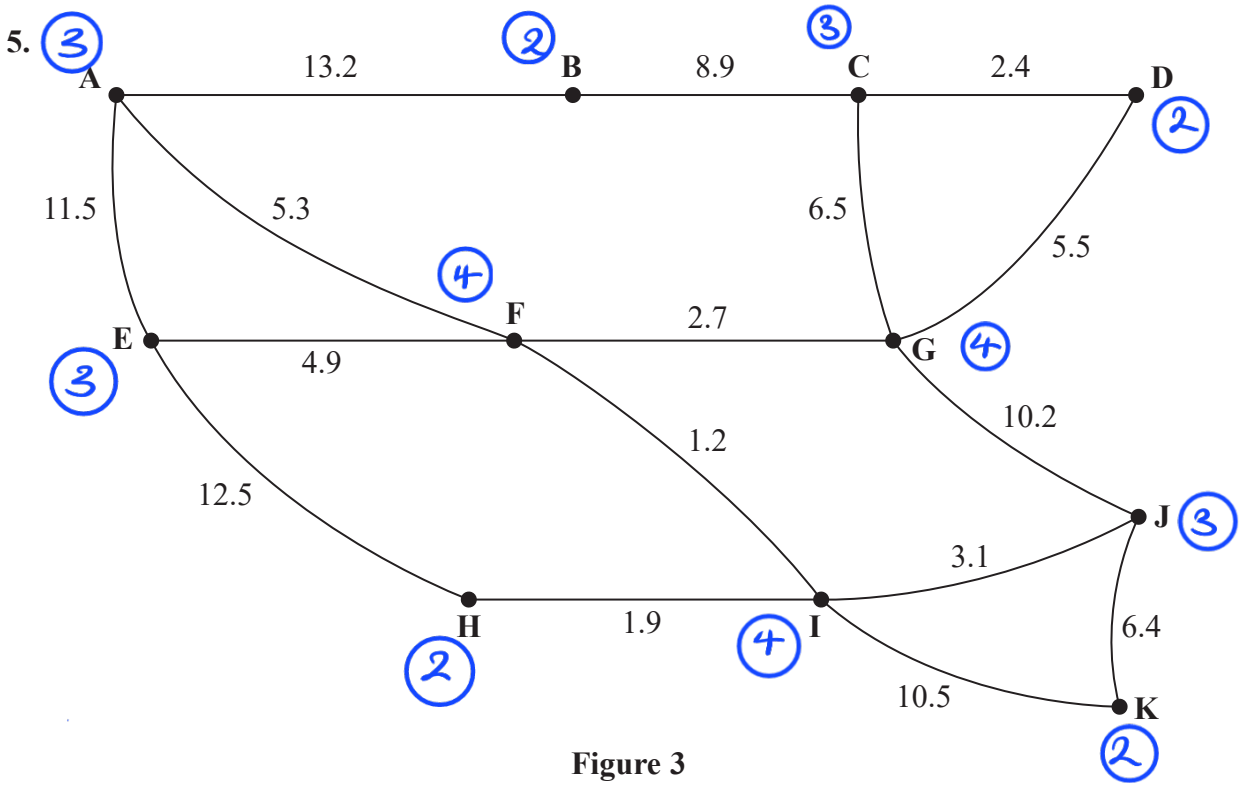

Figure 3

[*The total weight of the network is 106.7*]

a) Odd nodes : A , C , E , J

$\overset{(FG)}{AC} + \overset{(FI)}{EJ}$: $(5.3+2.7+6.5)+(4.9+1.2+3.1) = 23.7$

$\overset{(F)}{AE} + \overset{(GFI)}{CJ}$: $(5.3+4.9)+(6.5+2.7+1.2+3.1) = 23.7$

$\overset{(FI)}{AJ} + \overset{(GF)}{CE}$: $(5.3+1.2+3.1)+(6.5+2.7+4.9) = 23.7$

∴ Repeat AF , FG , GC , EF , FI , IJ

b) ABCDGCGJKIJIHEFIFGFEAFA

length : 106.7 + 23.7 = 130.4 km

c) A and C will become even nodes so only EF , FI and IJ are traversed twice (E and J are odd nodes).

15 + 4.9 + 1.2 + 3.1 = 24.2

24.2 − 23.7 = 0.5

Total distance is increased by 0.5 km

(Length of new route is 130.9 km)

6.

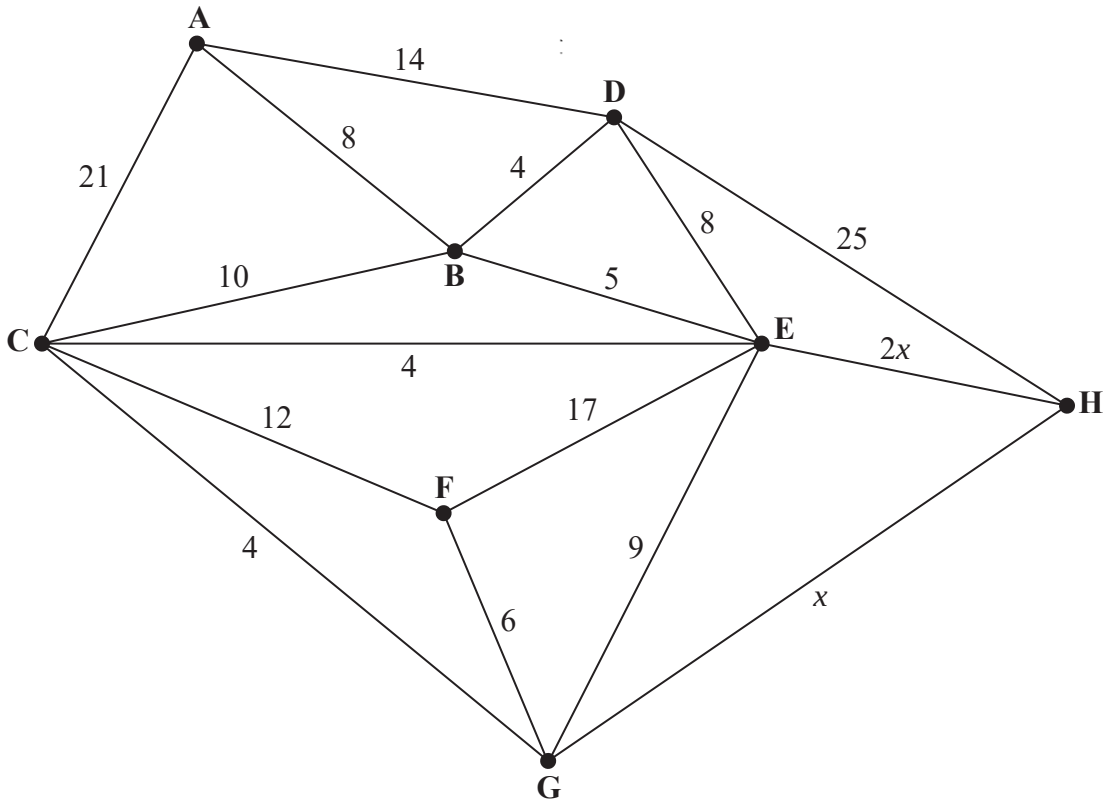

Figure 4

Figure 4 represents a network of roads. The number on each arc represents the time taken, in minutes, to drive along the corresponding road.

Stieg wishes to minimise the time spent driving from his home at A, to his office at H. The amount of traffic on two of the roads leading into H varies each day, and so the length of time taken to drive along these roads is expressed in terms of x, where $x > 7$

(a) Use Dijkstra's algorithm to find the possible routes that minimise the driving time from A to H. State the length of each route, leaving your answer in terms of x where necessary.

(7)

On a particular day, the quickest route from A to H via G is 2 minutes quicker than the quickest route from A to H via E.

(b) Calculate the value of x. You must make your method and working clear.

(2)

(Total 9 marks)

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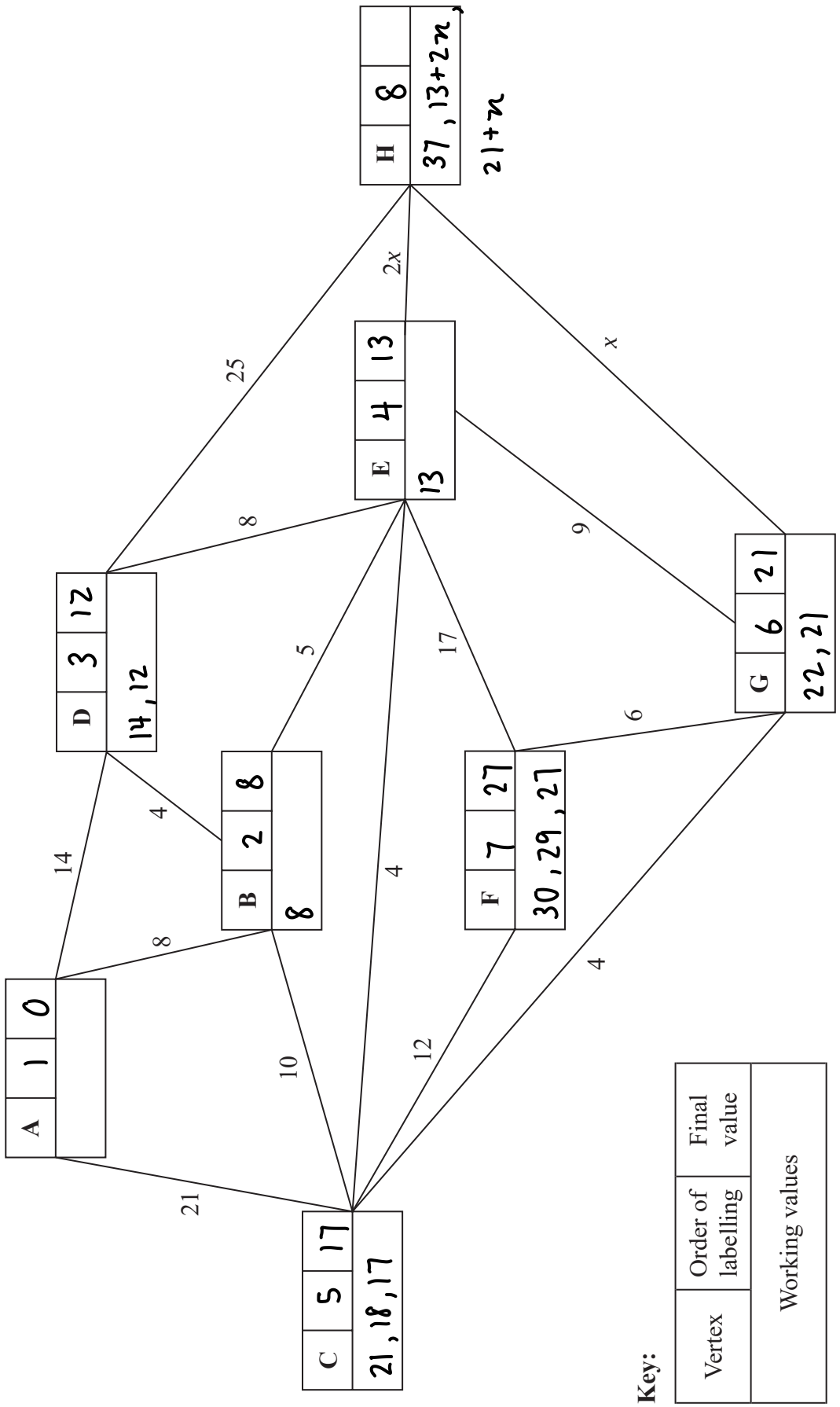
6.



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Question 6 continued

a) shortest path for A → H via D : ABDH length : 37
shortest path for A → H via E : ABEH length : 13+2n
shortest path for A → H via G : ABECGH length : 21+n

b) $21 + n + 2 = 13 + 2n$

$10 = n$

Q6

(Total 9 marks)

7.

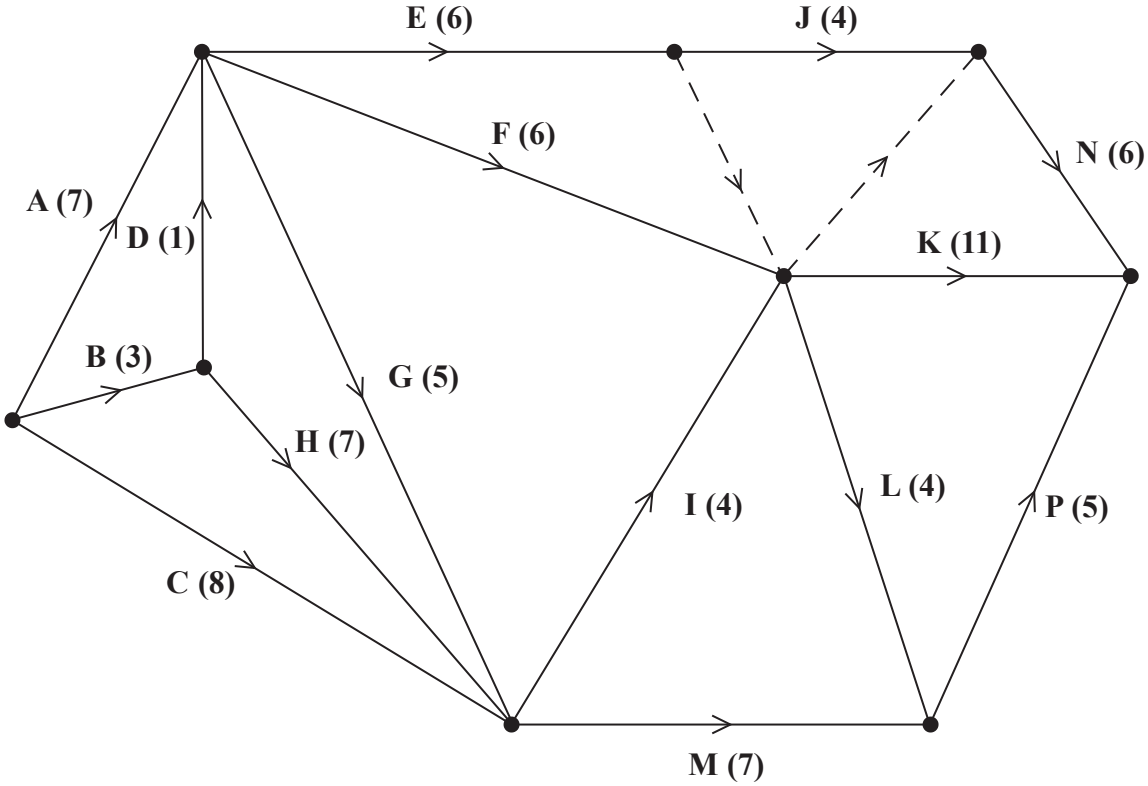

Figure 5

A project is modelled by the activity network shown in Figure 5. The activities are represented by the arcs. The number in brackets on each arc gives the time, in days, to complete the corresponding activity. Each activity requires exactly one worker. The project is to be completed in the shortest possible time.

(a) Complete Diagram 1 in the answer book to show the early event times and late event times.

(4)

(b) Explain what is meant by a critical path.

(2)

(c) List the critical path for this network.

(1)

(d) For each of the situations below, state the effect that the delay would have on the project completion date.

(i) A 4-day delay during activity J.
(ii) A 4-day delay during activity M.

(2)

The delays mentioned in (d) do not occur.

(e) Calculate a lower bound for the number of workers needed to complete the project in the minimum time. You must show your working.

(2)

(f) Schedule the activities using the minimum number of workers so that the project is completed in the minimum time.

(3)

(Total 14 marks)

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7. (a)

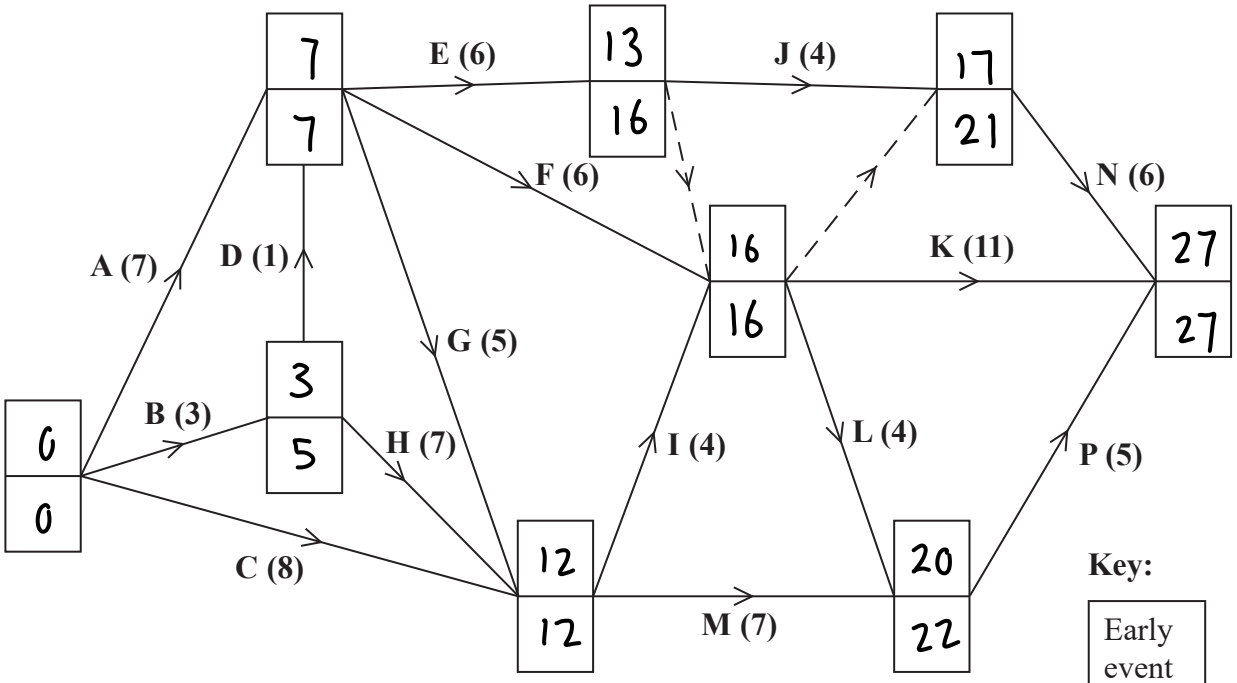


Diagram 1

b) A path from the source node to the sink node which entirely follows critical activities. Critical activities are activities with zero float.

c) A G I K

di) Float on J : 21-4-13 = 4 ∴ does not affect project completion date

ii) Float on M : 22-7-12 = 3 < 4 ∴ 1 day delay

e) (7+3+8+1+6+6+5+7+4+4+11+4+7+6+5) ÷ 27 = 84 ÷ 27

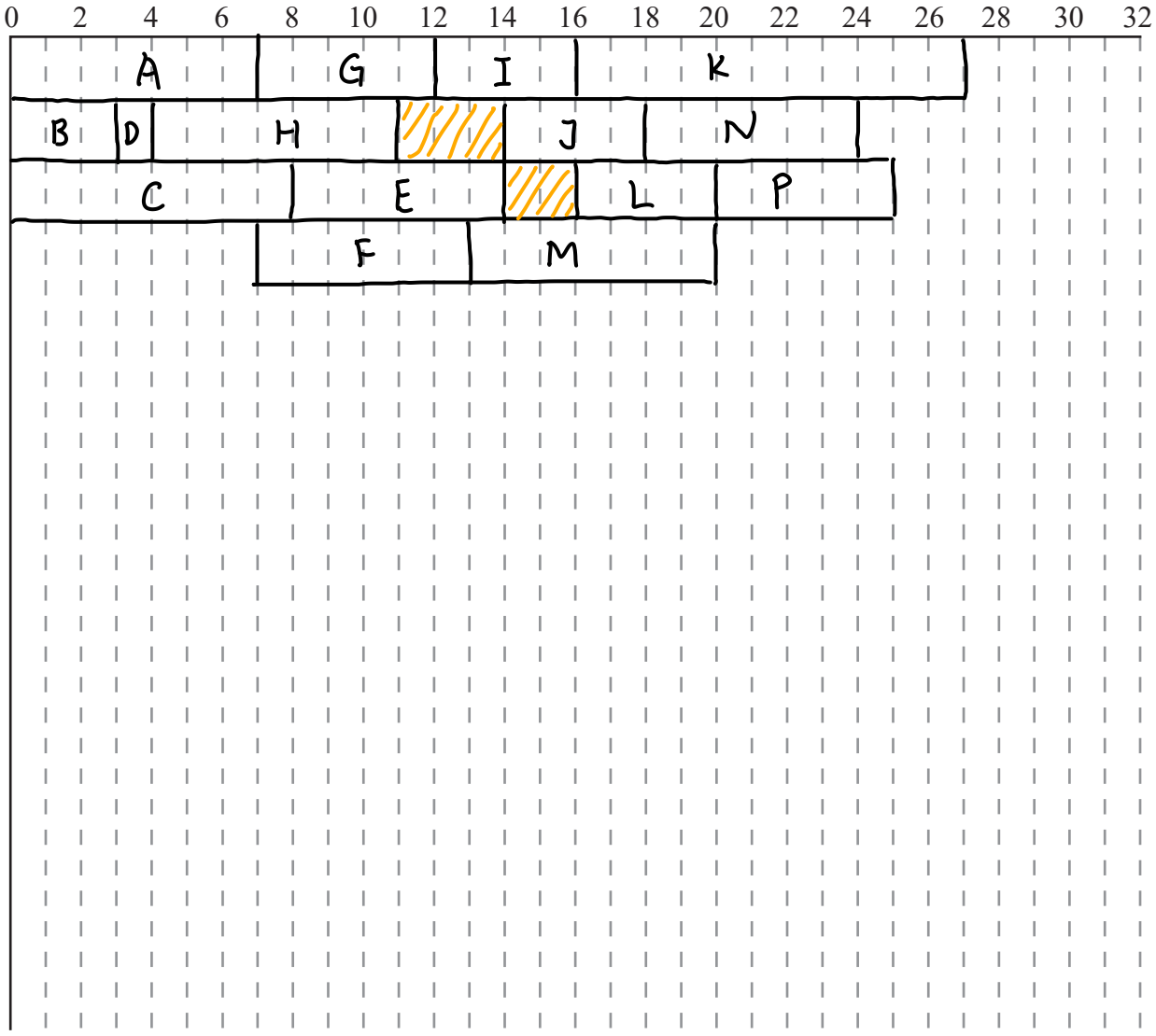
= 3.11

lower bound = 4 workers

Leave blank

Question 7 continued

(f)



Q7

(Total 14 marks)

8. A shop sells three types of pen. These are ballpoint pens, rollerball pens and fountain pens.

The shop manager knows that each week she should order

- at least 50 pens in total
- at least twice as many rollerball pens as fountain pens

In addition,

- at most 60% of the pens she orders must be ballpoint pens
- at least a third of the pens she orders must be rollerball pens

Each ballpoint pen costs £2, each rollerball pen costs £3 and each fountain pen costs £5

The shop manager wants to minimise her costs.

Let x represent the number of ballpoint pens ordered, let y represent the number of rollerball pens ordered and let z represent the number of fountain pens ordered.

(a) Formulate this information as a linear programming problem. State the objective and list the constraints as simplified inequalities with integer coefficients.

(7)

The shop manager decides to order exactly 10 fountain pens. This reduces the problem to the following

$$\begin{aligned} \text{Minimise} \quad & P = 2x + 3y \\ \text{subject to} \quad & x + y \geqslant 40 \\ & 2x - 3y \leqslant 30 \\ & -x + 2y \geqslant 10 \\ & y \geqslant 20 \\ & x \geqslant 0 \end{aligned}$$

(b) Represent these constraints on Diagram 1 in the answer book. Hence determine, and label, the feasible region R.

(4)

(c) Use the objective line method to find the optimal vertex, V, of the feasible region. You must make your objective line clear and label the optimal vertex V.

(3)

(d) Write down the number of each type of pen that the shop manager should order. Calculate the cost of this order.

(2)

(Total 16 marks)

TOTAL FOR PAPER: 75 MARKS

END

Leave blank

8.

Minimize $C = 2x + 3y + 5z$

Subject to $x + y + z \geqslant 50$

$y \geqslant 2z$

$x \leqslant \frac{3}{5}(x+y+z) \Rightarrow 2x \leqslant 3y + 3z$

$y \geqslant \frac{1}{3}(x+y+z) \Rightarrow 2y \geqslant x+z$

$x \geqslant 0, y \geqslant 0, z \geqslant 0$

Leave blank

Question 8 continued

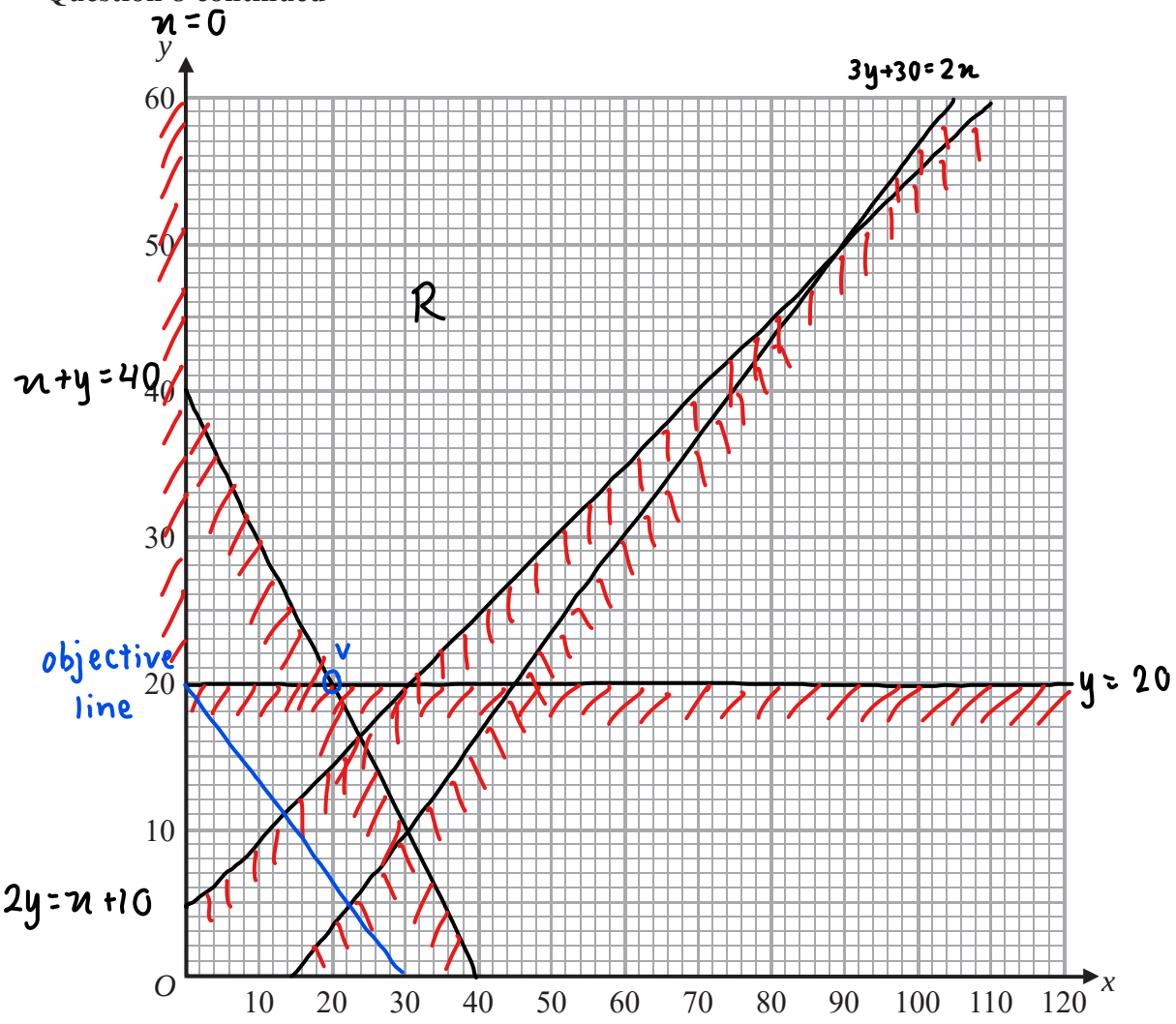


Diagram 1

d) $y = 20$

$n + y = 40$

$n + (20) = 40$

$n = 20$

The shop manager should order 20 ballpoint pens, 20 roller pens and 10 fountain pens

$C = 2(20) + 3(20) + 5(10)$

$= £150$