MODEL ANSWERS

Write your name here

Surname	Other names

Pearson Edexcel
International
Advanced Level

Centre Number | | | | | |

Candidate Number | | | | |

Decision Mathematics D1

Advanced/Advanced Subsidiary

Monday 1 February 2016 – Afternoon
Time: 1 hour 30 minutes

Paper Reference
WDM01/01

Answer Book
Do not return the question paper with the answer book.

Total Marks

P48000A

1/1/1/1/1/1/

P 4 8 0 0 0 A 0 1 1 6

PEARSON

Write your answers in the D1 answer book for this paper.

1.

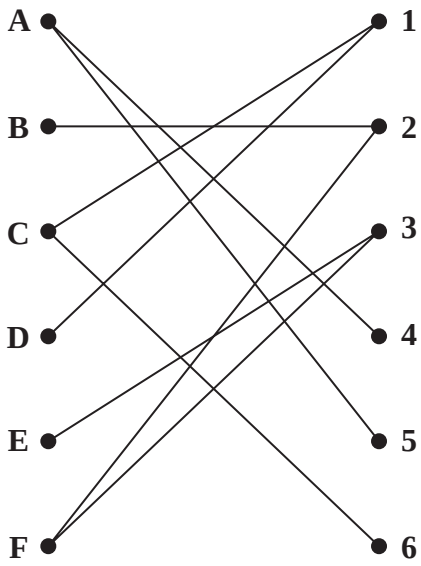

Figure 1

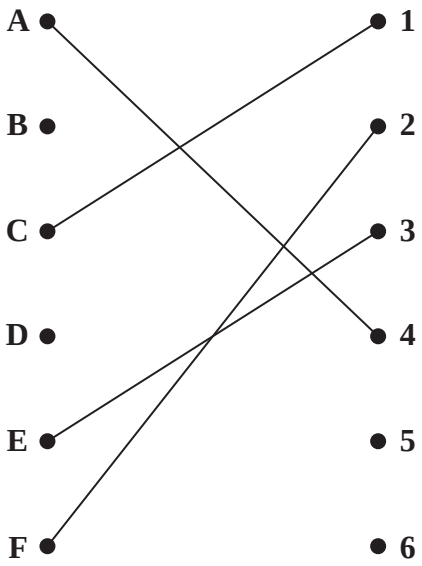

Figure 2

Figure 1 shows the possible allocations of six workers, A, B, C, D, E and F, to six tasks, 1, 2, 3, 4, 5 and 6.

Figure 2 shows an initial matching.

(a) Starting from this initial matching, use the maximum matching algorithm to find an improved matching. You must list the alternating path used and state your improved matching.

(2)

(b) Explain why it is not possible to find a complete matching.

(1)

Now, **exactly** one worker may be trained so that a complete matching becomes possible.
Either worker A can be trained to do task 1 **or** worker E can be trained to do task 4.

(c) Decide which worker, A or E, should be trained. Give a reason for your answer.

(2)

You may now assume that the worker you identified in (c) has been trained.

(d) Starting from the improved matching found in (a), use the maximum matching algorithm to find a complete matching. You must list the alternating path used and state your complete matching.

(3)

(Total 8 marks)

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1.

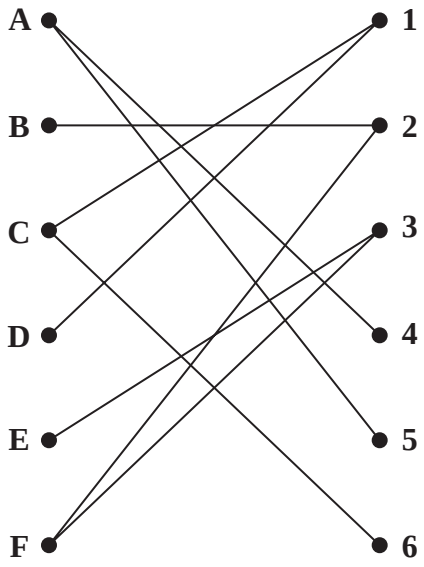

Figure 1

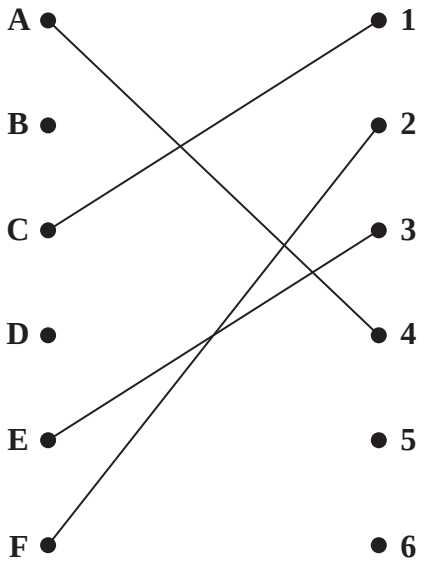

Figure 2

a) Alternating path: D-1=C-6

Change status: D=1-C=6

Improved matching:

A=4

B=

C=6

D=1

E=3

F=2

b) B can only do task 2 and E can only do task 3. F can only do tasks 2 and 3 so there are 3 workers who can only do tasks 2 and 3.

c) E. Tasks 4 and 5 can only be done by A. If E is trained, task 4 can be allocated to E and task 5 can be allocated to A.

d) Alternating path: B-2=F-3=E-4=A-5

Change status: B=2-F=3-E=4-A=5

Complete matching:

A = 5 D=1

B = 2 E=4

C = 6 F=3

2. Kruskal's algorithm finds a minimum spanning tree for a connected graph with n vertices.

(a) Explain the terms

(i) connected graph,

(ii) tree,

(iii) spanning tree.

(3)

(b) Write down, in terms of n, the number of arcs in the minimum spanning tree.

(1)

The table below shows the lengths, in km, of a network of roads between seven villages, A, B, C, D, E, F and G.

	A	B	C	D	E	F	G
A	–	17	–	19	30	–	–
B	17	–	21	23	–	–	–
C	–	21	–	27	29	31	22
D	19	23	27	–	–	40	–
E	30	–	29	–	–	33	25
F	–	–	31	40	33	–	39
G	–	–	22	–	25	39	–

(c) Complete the drawing of the network on Diagram 1 in the answer book by adding the necessary arcs from vertex C together with their weights.

(2)

(d) Use Kruskal's algorithm to find a minimum spanning tree for the network. You should list the arcs in the order that you consider them. In each case, state whether you are adding the arc to your minimum spanning tree.

(3)

(e) State the weight of the minimum spanning tree.

(1)

(Total 10 marks)

Leave blank

2.

a i) A path can be found between any 2 vertices

ii) A connected graph with no cycles

iii) A subgraph which includes all vertices and is also a tree

b) n-1 = number of arcs in a minimum spanning tree.

(c)

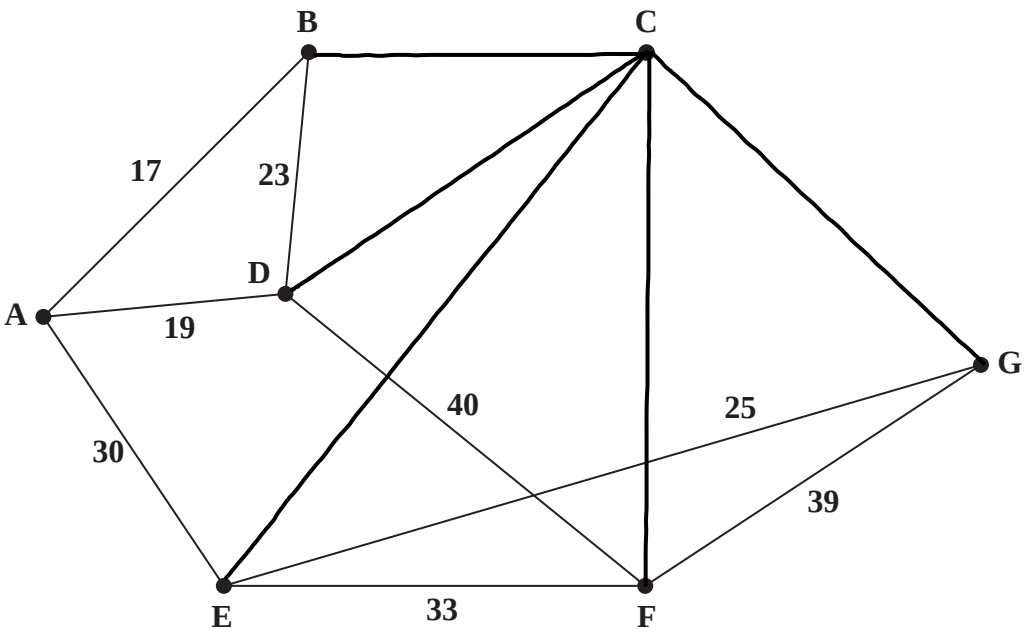

Diagram 1

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Question 2 continued

d) Order of arcs

AB 17 ✓
AD 19 ✓
BC 21 ✓
CG 22 ✓
BD 23
EG 25
CD 27
CE 29
AE 30
CF 31
EF 33

* Rejecting some arcs as they form a cycle (a spanning tree cannot include cycles) *

AB, AD, BC, CG, reject BD, EG, reject CD, reject CE, reject AE, CF, reject EF

e) 17 + 19 + 21 + 22 + 25 + 31 = 135 km

Q2

(Total 10 marks)

3. 12.1 9.3 15.7 10.9 17.4 6.4 20.1 7.9 8.1 14.0

(a) Use the first-fit bin packing algorithm to determine how the numbers listed above can be packed into bins of size 33

(3)

The list is to be sorted into **descending** order.

(b) (i) Starting at the left-hand end of the list, perform two passes through the list using a bubble sort. Write down the state of the list that results at the end of each pass.

(ii) Write down the total number of comparisons and the total number of swaps performed during your two passes.

(4)

(c) Use a quick sort on the **original** list to obtain a fully sorted list in **descending** order. You must make your pivots clear.

(4)

(d) Use the first-fit decreasing bin packing algorithm to determine how the numbers listed can be packed into bins of size 33

(3)

(e) Determine whether your answer to (d) uses the minimum number of bins. You must justify your answer.

(1)

(Total 15 marks)

Leave blank

3.

12.1 9.3 15.7 10.9 17.4 6.4 20.1 7.9 8.1 14.0

a) Bin 1: 12.1 9.3 10.9

Bin 2: 15.7 6.4 7.9

Bin 3: 17.4 8.1

Bin 4: 20.1

Bin 5: 14.0

bi) 12.1 15.7 10.9 17.4 9.3 20.1 7.9 8.1 14.0 6.4

15.7 12.1 17.4 10.9 20.1 9.3 8.1 14.0 7.9 6.4

ii) Comparisons: 9 + 8 = 17

Swaps: 7 + 5 = 12

c) 12.1 9.3 15.7 10.9 17.4 (6.4) 20.1 7.9 8.1 14.0

12.1 9.3 15.7 10.9 (17.4) 20.1 7.9 8.1 14.0 [6.4]

20.1 [17.4] 12.1 9.3 15.7 (10.9) 7.9 8.1 14.0 [6.4]

20.1 [17.4] 12.1 (15.7) 14.0 [10.9] 9.3 (7.9) 8.1 [6.4]

20.1 [17.4] [15.7] 12.1 (14.0) [10.9] 9.3 (8.1) [7.9] [6.4]

20.1 [17.4] [15.7] [14.0] 12.1 [10.9] 9.3 [8.1] [7.9] [6.4]

Sort complete

d) Bin 1: 20.1 12.1

Bin 2: 17.4 14.0

Bin 3: 15.7 10.9 6.4

Bin 4: 9.3 8.1 7.9

e) (20.1 + 17.4 + 15.7 + 14.0 + 12.1 + 10.9 + 9.3 + 8.1 + 7.9 + 6.4) ÷ 33

= 121.9 ÷ 33

= 3.69

≈ 4

lower bound = 4

∴ 4 bins is optimal

4.

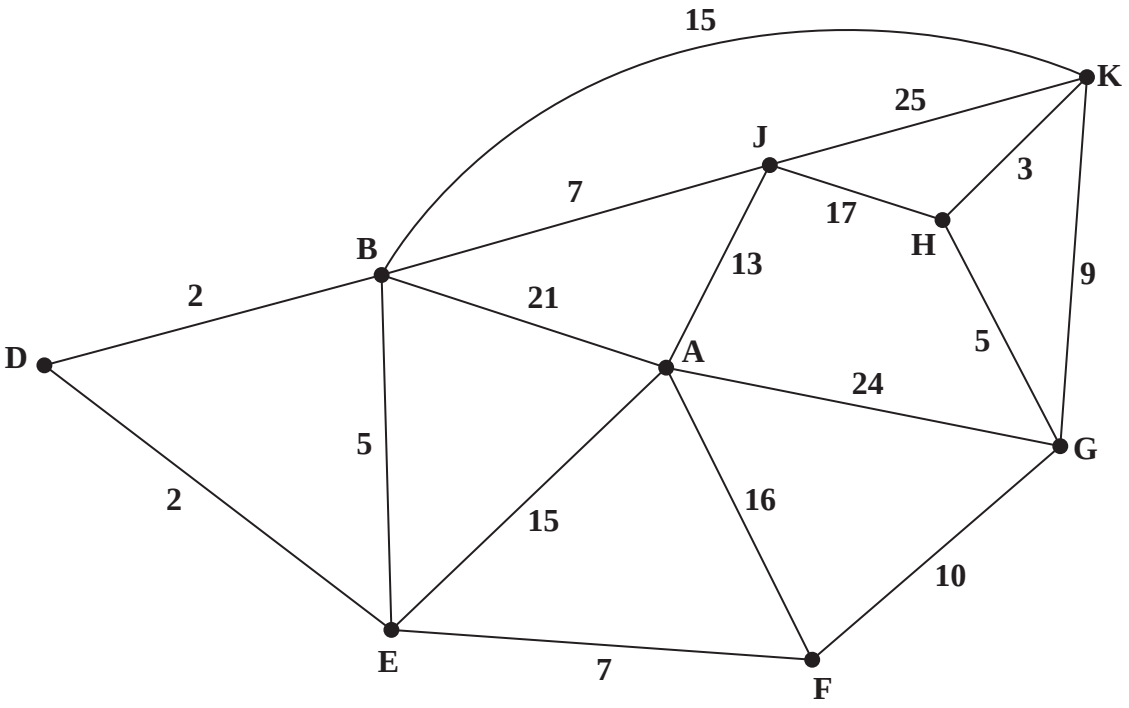


Figure 3

[*The total weight of the network is 196*]

Figure 3 models a network of roads. The number on each edge gives the time, in minutes, taken to travel along that road. Oliver wishes to travel by road from A to K as quickly as possible.

(a) Use Dijkstra's algorithm to find the shortest time needed to travel from A to K. State the quickest route.

(6)

On a particular day Oliver must travel from B to K via A.

(b) Find a route of minimal time from B to K that includes A, and state its length.

(2)

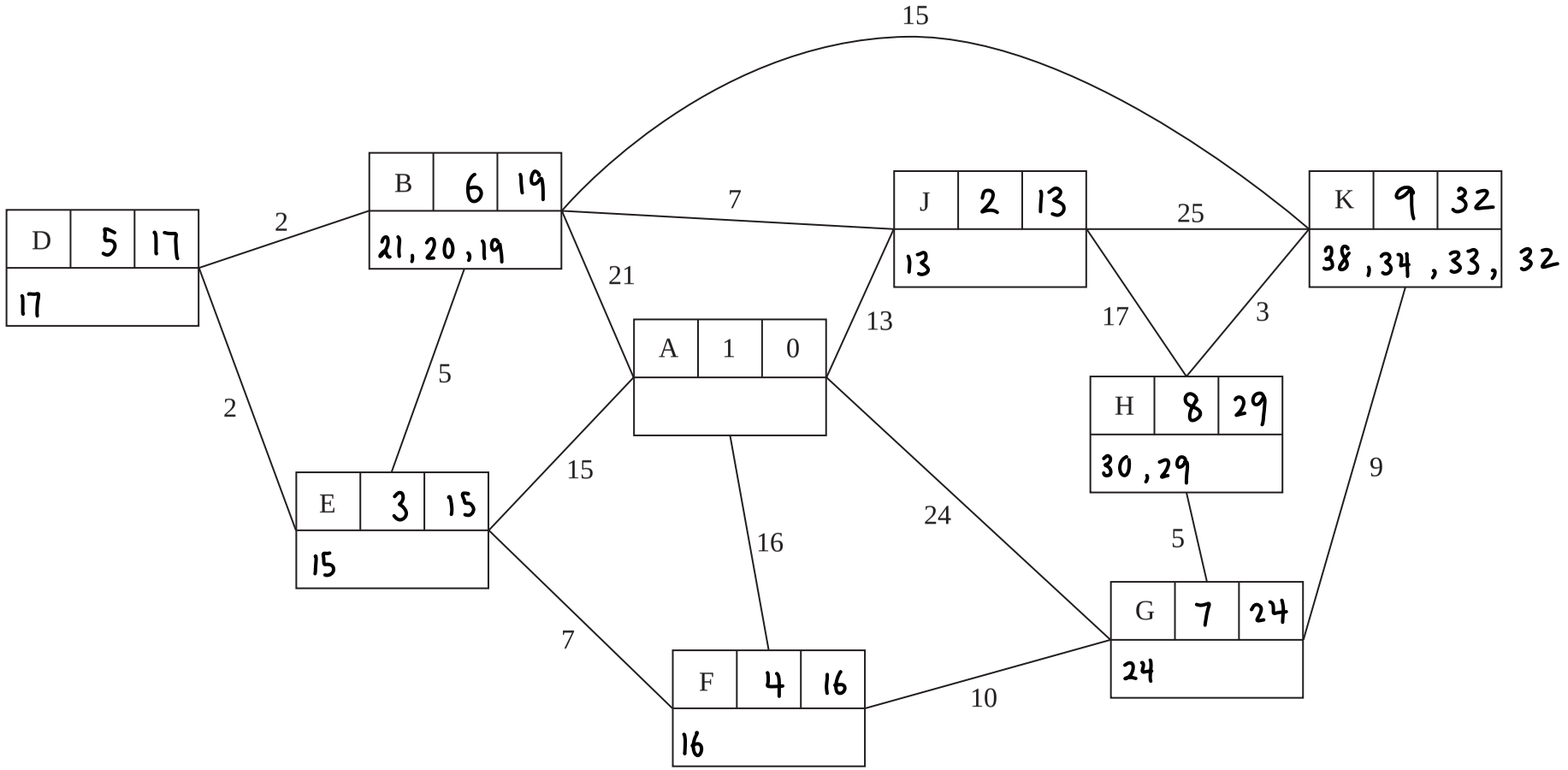
Oliver needs to travel along each road to check that it is in good repair. He wishes to minimise the total time required to traverse the network.

(c) Use the route inspection algorithm to find the shortest time needed. You must state all combinations of edges that Oliver could repeat, making your method and working clear.

(7)

(Total 15 marks)

4.



Key:

Vertex	Order of labelling	Final value
Working values		

Quickest route: A-G-H-K

Shortest time: 32 mins

Leave blank

Question 4 continued

b) BDEAGHK

19 + 32 = 51 mins

c) Odd nodes: A, B, F, H

AB + FH : 19 + (10+5) = 34

AF + BH : 16 + (15+3) = 34

AH + BF : 29 + (2+2+7) = 40

Oliver can repeat AE, ED, BD, FG & GH or AF, BK and KH

time needed : 34 + 196 = 230 mins

(Total 15 marks)

Q4

Leave blank

5. A linear programming problem in x and y is described as follows.

Maximise $P = 5x + 3y$

subject to: $x \geqslant 3$

$x + y \leqslant 9$

$15x + 22y \leqslant 165$

$26x - 50y \leqslant 325$

(a) Add lines and shading to Diagram 1 in the answer book to represent these constraints. Hence determine the feasible region and label it R.

(4)

(b) Use the objective line method to find the optimal vertex, V, of the feasible region. You must draw and label your objective line and label vertex V clearly.

(2)

(c) Calculate the exact coordinates of vertex V and hence calculate the corresponding value of P at V.

(3)

The objective is now to **minimise** $5x + 3y$, where x and y are **integers**.

(d) Write down the minimum value of $5x + 3y$ and the corresponding value of x and corresponding value of y.

(2)

(Total 11 marks)

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5.

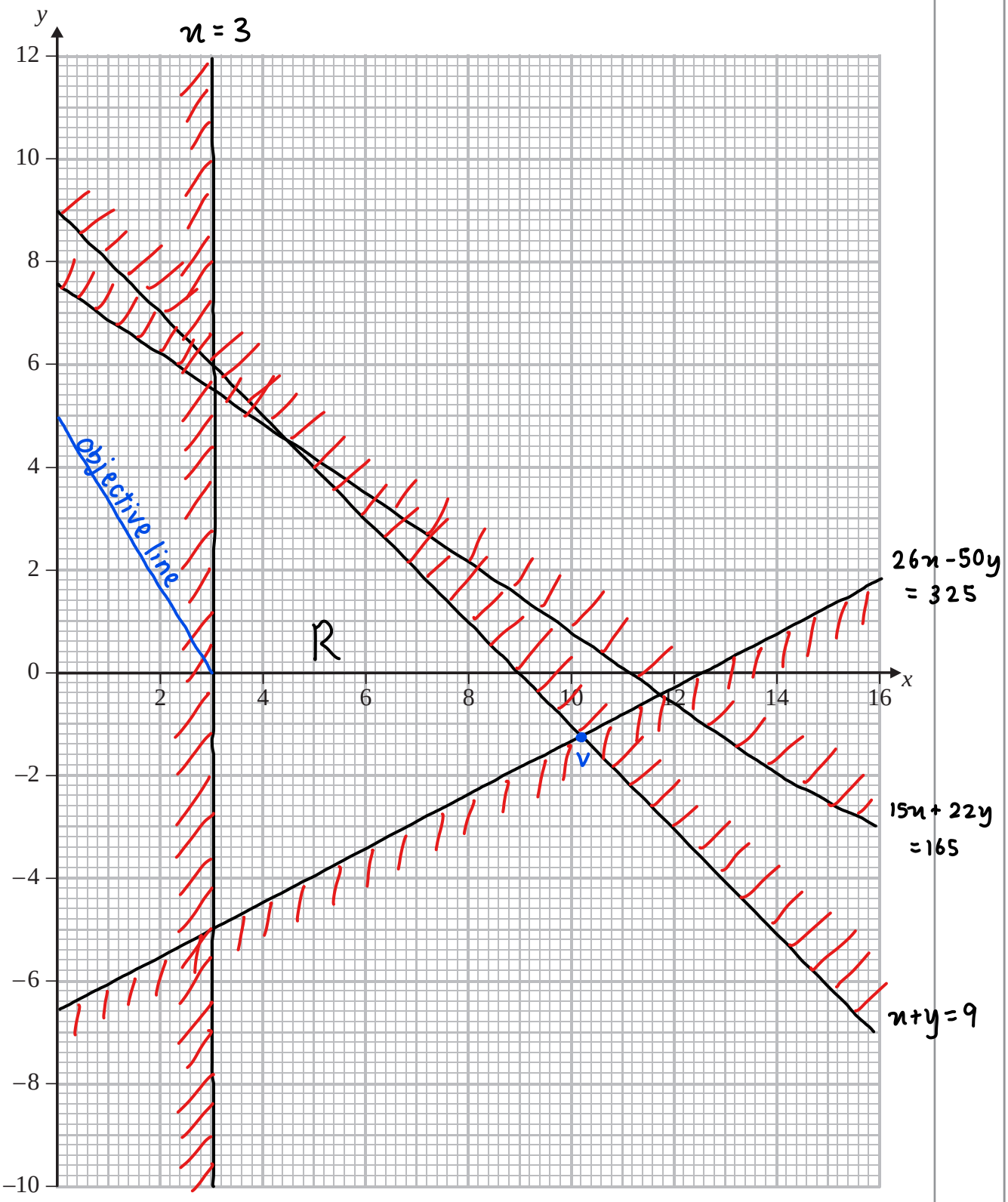


Diagram 1

c) $x + y = 9 \Rightarrow y = 9 - x$

$26x - 50y = 325$

$26x - 50(9 - x) = 325$

$76x = 775$

$x = \frac{775}{76}$

$y = 9 - \left(\frac{775}{76}\right)$

$= \frac{-91}{76}$

$\therefore V\left(\frac{775}{76}, \frac{-91}{76}\right)$

Leave blank

Question 5 continued

$P = 5\left(\frac{775}{76}\right) + 3\left(-\frac{91}{76}\right)$

$= \frac{1801}{38}$

d) $n = 3$

$26(3) - 50y = 325$

$y = -4.94$

n	y	$26n - 50y$
3	-4	$278 < 325$
3	-5	$328 > 325$ n/a

$\therefore n = 3, y = -4$

$C = 5(3) + 3(-4)$

$= 3$

Q5

(Total 11 marks)

6.

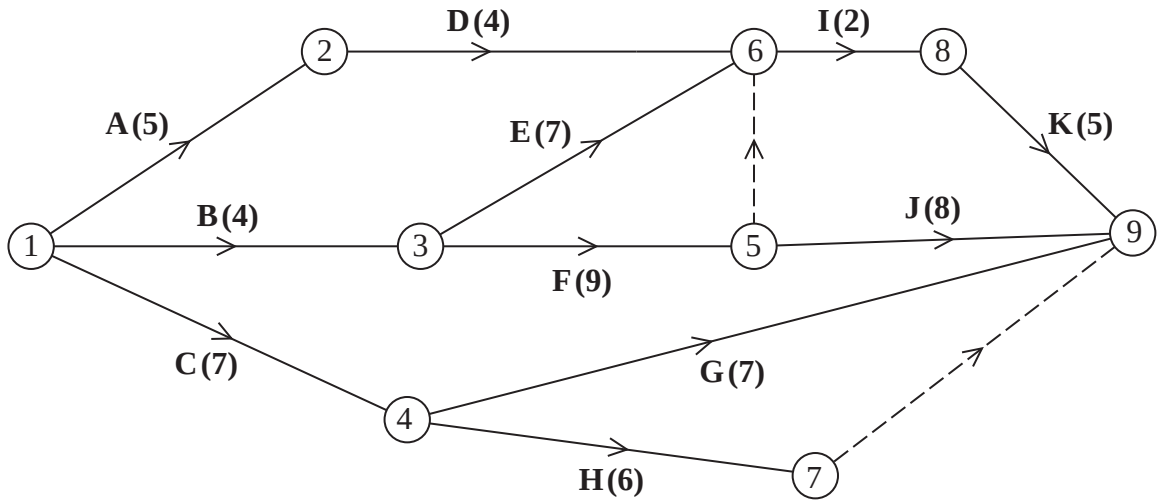


Figure 4

A project is modelled by the activity network shown in Figure 4. The activities are represented by the arcs. The number in brackets on each arc gives the time required, in hours, to complete the activity. The numbers in circles are the event numbers. Each activity requires one worker.

(a) Explain the significance of the dummy activity

(i) from event 5 to event 6

(ii) from event 7 to event 9

(2)

(b) Complete Diagram 1 in the answer book to show the early event times and the late event times.

(4)

(c) State the minimum project completion time.

(1)

(d) Calculate a lower bound for the minimum number of workers required to complete the project in the minimum time. You must show your working.

(2)

(e) On Grid 1 in your answer book, draw a cascade (Gantt) chart for this project.

(4)

(f) On Grid 2 in your answer book, construct a scheduling diagram to show that this project can be completed with three workers in just one more hour than the minimum project completion time.

(3)

(Total 16 marks)

TOTAL FOR PAPER: 75 MARKS

END

Leave blank

6. (a)

i) The dummy shows that activity I depends on activities D, E and F but activity J depends on activity F only.

ii) The dummy shows activities G and H uniquely in terms of events at each end.

(b)

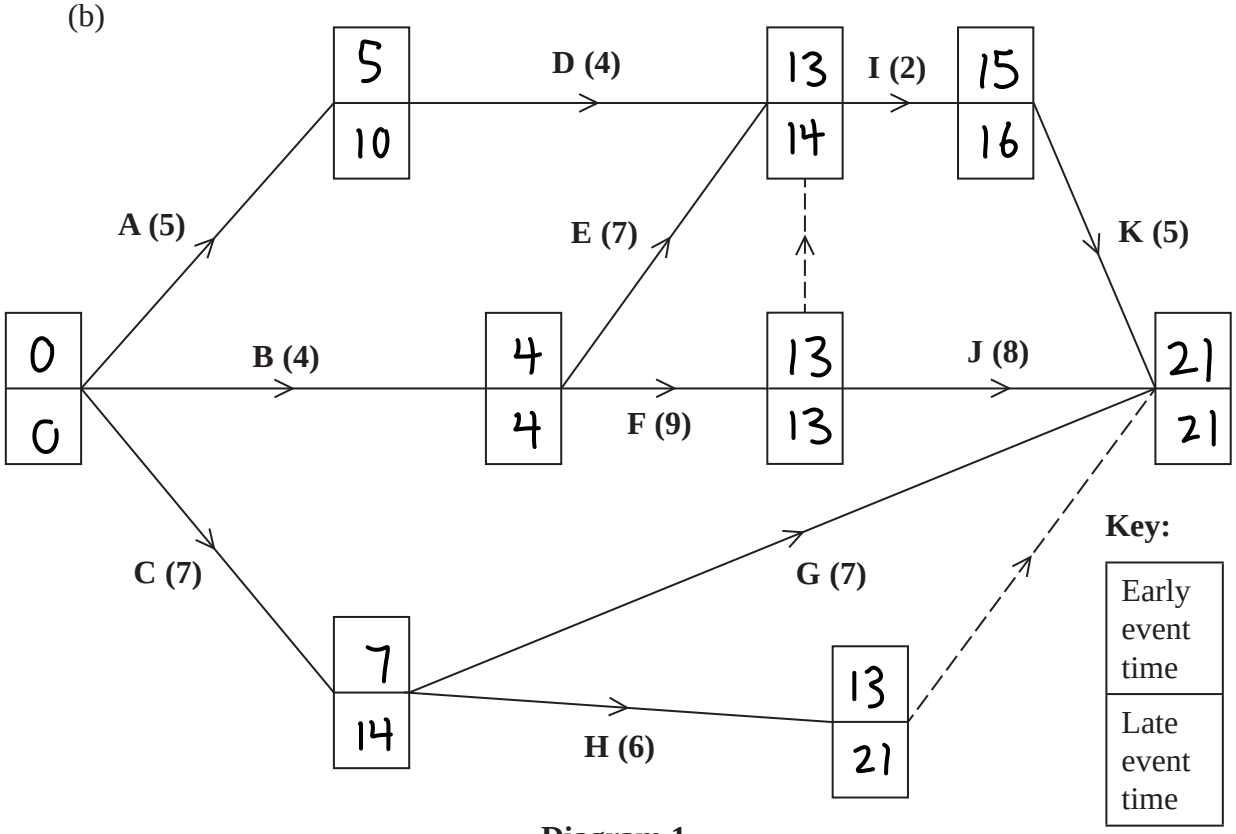


Diagram 1

c) 21 hours

d) $(5+4+7+4+7+9+7+6+2+8+5) \div 21 = 64 \div 21$

$= 3.048$

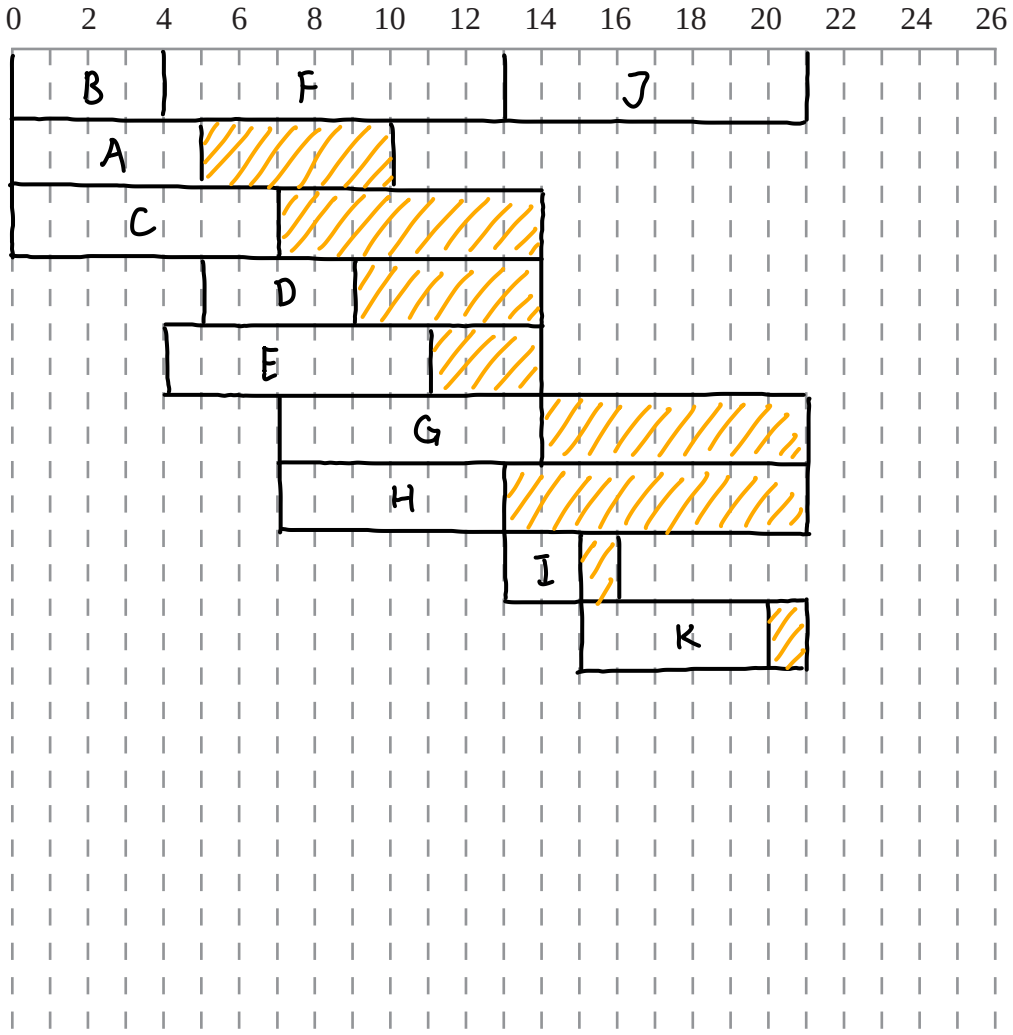
$\therefore$ lower bound:

4 workers

Leave blank

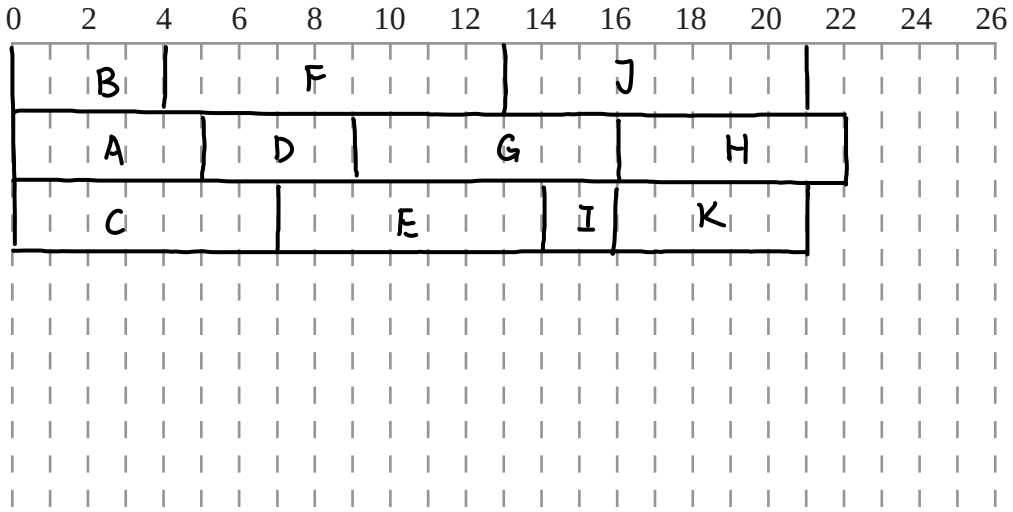
Question 6 continued

(e)



Grid 1

(f)



Grid 2

(Total 16 marks)

Q6

TOTAL FOR PAPER: 75 MARKS

END