Write your name here			
Surname		Other name	5
Pearson Edexcel International Advanced Level	Centre Number		Candidate Number
Further Pu Mathema Advanced/Advance	tics F		
Wednesday 3 June 2015 – I Time: 1 hour 30 minutes	Morning		Paper Reference WFM02/01

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer all questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
 there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for each question are shown in brackets
 use this as a guide as to how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶





x = 2	
$\frac{x}{x+2} < \frac{2}{x+5}$	
	(7)

2. (a) Express $\frac{1}{(r+6)(r+8)}$ in partial fractions.

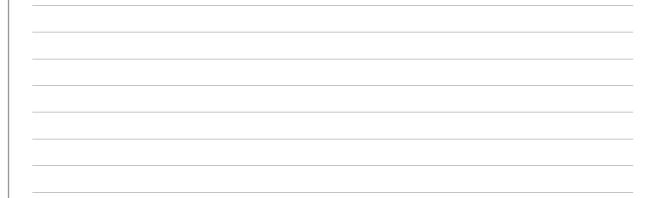
(1)

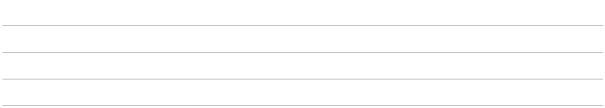
(b) Hence show that

$$\sum_{r=1}^{n} \frac{2}{(r+6)(r+8)} = \frac{n(an+b)}{56(n+7)(n+8)}$$

where a and b are integers to be found.

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uestion 2 continued	



(a) Show that the substitution $z = y^{-2}$ transforms the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} + 2xy = x\mathrm{e}^{-x^2}y^3 \quad (\mathrm{I})$$

into the differential equation

$$\frac{\mathrm{d}z}{\mathrm{d}x} - 4xz = -2x\mathrm{e}^{-x^2} \quad \text{(II)}$$

(b) Solve differential equation (II) to find z as a function of x.

(5)

(c) Hence find the general solution of differential equation (I), giving your answer in the form $y^2 = f(x)$.

(1)

uestion 3 continued		



A transformation T from the z-plane to the w-plane is given by

$$w = \frac{z-1}{z+1}, \quad z \neq -1$$

The line in the z-plane with equation y = 2x is mapped by T onto the curve C in the w-plane.

(a) Show that C is a circle and find its centre and radius.

(7)

The region y < 2x in the z-plane is mapped by T onto the region R in the w-plane.

(b) Sketch circle C on an Argand diagram and shade and label region R.

(2)



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uestion 4 continued	



- 5. Given that $y = \cot x$,
 - (a) show that

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = 2\cot x + 2\cot^3 x \tag{3}$$

(b) Hence show that

$$\frac{\mathrm{d}^3 y}{\mathrm{d}x^3} = p \cot^4 x + q \cot^2 x + r$$

where p, q and r are integers to be found.

(3)

(c) Find the Taylor series expansion of cot x in ascending powers of $\left(x - \frac{\pi}{3}\right)$ up to and including the term in $\left(x - \frac{\pi}{3}\right)^3$.

(3)

Question 5 continued	Leave blank



6. (a) Find the general solution of the differential equation

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} - 2\frac{\mathrm{d}y}{\mathrm{d}x} - 3y = 2\sin x \quad (I)$$

(8)

Given that y = 0 and $\frac{dy}{dx} = 1$ when x = 0

(b) find the particular solution of differential equation (I).

(5)

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Question 6 continued	



7.

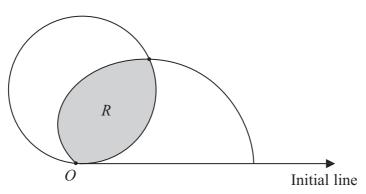


Figure 1

Figure 1 shows the two curves given by the polar equations

$$r = \sqrt{3}\sin\theta, \quad 0 \leqslant \theta \leqslant \pi$$

$$r = 1 + \cos \theta, \quad 0 \leqslant \theta \leqslant \pi$$

(a) Verify that the curves intersect at the point P with polar coordinates $\left(\frac{3}{2}, \frac{\pi}{3}\right)$. (2)

The region R, bounded by the two curves, is shown shaded in Figure 1.

(b) Use calculus to find the exact area of R, giving your answer in the form $a(\pi - \sqrt{3})$, where a is a constant to be found.

(6)

uestion 7 continued		
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8. (a) Show that

$$\left(z + \frac{1}{z}\right)^3 \left(z - \frac{1}{z}\right)^3 = z^6 - \frac{1}{z^6} - k\left(z^2 - \frac{1}{z^2}\right)$$

where k is a constant to be found.

(3)

Given that $z = \cos \theta + i \sin \theta$, where θ is real,

(b) show that

(i)
$$z^n + \frac{1}{z^n} = 2\cos n\theta$$

(ii)
$$z^n - \frac{1}{z^n} = 2i\sin n\theta$$

(3)

(c) Hence show that

$$\cos^3\theta \sin^3\theta = \frac{1}{32} (3\sin 2\theta - \sin 6\theta)$$
 (4)

(d) Find the exact value of

$$\int_0^{\frac{\pi}{8}} \cos^3 \theta \sin^3 \theta \, \mathrm{d}\theta$$

(4)



estion 8 continued		
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